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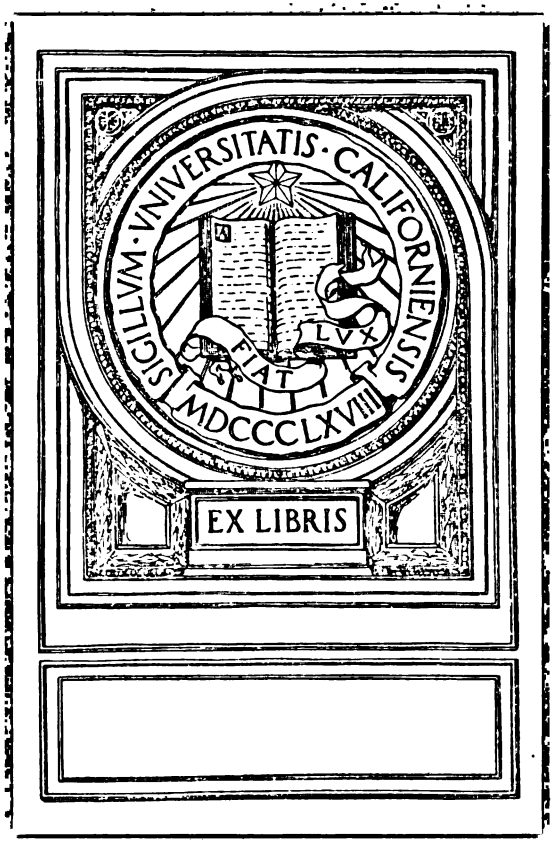
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# ALTERNATING CURRENTS

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# ALTERNATING CURRENTS

BY

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## PREFACE TO SECOND EDITION

All the material in the first edition has been retained essentially in its original form. The principal addition is an outline of laboratory experiments on alternating current machinery, correlated with the presentation of principles in the text. A number of new problems have been added, but it must not be forgotten that, in order to secure good results in the teaching of alternating currents, an abundance of problems with quantitative data from power systems and machines with which the student comes in direct contact must be provided by the instructor.

Helpful criticism and suggestions for improvement have been received from many sources. Special credit is due my colleague, Assistant Professor L. F. Curtis, for the computations for Line C, Chapter XXVII, and for much of the material in the laboratory experiments.

C. EDWARD MAGNUSSON.

UNIVERSITY OF WASHINGTON, SEATTLE, U. S. A.,
*November* 22, 1920.

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## PREFACE TO FIRST EDITION

This book is an outline of lectures and class-room discussions on alternating-current phenomena given by the author during the past ten years to students in the University of Washington. A text-book for undergraduates, in so extensive a field, must be limited to fundamental principles with a few illustrations of their application to industrial problems.

Energy transformation and energy transmission, with due emphasis on the storage features of the magnetic and dielectric fields, are the fundamental concepts in the study of all alternating-current phenomena. On the energy basis the apparently widely divergent phenomena of the magnetic, electric and dielectric circuits can be coördinated under the same general laws.

The topics discussed are arranged in the sequence that has proved advantageous to the student. The elemental principles are first applied to the transformer, as this is the pivotal apparatus of alternating-current systems and also the simplest of all alternating-current machines. The treatment of long-distance transmission lines in the last chapter is more extensive than is ordi-



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narily given in undergraduate courses. The author believes that after the student has completed a general survey of the main characteristics of alternating-current machinery, as given in Chapters XII to XXVI inclusive, it is highly desirable that at least one corner of the field be examined more thoroughly. For this purpose long-distance transmission lines offer special advantages in that the problem is complicated, the solution is particularly elegant and the theoretical equations must be used in the computations of commercial transmission lines.

The purpose of the book is to aid the student in gaining clear concepts of what actually takes place in alternating-current machinery, to explain the relations between the factors involved and to express the physical facts in mathematical forms in such manner that the student shall understand the equations and be able to use them rationally in the solution of every-day industrial problems. Graphical diagrams are used extensively to show the relations between the physical concepts and the algebraic equations. For convenience a few typical problems are included, but it is essential that the teacher supply an abundance of problems with quantitative data in accord with local conditions. The use of problems relating to nearby power systems or to machines in the laboratory is of great importance, for the center of interest must be on the real machines and on the alternating-current phenomena as they appear in industrial problems, otherwise the student merely recites from a text-book, which would be of little or no value.

In the preparation of the book material has been drawn from technical journals and other sources, while space limitations make it impossible to give the references except in a few special cases. Even a casual examination of this volume will show the author's indebtedness to the matchless works of Dr. C. P. Steinmetz and to Dr. A. S. McAllister's treatise on alternating-current motors. Credit should be given to my colleague, Assistant Professor F. K. Kirsten, for the computations on the long-distance transmission lines A and B, Chapter XXVII. The author also desires to acknowledge indebtedness to Dean A. S. Langsdorf, Dr. A. S. McAllister, Mr. L. F. Curtis, Mr. S. R. Burbank and Professor E. A. Loew for helpful criticism and for many valuable suggestions.

C. EDWARD MAGNUSSON.

UNIVERSITY OF WASHINGTON, SEATTLE,
*June* 28, 1916.

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# INTRODUCTION

A logical analysis of cause and effect in electrical phenomena shows that the *Principle of the Conservation of Energy* is the fundamental assumption, the basis upon which the argument rests. Electrical engineering deals with energy, not only in the electric, magnetic and dielectric forms and their interrelated phenomena, but also with its transformation into light, heat, mechanical work and chemical reactions. Moreover, during the many transformations which the energy can undergo while in the electric form, it is essential for a clear understanding of the processes involved that the student should take observations from the *energy point of view.* In general the problem of the electrical engineer consists of receiving energy in the mechanical form from a prime mover, transforming it into electric energy, transmitting in this form to the desired locality, reconverting the energy and delivering to the customer light, heat and power. Electric energy is the connecting link or chain by which energy can be economically transmitted and effectively controlled. To study readily the laws for the generation, transmission and control of electric energy it has been found expedient to group the phenomena into three divisions:

I. Direct Currents.
II. Alternating Currents.
III. Electric Transients.

This grouping is of special advantage to the student as it follows the natural order of first analyzing the simple problem and then proceeding to more complex phenomena.

**Direct Currents.**—In the field covered by direct currents the fundamental laws are fully established and the phenomena can be calculated with mathematical exactness. In direct currents the flow of energy is continually in one direction. The energy delivered is changed into heat, light and mechanical work, or stored chemically. The laws for direct currents state the relations of the several factors involved when the current and voltage have reached constant values and can not be correctly applied while the current or the voltage is increasing or decreasing.

**Electric Transients.**—While the current or voltage increases, as when the circuit is formed, energy is being stored in the magnetic and dielectric circuits; and part or all of this energy will be returned to the electric circuit when the current or voltage decreases. Phenomena that occur during the change of circuit conditions are of comparatively short duration and hence form part of Group III.

The chief characteristic of transients, as implied by the term, is their limited and usually short duration between two periods of stable conditions. Often the meaning given to the term *transients* is broadened so as to include other phenomena, such as unstable electric equilibrium, permanent instability and cumulative oscillations, that would not be included by the above definition.

Short circuits, lightning discharges, switching under load, opening field circuits, and similar sudden changes with their resultant effects on the system are included in this division. Any disturbance, even a small change in load or voltage, causes a readjustment of the energy content in the system and hence produces a transient condition. The interdependence of the impedance, dielectric stresses and other factors with the periodicity, voltage, current, etc., forms the vast and extremely important field of transient phenomena. This field is rapidly increasing in commercial importance and a better understanding of transients and related phenomena is now necessary for the successful operation of large power systems.

**Alternating Currents.**—In this volume are discussed the laws for energy transfer and transformation when the current and voltage periodically change directions; that is, for alternating currents. In the direct-current generator the voltage induced in the armature reverses in direction when passing from one pole to the next. In the armature the voltage, and hence the resulting current, is alternating, and it requires a continuous commutation to produce unidirectional voltage and current at the brushes. By eliminating the commutator and substituting collector rings, the alternating voltage generated in the armature causes alternating currents to flow through the circuit. With the rapidly changing magnitude of current and voltage many new factors must be taken into consideration. The laws as developed for direct currents must be modified so as to include these additional factors before application can be made to an alternating-

current system. The phenomena of magnetic and dielectric induction are of fundamental importance and become essential parts of the calculation. The flow of energy need not be continuous in one direction as for direct currents, but by virtue of the storage possibilities in the magnetic and dielectric fields it may change in direction during each cycle. The field covered by alternating currents has been thoroughly investigated and the fundamental laws are now well known and can be expressed in the form of quantitative equations. By the use of *effective values* and *complex quantities* alternating currents may be expressed like continuous phenomena and the laws stated as completely and concisely as for direct currents.

# ALTERNATING CURRENTS

## CHAPTER I

### THE ELECTRIC FIELD

In 1831 Joseph Henry in America and Michael Faraday in England showed that a conductor carrying an electric current was surrounded by magnetic and dielectric fields, and determined the fundamental relations existing between the current in the conductor and the magnetic field in the surrounding space. Any variation of the current in the conductor produces a corresponding change in the magnetic field. While the current is increasing, energy is stored in the magnetic field and while the current decreases the magnetically stored energy is returned to the electric circuit.

Similarly energy is stored dielectrically (electrostatically) in the space between conductors differing in electric potential. With increasing difference of potential, energy is stored in the dielectric, and while the potential decreases the dielectrically stored energy is returned to the electric circuit. These effects of the electric current and voltage on the surrounding space are termed magnetic induction and dielectric (electrostatic) induction; and the laws of the energy changes causing these magnetic and dielectric phenomena are of fundamental importance in the study of alternating currents.

**Lines of Force.**—In the description of his experiments Faraday developed the idea of "magnetic curves" and "lines of magnetic force." Although *lines of force* had been used in earlier publications, it was Faraday that applied the concept to magnetic and dielectric phenomena. By means of a system of lines of force he mapped the properties of the magnetic and dielectric fields. The space surrounding a magnet as well as the electric field surrounding a conductor carrying electric power was pictured as containing a definite number of lines of force possessing clearly defined properties. This method has proved extremely useful for both practical and theoretical purposes. The designer

of electrical machinery bases his quantitative calculations on the number of lines of force; theoretical investigations are based on the distribution and interaction of the lines of force; and even superficial electricians make use of the lines of force as a guide. The phenomena of magnetic and dielectric induction are closely related and both can be expressed by similar systems of lines of force having almost identical properties.

*Magnetic Lines of Force.*

(*a*) All magnetic lines of force or "lines of induction" are continuous and closed upon themselves.

(*b*) The direction of a magnetic line of force at any point follows the resultant of the magnetic forces acting on a north pole placed at the point.

(*c*) Magnetic lines of force in the same direction repel and in opposite directions attract one another.

(*d*) The strength of a magnetic field is measured by the density of the lines of force.

*Dielectric Lines of Force.*

(*a*) All dielectric lines of force are continuous and terminate at conductors.

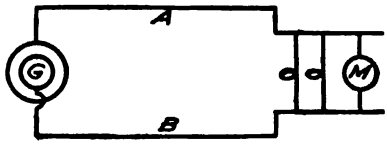


FIG. 1.—Electric circuit.

(*b*) The direction of a dielectric line of force at any point follows the resultant of the dielectric forces acting on a conductor of positive potential placed at the point.

(*c*) Dielectric lines of force in the same direction repel and in opposite directions attract one another.

(*d*) The strength of a dielectric field is measured by the density of the lines of force.

**Energy Stored in an Electric Field.**—Let Fig. 1 represent an electric circuit, in which a generator, *G*, supplies power to a load, *M*. Inside the conductors, *A* and *B*, some of the electric energy is converted into heat, as represented by the $ri^2$ losses. In the space surrounding the conductors magnetic and dielectric forces exist, and these can best be studied by means of magnetic and dielectric lines of force. In a single conductor, *A*, the magnetic properties of the electric field can be pictured by lines of force as in Fig. 2.

The conductor is surrounded by a magnetic field as shown by the continuous lines drawn concentric with the conductor. The field is strongest at the surface of the conductor and rapidly de-

creases with increasing distance from the conductor as is indicated by the spacing of the lines.

Likewise in Fig. 4, the dielectric stresses surrounding the conductor are represented by the lines drawn radially from the con-

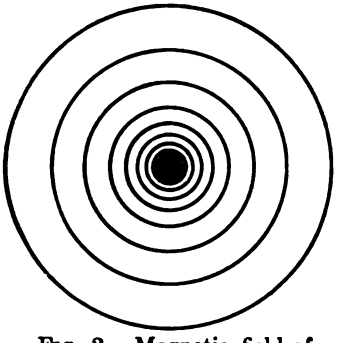


FIG. 2.—Magnetic field of single conductor.

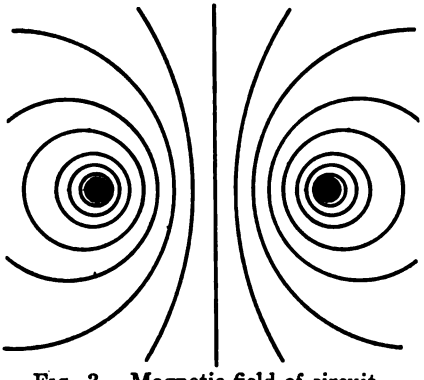


FIG. 3.—Magnetic field of circuit.

ductor. The strength of the dielectric field likewise decreases with the distance from the conductor as indicated by the widening of the space between the lines.

If both conductors are brought close together, both the magnetic and dielectric stresses in the electric field will be modified



FIG. 4.—Dielectric field of single conductor.

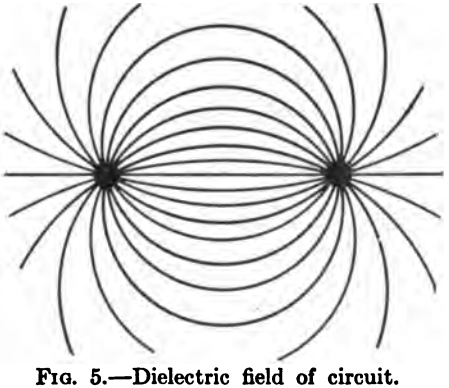


FIG. 5.—Dielectric field of circuit.

as represented by the distribution of the lines of force in Figs. 3 and 5. Superimposing the magnetic and dielectric lines a map of the *complete electric field* for the circuit is shown in Fig. 6.

**Magnetic Energy.**—The magnetic flux $\phi$ surrounding a conductor is proportional to the current as expressed by equation (1).

$$\phi = Li \tag{1}$$

The proportionality factor $L$ is called the *Inductance* of the circuit. The magnetic field represents energy, stored in the space surrounding the conductor, and hence requires energy from the electric circuit to produce it. Expressing power in terms of current and voltage we have equation (2) in which ${}_{L}e$ is called the inductance voltage.

$$p = {}_{L}ei \tag{2}$$

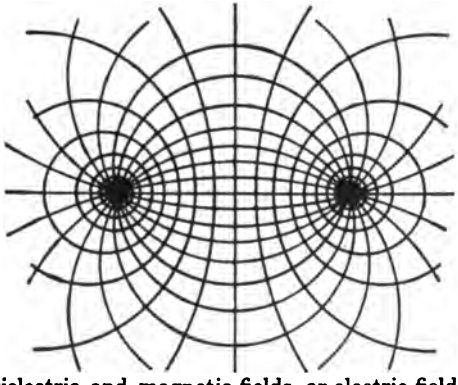


FIG. 6.—Dielectric and magnetic fields, or electric field of circuit.

The voltage ${}_{L}e$ required to form the field is directly proportional to the rate of change of the flux.

$${}_{L}e = \frac{d\Phi}{dt} \tag{3}$$

Hence by (1)

$${}_{L}e = L\frac{di}{dt} \tag{4}$$

The energy supplied by the power $p$ in forming the magnetic field is:

$$dW = pdt \tag{5}$$

By (2) and (4):

$$W = L\int_{o}^{i} idi \tag{6}$$

or

$$W = \frac{Li^2}{2} \tag{7}$$

The energy stored magnetically in the electric field surrounding a conductor is proportional to the square of the current. When the current decreases the energy is returned to the circuit, for if $i$ and therefore $\phi$ decrease, then $\frac{di}{dt}$ and hence ${}_{L}e$ are negative; which means that the energy is returned to the electric circuit.

The practical unit of inductance $L$ is the *henry*. In any consistent system of units a circuit possesses one unit of inductance if *a unit rate of change* of current in the circuit generates or consumes one unit of voltage. If the current changes *at the rate of one ampere per second* and the voltage generated or consumed is *one volt* then the inductance in the circuit is *one henry*.

**Dielectric Energy.**—For the dielectric field exactly similar relations exist. The dielectric flux $\Psi$ is proportional to the voltage.

$$\Psi = Ce \tag{8}$$

The proportionality factor $C$ is called the condensance (capacitance) of the circuit. The dielectric field represents energy stored in the dielectric. This energy must be supplied by the electric circuit when the dielectric field is established.

$$p = {}_{c}ie \tag{9}$$

In equation (9) the power required to produce the dielectric field surrounding the conductor is expressed in terms of the voltage $e$ and the condensance current ${}_{c}i$. The condensance current is directly proportional to the rate of change in the dielectric flux.

$${}_{c}i = \frac{d\Psi}{dt} \tag{10}$$

From (8) and (10):

$${}_{c}i = C\frac{de}{dt} \tag{11}$$

$$dW = pdt \tag{12}$$

$$W = C\int_{o}^{e} ede \tag{13}$$

$$W = \frac{Ce^2}{2} \tag{14}$$

Hence the energy stored dielectrically in the electric field surrounding a conductor is proportional to the square of the voltage.

When the voltage decreases the energy is returned to the electric circuit, for if $e$ and therefore $\Psi$ decrease, then $\frac{de}{dt}$ and hence $_ei$ are negative, which means that the energy is returned to the electric circuit.

The unit of condensance (capacity), $C$, is the *farad*. In any consistent system of units a circuit possesses one unit of condensance if *a unit rate of change* of voltage produces (or absorbs) one unit of current. If the voltage changes *at the rate of one volt per second* and the current produced (or absorbed) is *one ampere*, the condensance of the circuit is *one farad*. The farad is too large a unit for practical use and hence in most commercial problems the condensance is measured in microfarads.

$$1 \text{ farad} = 10^6 \text{ microfarads.}$$

**Kinetic Energy.**—In dealing with new phenomena it is well to compare the derived expressions with the laws of similar phenomena previously studied.

The storing of energy in a moving body follows relations exactly analogous to those just derived for dielectric and magnetic fields. Using the customary notation for mass ($M$), force ($F$), velocity ($v$), acceleration ($a$), power ($p$), and energy ($W$), we have the relations given in equations (15) to (21).

$$F = Ma \tag{15}$$

$$a = \frac{dv}{dt} \tag{16}$$

$$p = Fv \tag{17}$$

$$dW = pdt \tag{18}$$

$$dW = Mvdv \tag{19}$$

$$W = M\int_o^v vdv \tag{20}$$

$$W = \frac{Mv^2}{2} \tag{21}$$

The energy stored kinetically in a moving mass is proportional to the square of the velocity. If the moving body is connected to a machine, as the reciprocating parts of a steam

engine, when the velocity is reduced the energy is returned to the system. In such reciprocating parts energy is stored when the velocity of the mass increases and returned when the velocity decreases.

By comparing equations (7), (14) and (21) it is readily seen that inductance ($L$), condensance ($C$), and mass ($M$) are similar coefficients representing the amount of energy that can be stored in the system for any given current, voltage or velocity.

In order to use these equations for numerical calculations the proper units and reduction factors must be used. For commercial work the following units are in general use:

Energy in a Magnetic Field.—

$$W(\text{joules}) = \frac{L(\text{henrys})\ i^2\ (\text{amperes})}{2}.$$

Energy in a Dielectric Field.—

$$W(\text{joules}) = \frac{C(\text{microfarads})\ e^2\ (\text{volts})}{2 \times 10^6}.$$

Energy in a Moving Body.—

$$W(\text{ergs}) = \frac{M(\text{grams})\ v^2\ (\text{cm. per sec.})}{2}.$$

$$W(\text{joules}) = \frac{M(\text{kg.})\ v^2\ (\text{meters per sec.})}{2}.$$

$$W(\text{ft.-lb.}) = \frac{M(\text{lb.})\ v^2\ (\text{ft. per sec.})}{2 \times 32.2}.$$

1 joule = 1 watt-sec. = $10^7$ ergs = 0.7376 ft.-lb.
= 0.2389 g.-cal. = 0.102 kg.-m. = 0.0009480 B.t.u.
1 ft.-lb. = 1.356 joules = 0.3239 g.-cal. = 0.1383 kg.-m.
= 0.001285 B.t.u. = 0.0003766 watt-hour.
1 B.t.u. = 1,055 joules = 778.1 ft.-lb. = 252 g.-cal.
= 0.2930 watt-hour.

The close analogy existing between the magnetic and dielectric (electrostatic) fields can best be shown by arranging the corresponding quantities in tabular form as in Table I. The equations will also serve as definitions for terms used when discussing the properties of the magnetic and dielectric fields.

The magnetic and dielectric phenomena are caused by energy changes. The laws for storing and returning energy in the

TABLE I[1]

| Magnetic field | Dielectric field |
|---|---|
| Magnetic flux: $\Phi = Li10^8$ lines of magnetic force | Dielectric flux: $\Psi = Ce$ lines of dielectric force |
| Inductance voltage: $_Le = n\frac{d\Phi}{dt}10^{-8} = L\frac{di}{dt}$ volts | Condensance, permittance (capacity) current: $_Ci = \frac{d\Psi}{dt} = C\frac{de}{dt}$ amperes |
| Magnetic energy: $w = \frac{Li^2}{2}$ joules | Dielectric energy: $w = \frac{Ce^2}{2}$ joules |
| Magnetomotive force: $\mathfrak{F} = ni$ ampere-turns | Electromotive force: $e$ = volts |
| Magnetizing force: $f = \frac{F}{l}$ ampere-turns per cm. | Electrifying force or voltage gradient: $G = \frac{e}{l}$ volts per cm. |
| Magnetic-field intensity: $\mathfrak{H} = 4\pi f10^{-1}$ lines of magnetic force per $\overline{\text{cm.}}^2$ | Dielectric-field intensity: $K = \frac{G}{4\pi v^2}$ lines of dielectric force per $\overline{\text{cm.}}^2$ |
| Magnetic density: $\mathfrak{B} = \mu\mathfrak{H}$ lines of magnetic force. per $\overline{\text{cm.}}^2$ | Dielectric density: $D = \kappa K$ lines of dielectric force per $\overline{\text{cm.}}^2$ |
| Permeability: $\mu$ | Condensivity, permittivity or specific capacity: $\kappa$ |
| Magnetic flux: $\Phi = A\mathfrak{B}$ lines of magnetic force. | Dielectric flux: $\Psi = AD$ lines of dielectric force |

$v = 3 \times 10^{10}$ = velocity of light

magnetic field are exactly analogous to the laws for a like energy transformation in the dielectric field. Much confusion has been caused by erroneous metaphysical concepts of the nature of the so-called electrostatic (dielectric) phenomena. It should be kept clearly in mind that the energy or "charge" in all dielectric (electrostatic) phenomena is distributed in the dielectric outside the conductor, and does not consist of something smeared on the surface of the conductor. A serious difficulty in studying dielectric phenomena is found in the misleading terminology still in common use. Such terms as "electrostatic electricity," "capacity," "specific inductive capacity," and "charging current" lead the student astray by giving the impression that

[1] STEINMETZ: "Electric Discharges, Waves and Impulses," page 17.

dielectric (electrostatic) phenomena are radically different from those in the magnetic or electric circuits. By referring to Table I it is evident that by using the terms dielectric, condensance, condensive reactance, condensance current, and permittivity or condensivity, the close analogy between magnetic and dielectric phenomena may be more easily kept in mind, and the study of the complete electric field greatly simplified.

**Magnetic, Dielectric and Electric Circuits.**—Since all magnetic lines of force are closed upon themselves, forming a complete circuit, it is customary to speak of the *magnetic circuit* in

TABLE II[1]

| Magnetic circuit | Dielectric circuit | Electric circuit |
|---|---|---|
| Magnetic flux (magnetic current):<br>$\Phi$ = lines of magnetic force<br>Magnetomotive force:<br>$\mathfrak{F} = ni$ ampere-turns<br>Permeance:<br>$\mathfrak{G} = \frac{\Phi}{4\pi\mathfrak{F}}$<br>Inductance:<br>$L = \frac{n^2\Phi}{\mathfrak{F}} 10^{-8} = \frac{n\Phi}{i} 10^{-8}$ henry<br>Reluctance:<br>$\mathfrak{R} = \frac{\mathfrak{F}}{\Phi}$<br>Magnetic energy:<br>$w = \frac{Li^2}{2} = \frac{\mathfrak{F}\Phi}{2} 10^{-8}$ joules<br>Magnetic density:<br>$\mathfrak{B} = \frac{\Phi}{A} = \mu\mathfrak{H}$ lines per $\overline{\text{cm.}}^2$<br>Magnetising force:<br>$f = \frac{\mathfrak{F}}{l}$ amp.-turns per cm.<br>Magnetic-field intensity:<br>$\mathfrak{H} = 0.4\pi f$<br>Permeability:<br>$\mu = \frac{\mathfrak{B}}{\mathfrak{H}}$<br>Reluctivity:<br>$\rho = \frac{f}{\mathfrak{B}}$<br>Specific magnetic energy:<br>$w_0 = \frac{0.4\pi\mu f^2}{2} = \frac{f\mathfrak{B}}{2} 10^{-8}$ joules per $\overline{\text{cm.}}^3$ | Dielectric flux (dielectric current):<br>$\Psi$ = lines of dielectric force<br>Electromotive force:<br>$e$ = volts<br>Condensance, capacitance, permittance or capacity:<br>$C = \frac{\Psi}{e}$ farads<br>Elastance:<br>$S = \frac{1}{C} = \frac{e}{\Psi}$<br>Dielectric energy:<br>$w = \frac{Ce^2}{2} = \frac{e\Psi}{2}$ joules<br>Dielectric density:<br>$D = \frac{\Psi}{A} = \kappa K$ lines per $\overline{\text{cm.}}^2$<br>Dielectric gradient:<br>$G = \frac{e}{l}$ volts per cm.<br>Dielectric-field intensity:<br>$K = \frac{G}{4\pi v^2}$<br>Condensivity, permittivity or specific capacity:<br>$k = \frac{D}{K}$<br>Elastivity:<br>$\frac{1}{k} = \frac{K}{D}$<br>Specific dielectric energy:<br>$w_0 = \frac{kG^2}{4\pi v^2} = \frac{GD}{2}$ joules per $\overline{\text{cm.}}^3$ | Electric current:<br>$i$ = electric current<br>Voltage:<br>$e$ = volts<br>Conductance:<br>$g = \frac{i}{e}$ mhos<br>Resistance:<br>$r = \frac{e}{i}$ ohms<br>Electric power:<br>$p = ri^2 = ge^2 = ei$ watts<br>Electric-current density:<br>$I = \frac{i}{A} = vG$ amp. per $\overline{\text{cm.}}^2$<br>Electric gradient:<br>$G = \frac{e}{l}$ volts per cm.<br>Conductivity:<br>$\gamma = \frac{I}{G}$ mho-cm.<br>Resistivity:<br>$\rho = \frac{1}{\gamma} = \frac{G}{I}$ ohm-cm<br>Specific power:<br>$p_0 = \rho I^2 = G^2 = GI$ watts per $\overline{\text{cm.}}^3$ |

[1] STEINMETZ: "Electric Discharges, Waves and Impulses" (slightly modified) page 18.

a manner analogous to the *electric circuit.* The magnetic field in a dynamo or motor completes a circuit through the field poles, the yoke, air gap and armature iron, thus forming a magnetic circuit. Likewise all dielectric lines of force are continuous and terminate in conductors. The path taken by the dielectric line of force is called the *dielectric circuit.* The close analogy existing between magnetic, dielectric and electric circuits can best be shown by arranging the corresponding quantities in tabular form as in Table II. The three circuits are interdependent and in general exist simultaneously in all electromagnetic-dielectric phenomena. The relative magnitude may vary greatly and in many cases one or even two of the circuits may be negligibly small and may be omitted from the calculations, but intrinsically all three are present. For a unidirectional constant current the magnetic field remains constant; and similarly for a unidirectional constant voltage the dielectric field is constant. With both the current and the voltage unidirectional and constant, the electric circuit alone transfers energy and hence is the only circuit that enters in the problem. But in alternating currents both the voltage and the current are continually varying in magnitude and, moreover, for each successive half cycle reversing in direction. Therefore, in the study of alternating currents energy changes occur continuously and simultaneously in the interlinked magnetic, dielectric and electric circuits.

## PROBLEMS

**1.** A bullet weighing 8 oz. is projected from a gun with a velocity of 2,250 ft. per sec. An incandescent lamp takes 25 watts. How long must the lamp burn to consume an amount of energy equal to the kinetic energy of the bullet?

**2.** Find the height to which 1 ton must be raised in order to have the same amount of potential energy as is consumed by a 40-watt Mazda lamp in 1 hr.

**3.** An air-cored solenoid has an inductance of 0.052 henry. How much energy is stored in the magnetic field when 30 amperes flow through the coil? Compare the amount of magnetic energy in the coil to the amount consumed by a 25-watt incandescent lamp in 3 sec. In $\frac{1}{120}$ sec.

**4.** Let the current flowing through the solenoid in problem 3 vary from 0 to 100 amperes. Plot a curve with amperes as abscissæ and joules as ordinates.

**5.** A condenser has 280 microfarads condensance. How much energy is stored in the condenser when 625 volts are impressed across its terminals?

100 volts? 2,250 volts? Express the answers in joules, gram-calories and foot-pounds.

**6.** Let the voltage impressed on the condenser in problem 5 vary from 0 to 3,000 volts. Plot a curve with volts as abscissæ and joules as ordinates.

**7.** In a water-cooled transformer a system of pipes is placed in the oil and water passed through to remove the heat produced by the copper and iron losses. Given a 300-k.v.a. transformer operating at full load with an efficiency of 98.2 per cent. The temperature of the water on entering the transformer is 16°C. and on leaving 54°C. How much water is flowing through the transformer pipes? Give answer in pounds per hour.

**8.** What must be the velocity of a body weighing 6 oz. if its kinetic energy is equal to the energy stored in a condenser of 750 microfarads when 850 volts are impressed on its terminals? For 2,500 volts?

**9.** Find the condensance of a condenser that would at 1,250 volts store the same amount of energy as is consumed by a 40-watt incandescent lamp in 7 sec. In $\frac{1}{50}$ sec.

**10.** The field of a certain dynamo has 50 henrys inductance. Find the energy stored magnetically when a current of 3.2 amperes is flowing in the field winding.

**11.** An electromagnet has a self-inductance of 180 henrys and takes 53.2 amperes. What energy is stored in the field? Find the condensance of a condenser that would store the same amount of energy at 1,200 volts.

## CHAPTER II

## GENERATION OF ELECTROMOTIVE FORCE

### WAVE FORMS

When the magnetic lines of force are cut by an electrical conductor an electromotive force is generated in the conductor. The voltage produced is proportional to the length of the conductor, the intensity of the magnetic field and to the speed of the motion perpendicular to the magnetic field and the direction of the conductor, or, to the rate of cutting lines of force.

$$e \propto \frac{d\phi}{dt} \tag{25}$$

$$1 \text{ volt} = 10^8 \text{ lines of force per sec.} \tag{26}$$

If the conductor forms a closed circuit the voltage produces a current. Faraday showed that the *relative* motion of the field and the conductor is the determining factor. In small alternators the field is usually stationary as in direct-current generators while the conductor on the rotating armature cuts the lines of force, thus generating voltage. In larger generators more economical designs are obtained by placing the fields on the rotating spider, while the armature conductors are stationary, surrounding the moving part. In the transformer the field moves, due to the periodic increase and decrease of the current, while both the primary and secondary conductors are stationary.

**Direction of Generated Voltage.**—In determining the direction of the generated voltage several rules or devices have been suggested.

*Fleming's Rule.*—"If the thumb, forefinger and middle finger of the right hand are all set perpendicularly to each other so as to represent the three coördinates in space, the thumb pointed in the direction of the motion of the conductor relative to the magnetic field, and the forefinger in the direction of the lines of force, then the middle finger will point in the direction

in which the generated e.m.f. tends to send the current of electricity" (Fig. 7).




FIG. 7.

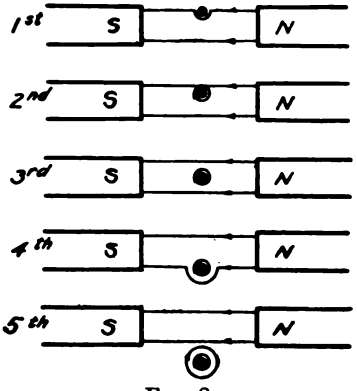



FIG. 8.

*Lenz's Law.*—"The generated e.m.f. always tends to send a current in such a direction as to oppose the change in flux which produces it."

An easily remembered method is found in visualizing the relative motion of the conductor and the field and referring to the properties already ascribed to the lines of force. In Fig. 8 let the conductor relative to the field assume successively the first to the fifth positions. When the conductor moves across the lines of force these bend and form small circles across the conductor indicated in the figure. Given the direction of the field it is evident that the conductor reaches the second and following positions encircled by lines of force whose direction indicates that the voltage generated tends to cause a current to flow downward in the conductor.

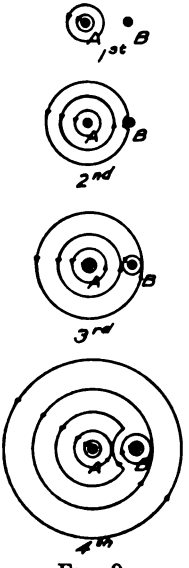



FIG. 9.

It is apparent that if either the direction of the field or the relative direction of the motion of the conductors and the flux were reversed, the circular lines around the conductor would also be in the opposite direction, and hence indicate a voltage tending to send a current upward in the conductor. By increasing or decreasing the current in a conductor the field ex-

pands and contracts. In Fig. 9 let $A$ and $B$ represent the parallel conductors, and let a current of increasing strength be sent through $A$, in the upward direction. Then the direction of the field around $A$ is indicated by the arrows, and the intensity increases with the strength of the current by the number of lines in the four successive time intervals.

With increasing current the field around $A$ expands and the lines of force cut conductor $B$. The direction of the arrows in the lines around $B$ shows that the induced voltage is in the opposite direction to the voltage in $A$. With decreasing current in $A$ the lines of force contract and move toward the conductor. Hence lines will encircle $B$ in the opposite direction, indicating an induced voltage in $B$ in the same direction as the voltage in $A$. By visualizing the direction and relative motion of the lines of force and the conductors, the direction of the induced voltage can readily be determined.

**Wave Forms.**—Since the electromotive force at any instant depends simply on the rate of cutting lines of force, this deter-

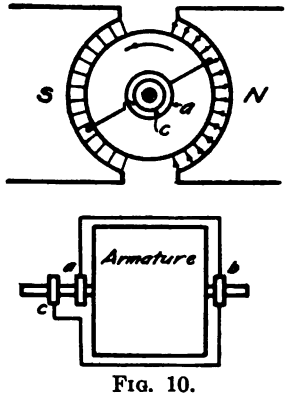



FIG. 10.



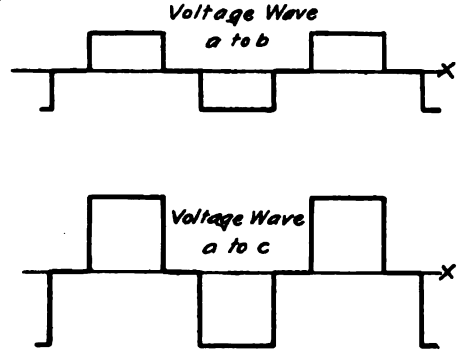







FIG. 11.

mines the shape of the wave from an alternator. The rate of cutting will depend upon the distribution and direction of the magnetic flux, the arrangement and the number of conductors in series, and the speed. From equation (25) it is evident that while a conductor moves at a uniform speed across a uniform field the voltage will be constant. If the field is reversed for half the revolution, as in Fig. 10, the voltage also reverses. In Fig. 11 is shown the form of the wave produced by a single conductor moving at right angles in a uniform field as illustrated

in Fig. 10 between rings *a* and *b*. Evidently the voltage wave between *a* and *c* will have the same rectangular shape but of double amplitude as shown in Fig. 11 since the two conductors are in symmetrical positions and connected in series. In those

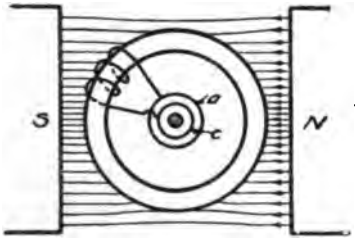



Fig. 12.

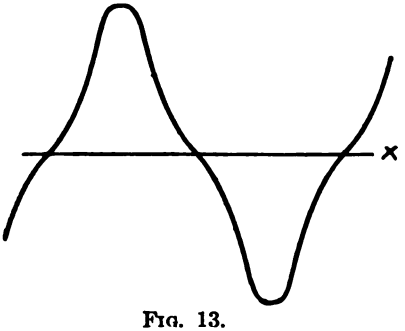



Fig. 13.

portions not covered by the poles no lines appear and hence no voltage is produced.

If the distribution of the flux be not uniform but concentrated at the center of the poles as in Fig. 12, the resulting wave for a concentrated winding becomes peaked as shown in Fig. 13. The

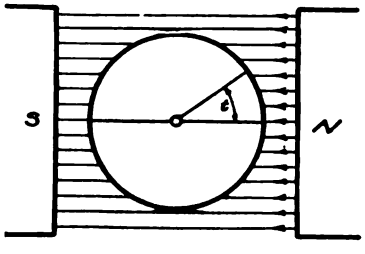



Fig. 14.

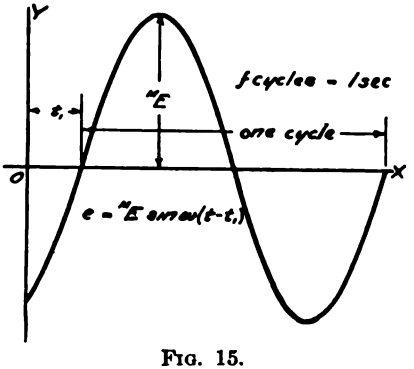



Fig. 15.

shape of the wave is affected by the number and spacing of the conductors between the rings as well as by the density and direction of the lines of force. In a uniform field, as in Fig. 14, the rate of cutting magnetic lines by a single conductor will depend upon the angular position of the moving conductor. The voltage wave will under these conditions be a simple sine wave,

Fig. 15, and the relation between the instantaneous and maximum values is given by equation (27).

$$e = {}^{M}E \sin 2\pi f (t - t_1) \tag{27}$$

$e$ = instantaneous voltage.
${}^{M}E$ = maximum voltage.
$f$ = number of complete cycles in 1 sec., or frequency.
$t_1$ = epoch or phase of wave.
$t$ = time in seconds.

Similarly, $i = {}^{M}I \sin 2\pi f(t - t_1)$ (28)

gives the instantaneous values of a sine current wave.

On small systems of low voltage and few machines, the shape of the generated voltage wave may be of little importance.

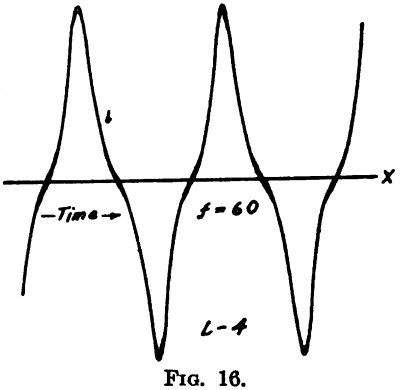



FIG. 16.

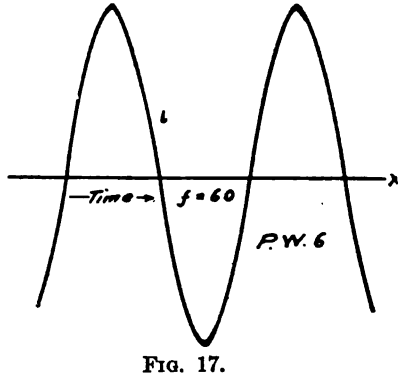



FIG. 17.

Thus in Fig. 16 is shown an oscillogram of a voltage wave from a 60-kw., 1,100-volt, 1-$\phi$, 60-cycle alternator supplying an independent lighting load in a satisfactory manner. In larger systems with several generators operating in parallel and with higher potential on the system a better wave is necessary. With the increase in the size of the system the shape of the generated voltage wave becomes of increasing importance. For several reasons, as will be explained in detail[1] later, the ideal wave form is simple harmonic or sinusoidal as expressed by equations (27) and (28). Modern alternators, especially for large systems, closely approach the ideal form. In Fig. 17 is shown an oscillogram of a 10,000-kw., 6,600-volt, 3-$\phi$, 60-cycle alternator. The shape of the wave is very nearly simple harmonic.

[1] See Chap. XXIII.

In commercial systems the voltage and current waves are modified by many factors in the system outside of the machine itself. The relative amounts, and their location in the system, of inductance, condensance and resistance, the iron in transformers and other apparatus; the location, size and design of synchronous and induction motors; reaction of leading and lagging currents; load variations and many other factors modify and distort the original wave supplied from the generator into an endless variety of forms. As an illustration of distorted waves, Fig. 18 shows an oscillogram of busbar voltage, curve *e*, and the no-load current, curve *i*, of a 500-kv.a., 2,300-volt, 60-cycle, synchronous motor floating on the line. While the

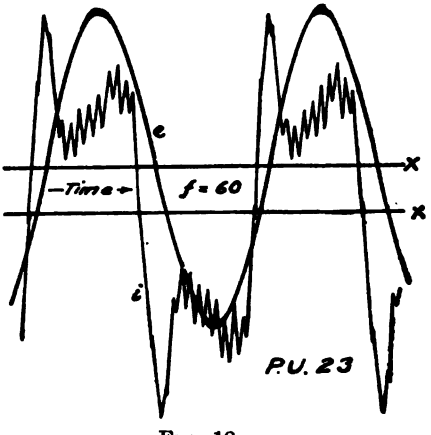



FIG. 18.

voltage is nearly of the sine form the current wave is so greatly distorted as to bear no resemblance to the simple sine wave. Under full load the current wave, curve *i*, Fig. 19, approaches the form of the voltage wave curve *e*. While the voltage or current waves may be greatly distorted in certain parts of the system, as illustrated by the no-load current, on the synchronous motor the power involved is relatively small compared to the total power of the system. The busbar voltage wave should closely approach the ideal sine form as shown by curve *e*, Fig. 18.

As a rule, operating difficulties as well as losses from cross-current are reduced to a minimum when the voltage wave in all parts of the system closely approaches the simple sine form. To

gain this much desired end a number of factors must be given careful consideration both in the design of the generators and motors as well as in every part of the transmission line and distribution network. The more important factors are discussed in later chapters.

To appreciate the need for eliminating these distortions it is first necessary to fully understand the laws that govern alternating-current phenomena, and to be thoroughly familiar with their application to simple sine waves.

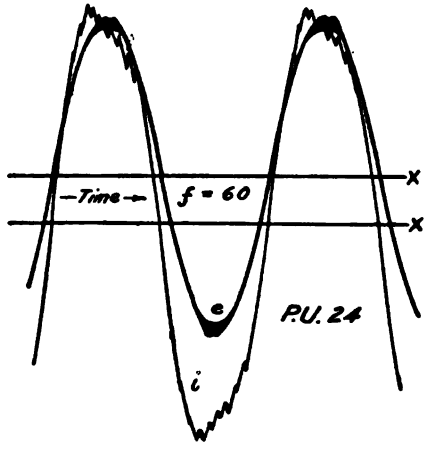



FIG. 19.

## SINE WAVES

**Instantaneous and Maximum Values.**—An alternating current or voltage wave passes through an infinite number of values in each cycle. In engineering problems the instantaneous, maximum, average and effective values are of importance.
Let:

$i, e$ = instantaneous values.
$^{m}I, ^{m}E$ = maximum values.
$^{av}I, ^{av}E$ = average values.
$I, E$ = effective values.

The relation between the instantaneous and maximum value in simple sine waves is given from the equations, as shown in (29) and (30).

$$i = {}^{m}I \sin(\omega t \pm \gamma) \tag{29}$$
$$e = {}^{m}E \sin(\omega t \pm \gamma). \tag{30}$$

**Average Value.**—The average value, Fig. 20, is found by integrating the area for one-half wave and dividing by the base. Letting $(\omega t)$ be the variable,

$$^{av}I = \frac{^{m}I}{\pi}\int_{o}^{\pi} \sin(\omega t)\, d(\omega t) = 0.636\ ^{m}I \tag{31}$$

**Effective Value.**—In 1841 Joule proved that the energy converted into heat when an electric current flows in a conductor is proportional to the square of the current. Stated in the form

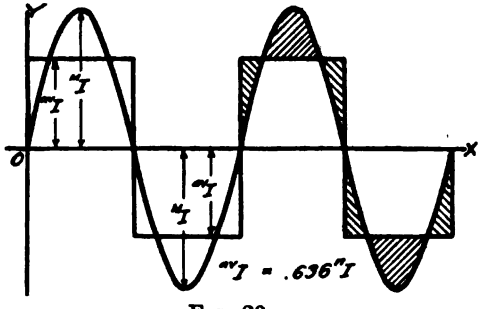



FIG. 20.

of an algebraic equation Joule's law is expressed by (32) for direct currents.

$$J = RI^2T \tag{32}$$

$J$, in joules, is the energy expended by $I$ amperes flowing through a circuit having $R$ ohms resistance, in $T$ seconds.

In watts,
$$P = RI^2 \tag{33}$$

The quantity of energy converted into heat, expressed in calories, for the time $T$ in seconds, is given by (34) for direct currents.

$$H = 0.24I^2RT \tag{34}$$

In alternating currents, the instantaneous current is continually varying throughout the cycle. Since the complete cycle repeats again and again, the average heating value may be found by taking the sum of the instantaneous heating values through one cycle and dividing by the time for one complete wave.

$$H = 0.24R\int_{o}^{T} i^2dt \tag{35}$$

$$J = R\int_{o}^{T} i^2dt \tag{36}$$

Equation (35) gives the heat in calories generated in one cycle, and equation (36) gives the heat generated, in joules, during the cycle. The power expended in heating the conductor is therefore proportional to the average of the squared instantaneous currents, as shown in equation (37).

$$P = \frac{R}{T}\int_o^T i^2 dt \tag{37}$$

The direct current that would produce the same heating effect in a circuit having the same resistance would be equivalent to the *square root of the mean square of the instantaneous currents.* This

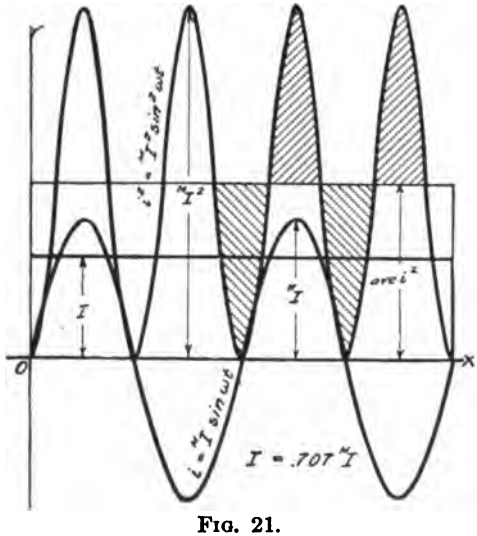



FIG. 21.

value is termed the *effective* alternating current, or the *root mean square.*

$$RI^2T = R\int_o^T i^2 dt \tag{38}$$

$$I = {}^{\prime\prime}I\sqrt{\frac{1}{2\pi}\int_o^{2\pi}\sin^2(\omega t)\, d(\omega t)} = \frac{{}^{\prime\prime}I}{\sqrt{2}} = 0.707\,{}^{\prime\prime}I \tag{39}$$

Hence by using *effective* values, Joule's law applies to alternating currents in the same form as for direct currents.

In Fig. 21 the squared and effective values are compared to the current wave. It is of importance to note that the effective

values appear as continuous positive values and not half positive and half negative as in Fig. 20 for the average values.

Alternating current indicating voltmeters and ammeters give effective values, and in commercial work, unless specifically stated otherwise, volts and amperes always refer to effective values.

**Form Factor.**—The ratio between the effective and average values is called the form factor as this gives an indication of the shape of the wave. For sine waves the form factor is 1.11.

**Fundamental Equation.**—Since voltage is generated by cutting lines of force electrical generators are arranged so that the coil in which the e.m.f. is produced alternates in position relatively to a magnetic field, or flux, $\phi$. Either the coil revolves in a field or the flux passes in and out of the turns. In a complete cycle the total flux would be cut four times by each turn of wire in the coil. Hence, if $f$ is the number of complete cycles per second, ${}^{M}\Phi$ the maximum flux and $n$ the number of turns in series, the average voltage generated would be,

$$^{av}E = 4n{}^{M}\Phi f 10^{-8} \text{ volts} \tag{40}$$

This is the fundamental equation of electrical engineering.

In the preceding paragraphs the ratios between the average, maximum and effective values have been derived.

$$^{M}E = \frac{\pi}{2}{}^{av}E = 2\pi n{}^{M}\Phi f 10^{-8} \text{ volts} \tag{41}$$

$$E = \frac{^{M}E}{\sqrt{2}} = \sqrt{2}\pi n f {}^{M}\Phi 10^{-8} = 4.44 n f {}^{M}\Phi 10^{-8} \text{ volts} \tag{42}$$

Equation (42) is the fundamental relation for alternating current induction for sine waves.

**Frequency.**—In alternators the armature conductors pass alternately under north and south poles and hence the generated voltage wave completes a cycle for each pair of poles. The number of cycles per second is termed the frequency of the alternating current. In the United States 60 and 25 cycles per second are standard frequencies for light and power. In Europe 50 and 25 cycles per second are in general use. Other frequencies are in commercial use both in America and Europe, but the advantages that come from standardization in frequency are very great so that by far the greater portion of industrial power plants operate at frequencies of 60, 50, or 25 cycles.

## PROBLEMS

**1.** An alternator has 36 poles. At what speed must it run to give a current of 25 cycles frequency? 60 cycles?

**2.** A 10-pole alternator has a normal speed of 1,650 r.p.m. What is the frequency of the generated voltage?

**3.** What is the normal speed of a 60-cycle, eight-pole alternator?

**4.** A 110-volt battery is connected to a circuit containing resistance only. The direction of the voltage is reversed 50 times per second. The resistance is 15 ohms. Plot the voltage and current waves to scale with time as abscissæ.

**5.** The maximum value of a voltage wave of sinusoidal shape is 127 volts.

(*a*) What is the effective value?

(*b*) What is the average value?

**6.** In a circuit having 15 ohms resistance 2,174 cal. of heat are generated in 5 min. by a 25-cycle sinusoidal current.

(*a*) Find the average, effective and maximum values of the current in the circuit.

(*b*) Find the average, effective and maximum values of the voltage impressed on the circuit.

**7.** A 60-cycle sine wave, whose average value is 245 volts is impressed on a circuit having 32 ohms resistance.

(*a*) Find the effective, average and maximum values of the current.

(*b*) Find the amount of heat (calories) generated in 30 minutes.

For Experiments see page 526.

## CHAPTER III

## FORMS OF REPRESENTATION

The importance of the simple sine wave in the study of alternating-current phenomena has made it desirable to use several forms of representation in order that the inter-relations between the several factors in a given problem may be more easily understood and the resultant effect readily calculated. The methods in common use may be grouped as follows:

I. Graphic
1. Rectangular coördinates.
2. Polar coördinates.
3. Vector diagrams. (*a*) Crank-phase diagrams. (*b*) Polar-phase diagrams.
4. Topographic (see Chap. XXVI).

II. Analytic
1. Trigonometric functions.
2. Expotential functions (see Chap. XXVI).
3. Symbolic or complex algebra.

**Rectangular Coördinates.**—In Chap. II several simple sine waves and their corresponding trigonometric expressions have been discussed. In elementary alternating currents equations of the simplest form as (45) and (46) and Fig. 22 give the desired relations.

$$i = {}^{\prime\prime}I \sin(\omega t) \tag{45}$$

$$e = {}^{\prime\prime}E \sin(\omega t + \theta) \tag{46}$$

The current in this case reaches a maximum, $\theta°$ later than the corresponding maximum of the voltage, or the current lags $\theta°$ behind the voltage.

**Polar Coördinates.**—In polar coördinates time is measured by the angle between the rotating radius vector and the reference axis; and the instantaneous value is given by the length of the radius vector. Rotation in the counter-clockwise direction is positive, and time is usually measured from the $X$-axis in the first quadrant. In Fig. 23 is shown the polar diagram of equations (45) and (46) with the same scale and maximum values as in Fig. 22.

The locus of a simple sine wave is a circle passing through the origin with a diameter equal to the maximum value. It is evident that the maximum value of the current will occur when $\omega t = \frac{\pi}{2}$ or 90°, and hence the diameter of the current locus lies along the $Y$-axis. The voltage reaches its maximum before the current and its locus is a circle whose diameter lies $\theta°$ from the $Y$-axis in the first quadrant.

In revolving from 0° to 180° the radius vector traces the circular locus and this is equivalent to the positive half wave in the rectangular diagram. From 180° to 360° the same series repeats with negative values, and hence is laid off in the negative

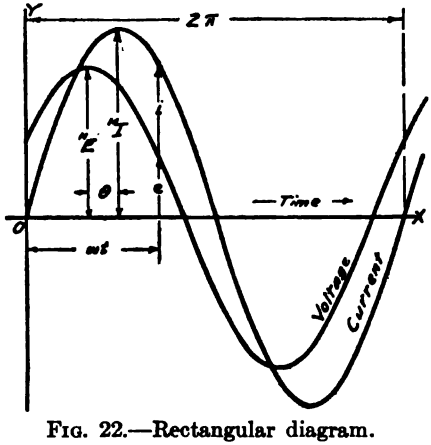



FIG. 22.—Rectangular diagram.

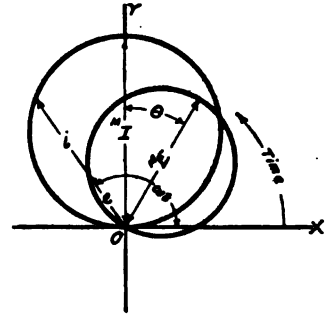



FIG. 23.—Polar diagram.

direction on the radius vector, and therefore retraces the same locus as for the first half cycle.

**Vector Diagrams.**—Quantities that can be fully determined by giving the value of one factor only are termed *Scalar*. Examples: Mass, length, time, common numbers. Such quantities have magnitude only and can be represented graphically for comparison by straight lines of proportionate lengths. Scalar quantities may be grouped or divided by the usual arithmetical operations.

The sum of certain other quantities can not be determined from the known magnitude of the components. For example: If a person walks 3 miles east and 2 miles south, his distance from the starting point is not determined by stating that he

walked 5 miles. If three forces act on a body the resultant would equal the sum of the three only under the condition that all were applied at the same point and acting in the same direction. If a force of 6 lb. acts horizontally due north, a second force of 2 lb. due west while a third of 9 lb. is applied in a vertical direction, the resultant force acting on the body is not 6 + 2 + 9, but 11 lb. If two simple harmonic waves of the same frequency are combined, the resultant maximum value will be equal to the sum of the component maxima only if the two reach their maximum values at the same time. For every other phase relation the resultant maximum will be less. A physical quantity that has both magnitude and either time-phase or space-phase position and direction is called a *Vector quantity.* A line which by its position and direction represents the time phase or space phase, and by its length the magnitude of the physical quantity is termed a *Vector.* In all vector diagrams it is customary to measure time as positive when the vector rotates in the counter-clockwise direction.

**Polar-phase Diagram.**—In polar coördinates the length of the radius vector represents the instantaneous values of the function.

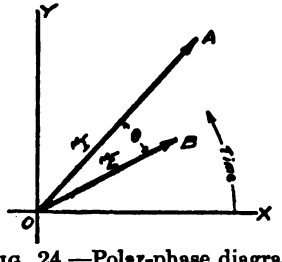



FIG. 24.—Polar-phase diagram current lagging $\theta°$.

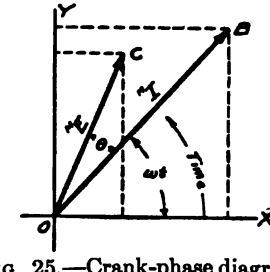



FIG. 25.—Crank-phase diagram current lagging $\theta°$.

For sine waves the loci are circles, and in order to keep the diagram as simple as possible it is customary to draw only the diameter of the locus; that is, the maximum value of the current or voltage. Since the current and voltage have the same frequency the condition of chief importance is their *relative* time position. In most problems in alternating currents the only factors under consideration are the relative time-phase positions and the maximum or effective values of the currents and voltages under discussion. The phase-vector diagram is thus independent of ($\omega t$) and can be drawn in any position. Thus in Fig. 24 the mag-

nitude and relative phase position of a current and voltage are shown in the first quadrant.

**Crank Diagrams.**—In dealing with simple harmonic functions it is often convenient to keep the rotating vector a constant in magnitude and let the instantaneous values be represented by the projections of the vector upon any fixed line through the origin selected as reference axis. This representation is called a *Crank- or Clock-phase or Vector* diagram. Thus in Fig. 25 let the line $B$, of constant length, rotate in a counter-clockwise direction. The time is measured by the angle ($\omega t$) while the instantaneous values of the sine function are represented by the projection of the line upon either the $X$- or the $Y$-axis. Com-

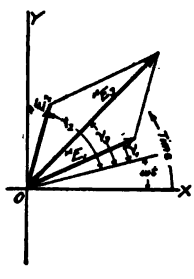


FIG. 26.—Crank-phase diagram.

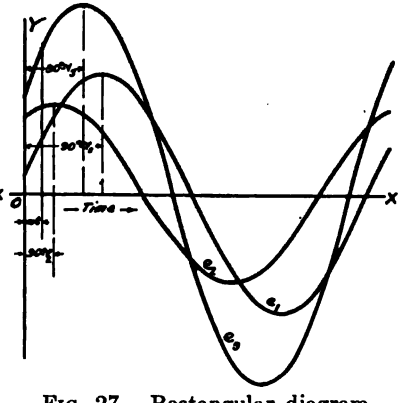


FIG. 27.—Rectangular diagram.

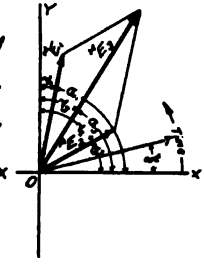


FIG. 28.—Polar-phase diagram.

paring the quantites in the crank diagram with an equation for a current wave of sine form: $i = {}^{\prime\prime}I \sin(\omega t)$, we have as follows: The vector $OB$ represents the maximum value ${}^{\prime\prime}I$; the angle $XOB$ represents the time-angle $\omega t$; and the projection upon the $Y$-axis represents $i$.

The resultant of two sine waves having the same frequency but differing in phase is also a simple harmonic function. The plotting of these waves in rectangular coördinates is tedious and the values of the resultant can be more easily found by using the crank- or polar-vector diagrams.

In Figs. 26, 27 and 28 are represented sine waves as given by equations (47) and (48); and the resultant sine wave as given by equation (49).

$$e_1 = {}^{\times}E_1 \sin(\omega t + \gamma_1) \quad (47)$$
$$e_2 = {}^{\times}E_2 \sin(\omega t + \gamma_2) \quad (48)$$
$$e_3 = {}^{\times}E_3 \sin(\omega t + \gamma_3) \quad (49)$$

The quantitative values used in Figs. 26, 27, and 28 are:

$${}^{\times}E_1 = 80 \text{ volts.}$$
$${}^{\times}E_2 = 60 \text{ volts.}$$
$${}^{\times}E_3 = 127 \text{ volts.}$$
$$\gamma_1 = 10^\circ = \frac{\pi}{18} \text{ radians.}$$
$$\gamma_2 = 60^\circ = \frac{\pi}{3} \text{ radians.}$$
$$\gamma_3 = 31^\circ = \frac{31\pi}{180} \text{ radians.}$$

For both polar and crank diagrams, time is measured by the angle from the $X$-axis and counted as positive in the counter-clockwise direction. It should be noted that vectors $E_1$ and $E_2$ reverse their position relative to $E_3$ in Figs. 26 and 28. This follows directly from what the diagrams represent. In the polar diagram, Fig. 28, the length of the radius vector represents the instantaneous value and hence the position of the vectors $OA$, $OB$, and $OC$ representing the maximum values of the three functions are determined by the values for $\alpha_1$, $\alpha_2$ and $\alpha_3$. With the line $OT$ representing time rotating in the counter-clockwise direction the function having the smallest value for $\alpha$ will be first to reach its maximum, and likewise the function having the largest value for $\alpha$ will be last in order to reach its maximum value. Hence the vectors $E_2$, $E_3$, $E_1$ lie in the order given above the $X$-axis. In the crank diagram, Fig. 26, the instantaneous values are represented by the projections of the vectors upon the $Y$-axis. Hence the maximum values rotate as vectors with the time, and therefore the vector $E_1$ with the smallest $\gamma$ leads and $E_2$ comes last in order, and the relative positions of $E_1$ and $E_2$ are reversed when compared with the polar diagram. This reversal in position of vectors in the polar and crank diagrams requires a reversal in signs when the functions are expressed by complex algebraic symbols as explained in the next paragraph. This change in sign has led to confusion. To remove this difficulty the International Electrical Congress in 1912 adopted as standard the complex notation corresponding to the crank-phase diagram.

**Symbol of Rotation.**—Mathematical symbols are useful for expressing concise statements in compact form. Such symbols as $+$, $-$, $\int$, log, $\surd$, etc., state completely more or less complicated operations. In dealing with scalar quantities the minus sign means simply subtraction; that is, part of the positive quantity has been removed. In vector quantities the minus sign indicates opposite direction from the vector with the positive sign. Since the application of the minus sign reverses the direction of the vector, it is evident that the symbol $(-)$ may be considered as an operator indicating rotation through 180°. Two applications of this operator would result in a rotation of twice 180° or 360°. In algebra $(-) \times (-) = (+)$. Successive applications make the direction of the vector alternately positive and negative. While working with harmonic functions, it is convenient to use an operator indicating a rotation of 90°. In the notation generally used when discussing alternating-current phenomena the symbol "$j$" indicates a rotation of 90° in the counter-clockwise direction. Thus, if in Fig. 29 $F$ represents a vector quantity, a force for instance, then $-F$ would represent a force along the $X$-axis but in the opposite direction from $F$. Likewise $jF$ would indicate an equal force turned 90° so as to lie in the positive direction along the $Y$-axis. It follows that $-jF$ must denote a like force in the negative direction along the $Y$-axis. If the operator "$j$" indicates a rotation of 90° then two applications must result in a rotation of 180°. Stated in the form of an equation:

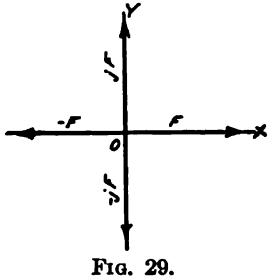


FIG. 29.

$$j \times j = j^2 = (180°) = -1 \tag{50}$$

Hence

$$j = \sqrt{-1} \tag{51}$$

$$j^4 = 1 \tag{52}$$

In ordinary algebra, quantities having the factor $\sqrt{-1}$ are often called "imaginary." The term is misleading by giving the impression of lack of reality of these quantities. The term *Quadrature component* is more appropriate as the relations between quantities having the factors $\sqrt{-1}$ and the real values

may be shown graphically by placing the two quantities in quadrature. The factor $\sqrt{-1}$ or "$j$" may therefore be considered as an operator indicating a counter-clockwise rotation of 90° and is used to indicate a quadrature component or a quantity in time or space quadrature to the reference axis.

**Complex Quantities.**—Expressions containing both the *real* and the "imaginary" or *quadrature* components are termed *Complex*, and the system of consistent operations for these quantities the *Algebra of Complex Quantities*. Only the few simple operations used in alternating currents will be discussed.

The laws of ordinary addition and subtraction apply with the one proviso that *real* and *quadrature components* must be taken separately, or graphically. The $X$ and $Y$ components taken separately may be added and subtracted in the same way

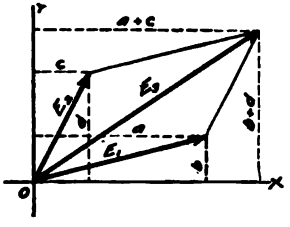



FIG. 30.

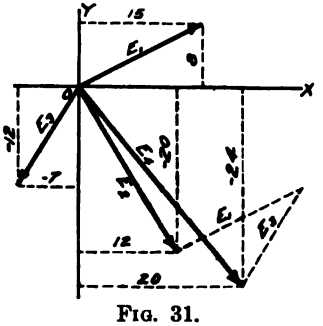



FIG. 31.

as if there were only $X$ or only $Y$ quantities. In Fig. 30 let the horizontal and vertical components of $E_1$ and $E_2$ be $a$, $c$ and $b$, $d$ respectively. The corresponding complex equations would be (53) and (54).

$$\dot{E}_1 = a + jb \tag{53}$$

$$\dot{E}_2 = c + jd \tag{54}$$

The sum of the $X$ or *real* components is $a + c$; and the sum of the $Y$ or *quadrature* components is $b + d$. Hence the resultant vector $E_3$ is a complex quantity having $a + c$ for the $X$ component and $c + d$ along the $Y$-axis.

$$\dot{E}_3 = a + c + j(b + d)$$

It is readily seen that this method may be extended for three or more vector quantities and that the resultant can be found

both graphically and algebraically. This is illustrated in Fig. 31 and equations (56), (57), (58) and (59).

$$\dot{E}_1 = 15 + j8 \tag{56}$$
$$\dot{E}_2 = 12 - j20 \tag{57}$$
$$\dot{E}_3 = -7 - j12 \tag{58}$$

The resultant:

$$\dot{E}_4 = 20 - j24 \tag{59}$$

The rules for ordinary multiplication and division can be applied to complex quantities with the additional factor $j$ or $\sqrt{-1}$ indicating a rotation of 90°.

Multiplying the complex equations (60) and (61) gives the resultant complex equation (62). The corresponding graphical representation is shown in Fig. 32.

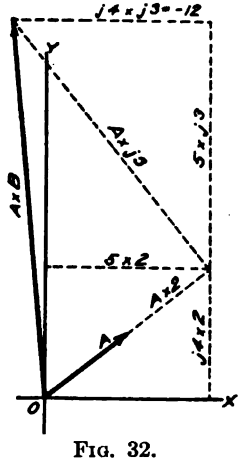



FIG. 32.

$$A = 5 + j4 \tag{60}$$
$$B = 2 + j3 \tag{61}$$
$$A \times B = (5 + j4)\,(2 + j3)$$
$$= 10 + j8 + j15 + j^2 12 = -2 + j23 \tag{62}$$

In division the operation is stated in the form of a fraction. The denominator is rationalized by multiplying both numerator and denominator with the conjugate quantity as shown in equation (63).

$$\frac{A}{B} = \frac{a + jb}{c - jd} = \frac{(a + jb)\,(c + jd)}{(c - jd)\,(c + jd)} = \frac{(a + jb)\,(c + jd)}{c^2 + d^2}$$
$$= \frac{ac - bd}{c^2 + d^2} + \frac{j(cb + ad)}{c^2 + d^2} \tag{63}$$

## PROBLEMS

**1.** Given: $e = 80 \sin (\omega t - 15°)$ volts.
$i = 60 \sin (\omega t + 40°)$ amperes.

(*a*) Draw curves for $e$ and $i$ in rectangular coördinates.
(*b*) Draw the corresponding polar-phase diagram.
(*c*) Draw the corresponding crank diagram. Note on all the diagrams the angles, $\omega t$, 15° and 40°.

**2.** Two alternators of the same frequency are connected in series.

$e_1 = 120 \sin (\omega t)$
$e_2 = 100 \sin (\omega t - 45°)$

(*a*) Draw the polar-phase diagram of the three voltages.
(*b*) Draw the crank-phase diagrams.
(*c*) Find graphically the value of $^{\sim}E_3$.
(*d*) Find the value of $E_3$.
(*e*) Write the equation for $e_3$, similar to those given for $e_1$ and $e_2$.

**3.** Given: $\dot{E}_1 = 20 + j15$.
$\dot{E}_2 = -12 - j8$.
$\dot{E}_3 = -17 + j10$.

(*a*) Find $\dot{E}_4$ the resultant of $\dot{E}_1$, $\dot{E}_2$ and $\dot{E}_3$.
(*b*) Draw the corresponding vector diagram (crank) for $\dot{E}_1$, $\dot{E}_2$, $\dot{E}_3$, and $\dot{E}_4$.

**4.** Given: a circuit with $z = 5 + j3$ ohms.

$\dot{I} = 8 - j7$ amperes.
$\dot{E} = z\dot{I}$ volts.

(*a*) Find the value of $\dot{E}$ and $E$.
(*b*) Draw the vector diagram for $\dot{E}$ and $\dot{I}$.

**5.** Given: a circuit with $z = 4 + j3$.

$\dot{E} = 50 - j20$.
$\dot{I} = \frac{\dot{E}}{z}$.

(*a*) Find the value of $\dot{I}$, $I$ and $^{\sim}I$.
(*b*) Draw the crank-vector diagram for $\dot{I}$, $^{\sim}\dot{I}$, $\dot{E}$ and $^{\sim}\dot{E}$.

6: Given a circuit in which:
$e = 140 \sin(\omega t + 10°)$ volts.
$i = 42 \sin(\omega t - 12°)$ amps.
(a) Find the effective values E & I
(b) Draw curves for e and i in rectangular coordinates.
(c) Draw the corresponding polar-phase diagram, (b), (a), and (d) where $^{\sim}E$ $^{\sim}I$ a

## CHAPTER IV

## RESISTANCE, REACTANCE, IMPEDANCE

### SERIES CIRCUITS

In direct currents the relation between the current and voltage is given by the equation, $E = RI$. In alternating currents magnetic and dielectric induction enter as essential factors and must be taken into consideration with the resistance in determining both the quantitative and phase relations of the currents and voltages. Most commercial systems consist of networks with the three factors, resistance, inductance and condensance (capacity) distributed and intermingled in a more or less complex manner. It may be looked upon as a combination of a number of simple series and parallel circuits. In a simple-series circuit the current is the same throughout while the voltage may be separated into parts. The voltage consumed by the resistance, inductance and condensance (capacity) may be found separately and the total obtained from the resultant of the several components.

**Resistance.**—The effect of the resistance factor is the same for alternating as for direct currents.

Assuming a sine wave

$$i = {}^{m}I \sin(\omega t) \tag{66}$$

For instantaneous values:

$${}_{r}e = ri = r{}^{m}I \sin(\omega t) \tag{67}$$

For maximum values:

$${}^{m}_{r}E = r{}^{m}I \tag{69}$$

For effective values:

$${}_{r}E = rI \tag{70}$$

The voltage is similar to, and in time phase with, the current.

Let the current vector be placed along the $X$-axis. Since the voltage is in time phase with the current the voltage vector will also lie along the $X$-axis, Fig. 33.

As already stated the length of the vector represents the

maximum value of the voltage or current. Since the ratio between the maximum and effective values for sine waves is $\sqrt{2}$, a constant, it is evident that the length of the vector may also be drawn so as to represent effective values of voltage or current. It should be noted that when effective values are used the diagram represents in magnitude constant quantities and also the phase relations of the maximum values.

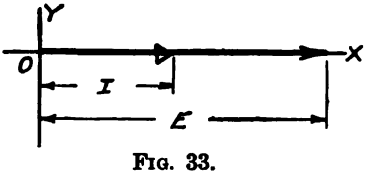


FIG. 33.

**Inductive Reactance.**—The magnetic field surrounding a conductor is proportional to the current.

$$\phi = Li \tag{71}$$

The voltage consumed by the inductance in the circuit is proportional to the rate of change in the lines of force.

$$_{L}e = \frac{d\phi}{dt} \tag{72}$$

From 71 and 72:

$$_{L}e = L\frac{di}{dt} \tag{73}$$

Assuming the current a sine wave from (66)

For instantaneous values:

$$_{L}e = L\frac{d(^{M}I \sin(\omega t))}{dt} = \omega L^{M}I \cos(\omega t) = {}_{L}x^{M}I \cos(\omega t) \tag{74}$$

For maximum values:

$$_{L}^{M}E = j\omega L^{M}I = j_{L}x^{M}I \tag{75}$$

For effective values:

$$_{L}E = j\omega LI = j_{L}xI \tag{76}$$

The voltage consumed by the inductance is in time quadrature with the current; that is, the voltage will reach its positive maximum value of 90° before the current reaches the corresponding maximum.

For the maxima and effective values the phase relation is indicated by the symbol $j$. Placing the current vector along the $X$-axis, the voltage vector will be at right angles and lie upward along the $Y$-axis, Fig. 34. By comparing the vector diagrams, Figs. 33 and 34, it is seen that for the same current the

voltage consumed by the inductance is in time quadrature with the voltage taken by the resistance. The quantity $\omega L$ or $2\pi fL$ bears the same relation to the voltage consumed by the inductance as $r$ to the resistance voltage. It is called the *inductive reactance* of the circuit and is represented by the symbol ${}_Lx$. It should be noticed that ${}_Lx$ is directly proportional to the frequency while the resistance is independent of the frequency.

In a circuit having resistance and inductive reactance in series,

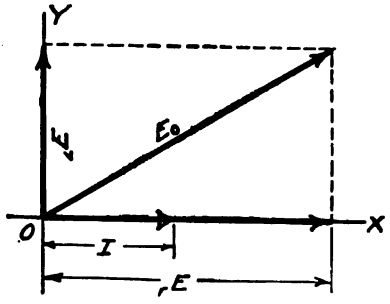



FIG. 34.

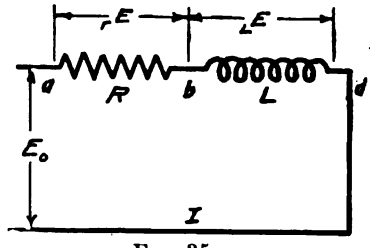



FIG. 35.

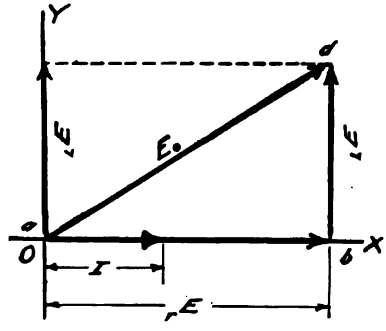



FIG. 36.

as indicated in Fig. 35, the voltage consumed by the inductive reactance is in time quadrature with the resistance voltage. Hence the total voltage is the vector sum of the two component vectors.

For instantaneous values:

$$e_0 = r^{\prime\prime}I \sin(\omega t) + {}_Lx^{\prime\prime}I \cos(\omega t) \tag{77}$$

For maximum values:

$$\begin{aligned} {}^{\prime\prime}\dot{E}_0 &= r^{\prime\prime}I + j_L x^{\prime\prime}I = {}^{\prime\prime}_rE + j^{\prime\prime}_LE \\ &= {}^{\prime\prime}\dot{E}_{ab} + {}^{\prime\prime}\dot{E}_{bd} \end{aligned} \tag{78}$$

For effective values:

$$\dot{E}_0 = rI + j_L xI = {}_rE + j_L E = \dot{E}_{ab} + \dot{E}_{bd} \tag{79}$$

In Fig. 36 is shown the vector diagram of the voltages and current in a circuit having a resistance of 8 ohms in series with an inductive reactance of 5 ohms. The current is 12 amperes.

Since the vectors for ${}_re$ and ${}_le$ are in phase quadrature their sum equals the square root of the sum of the squares.

$$E_0 = \sqrt{{}_rE^2 + {}_lE^2} = \sqrt{r^2I^2 + {}_lx^2I^2} = I\sqrt{r^2 + {}_lx^2} \tag{80}$$

Hence the three voltmeters, connected as indicated in Fig. 35 would read ${}_rE = 96$, ${}_lE = 60$ and $E_0 = 112.7$ volts.

**Condensive Reactance.**—The dielectric flux between two conductors is proportional to their difference of potential.

$$\psi = Ce \tag{81}$$

Any increase or decrease in the dielectric flux requires or produces a corresponding flow of current in the condenser circuit.

$$i = \frac{d\psi}{dt} \tag{82}$$

$$i = C\frac{d_ce}{dt} \tag{83}$$

Hence:

$${}_ce = \int \frac{idt}{C} = \frac{{}^mI}{C}\int \sin(\omega t)dt \tag{84}$$

For instantaneous values:

$${}_ce = -\frac{{}^mI}{\omega C}\cos(\omega t) = -{}_cx\,{}^mI\cos(\omega t) \tag{85}$$

For maximum values:

$${}^m_cE = -j\frac{{}^mI}{\omega C} = -j_cx\,{}^mI \tag{86}$$

For effective values:

$${}_cE = -j\frac{I}{\omega C} = -j_cxI \tag{87}$$

The factor $\frac{1}{\omega C}$ is called the *condensive reactance* and is represented by the symbol ${}_cx$.

A circuit having resistance and condensance in series is represented in Fig. 37. The corresponding vector diagram is shown in Fig. 38. The voltage consumed by the condensance

is represented by a vector along the $Y$-axis and 90° behind the resistance voltage. The total voltage is the resultant of the two component voltages.

The circuit represented in Figs. 37 and 38 has 6 ohms resistance, 12 ohms reactance and a current of 5 amperes.

For instantaneous values:

$$e_0 = r^{\nu}I \sin(\omega t) - {}_c x^{\nu}I \cos(\omega t) \tag{88}$$

For maximum values:

$${}^{\nu}E_0 = r^{\nu}I - j_c x^{\nu}I = {}^{\nu}_{r}E - j^{\nu}_{c}E = {}^{\nu}\dot{E}_{ab} + {}^{\nu}\dot{E}_{bd} \tag{89}$$

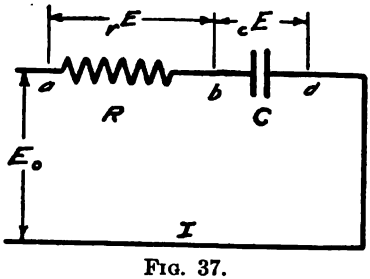



FIG. 37.

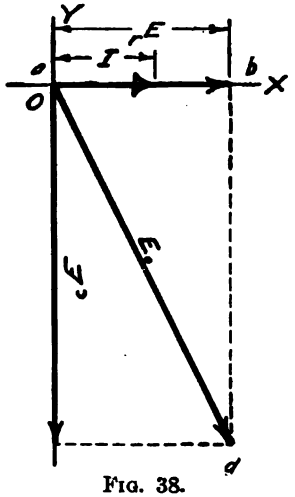



FIG. 38.

For effective values:

$$E = rI - j_c xI = \dot{}_{r}E - j_c E = \dot{E}_{ab} + \dot{E}_{bd} \tag{90}$$

The absolute or numerical value of the total voltage is the square root of the sum of the squares of the voltages consumed by the resistance and reactance.

$$E_0 = \sqrt{r^2I^2 + {}_c x^2 I^2} = I\sqrt{r^2 + {}_c x^2} = \sqrt{{}_r E^2 + {}_c E^2} \tag{91}$$

**Impedance.**—In Fig. 39 is represented a circuit having resistance, inductance and condensance in series.

Let $I$ = 7.5 amperes; $r$ = 5.4 ohms; ${}_l x$ = 4.0 ohms; and ${}_c x$ = 8.2 ohms.

The equation of the circuit is:

For instantaneous values:

$$e_0 = r^{\omega}I \sin(\omega t) + {}_{L}x^{\omega}I \cos(\omega t) - {}_{c}x^{\omega}I \cos(\omega t) \tag{92}$$

$$e_0 = 5.4 \cdot \sqrt{2} \cdot 7.5 \sin(\omega t) + 4.0 \cdot \sqrt{2} \cdot 7.5 \cos(\omega t) - 8.2 \cdot \sqrt{2} \cdot 7.5 \cos(\omega t) \text{ volts.}$$

For maximum values:

$$^{\omega}\dot{E}_0 = r^{\omega}I + j_{L}x^{\omega}I - j_{c}x^{\omega}I \tag{93}$$

$$= 5.4 \cdot \sqrt{2} \cdot 7.5 + j4.0 \cdot \sqrt{2} \cdot 7.5 - j8.2 \cdot \sqrt{2} \cdot 7.5 \text{ volts}$$

For effective values:

$$\dot{E}_0 = rI + j_{L}xI - j_{c}xI = rI + jI({}_{L}x - {}_{c}x) \tag{94}$$

$$= 5.4 \cdot 7.5 + j7.5(4.0 - 8.2) \text{ volts.}$$

$$= \dot{E}_{ab} + \dot{E}_{bd} + \dot{E}_{dc}$$

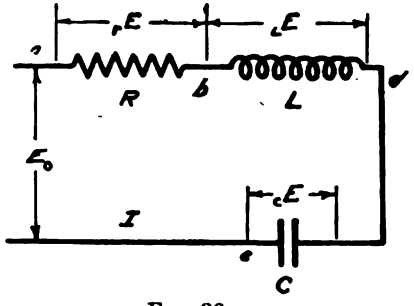



FIG. 39.

$$\text{Let } x = {}_{L}x - {}_{c}x = -4.2 \tag{95}$$

$$\dot{E}_0 = I(r + jx) \tag{96}$$

$$= 7.5(5.4 - j4.2) \text{ volts.}$$

$$\text{Let } z = r + jx \text{ or, } Z = \sqrt{r^2 + x^2} \tag{97}$$

$$= 5.4 - j4.2, \; 6.85 = \sqrt{5.4^2 + 4.2^2}$$

$$\dot{E}_0 = z\dot{I} \text{ or } E_0 = ZI \tag{98}$$

$$= 6.85 \cdot 7.5 = 51.4 \text{ volts.}$$

This factor $Z$ or $z$ in equation (98) is called the *impedance* of the circuit. By replacing the resistance as used in direct currents with the impedance, *Ohm's law is reëstablished for alternating currents*, or *stated in its complete form for all electric circuits*. In Fig. 40 is shown the vector diagram corresponding to the circuit in Fig. 39. The resultant voltage vector along the

$Y$-axis is the difference of the voltages consumed by ${}_{L}x$ and ${}_{c}x$. The total voltage is the vector sum of the resistance voltage on the $X$-axis and the difference of the two reactance voltages. Instead of drawing the voltage vectors through the origin, as in Fig. 40, it is sometimes less labor to draw a triangle of the voltages as shown in Fig. 41.

Since $I$ is a common factor, a right triangle may also be formed from $x$, $r$ and $z$ as shown in Fig. 41. This is called the *impedance triangle*. The complex expression $z = r + jx$, from equation (97), indicates the same relation. Resistance, reactance and impedance are, however, not vector quantities like current and voltage, but merely act as operators.

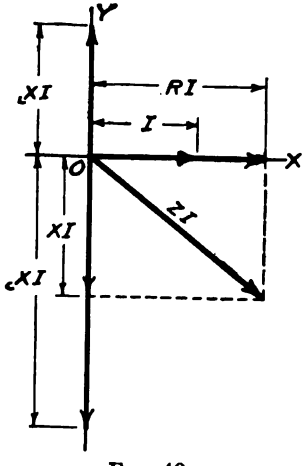



FIG. 40.

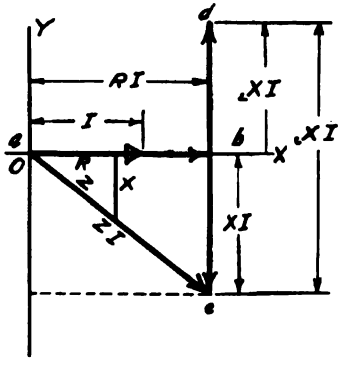



FIG. 41.

## PROBLEMS

**1.** Given a circuit as shown in Fig. 35. $I = 7$ amperes; $r = 5$ ohms; $L = 0.02$ henry; $f = 60$ cycles

$$z = r + j(2\pi fL) = r + j_{L}x.$$

(*a*) Find ${}_{r}E$, ${}_{L}E$, $E_0$.

(*b*) Draw vector diagram for $\dot{I}$, ${}_{r}\dot{E}$, ${}_{L}\dot{E}$, and $\dot{E}_0$.

(*c*) Draw the impedance triangle.

**2.** Same circuit and data as for problem 1, except the frequency is 25 cycles.

**3.** Circuit diagram as in Fig. 37. The impressed voltage, $E_0 = 220$ volts; $r = 8$ ohms; $C = 320$ microfarads; $f = 60$ cycles.

$$z = r - j\left(\frac{1}{2\pi fC}\right) = r - j_{c}x.$$

(*a*) Find $I$, ${}_rE$ and ${}_cE$.

(*b*) Draw vector diagram showing $\dot{I}$, ${}_r\dot{E}$, ${}_c\dot{E}$ and $\dot{E}_0$.

(*c*) Draw the impedance triangle.

**4.** Same circuit and data as for problem 3 except the frequency is 25 cycles.

**5.** Circuit diagram as in Fig. 39. The impressed voltage, $E_0 = 240$ volts; $r = 6$ ohms; $L = 0.05$ henry; $C = 273$ microfarads; $f = 60$.

$$z = r + j\left(2\pi fL - \frac{1}{2\pi fC}\right) = r + j({}_lx - {}_cx).$$

(*a*) Find $\dot{I}$, ${}_rE$, ${}_lE$, ${}_cE$ and ${}_zE$.

(*b*) Draw a vector diagram showing $\dot{I}$, ${}_r\dot{E}$, ${}_l\dot{E}$, ${}_c\dot{E}$, ${}_z\dot{E}$, $\dot{E}_0$.

(*c*) Draw the impedance triangle.

**6.** Same circuit and data as in problem 5 except the frequency is 25 cycles.

**7.** Readings taken in the laboratory with an alternating-current voltmeter in series with a telephone condenser and directly across a 60-cycle line are respectively 47.0 and 114.0 volts. The resistance of the voltmeter is 3500 ohms and its reactance is negligible. What is the impedance and condensance of the condenser, assuming that there is no energy loss in it?

For Experiments see page 527.

## CHAPTER V

## ADMITTANCE, CONDUCTANCE, SUSCEPTANCE

### PARALLEL CIRCUITS

In a series circuit the same current passes through all the parts, while in parallel circuits the voltage is the common factor. In drawing the vector diagrams for the series circuits the current is generally taken as the reference vector since it is the common factor. Similarly, it is usually desirable to take the voltage as reference vector in parallel circuits. Just as in dealing with series circuits in Chap. IV the total voltage was composed of several components, so in parallel circuits the total current may be considered as consisting of vector components from the several parallel circuits. In equation (101) and Fig. 42 the voltage is separated into one part, $rI$, in phase with the current and another, $jxI$ leading the current by 90°. Solving the equation for current we have (102).

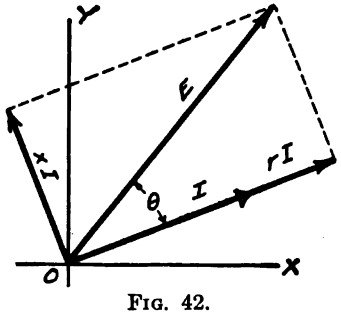


FIG. 42.

$$E = rI + jxI \tag{101}$$

$$I = \frac{E}{r + jx} \tag{102}$$

Rationalizing the fraction by multiplying both numerator and denominator by $r - jx$ gives,

$$I = E\frac{(r - jx)}{r^2 + x^2} = \frac{rE}{r^2 + x^2} - \frac{jxE}{r^2 + x^2} \tag{103}$$

For convenience let:

$$\frac{r}{r^2 + x^2} = g; \text{ and } \frac{x}{r^2 + x^2} = b \tag{104}$$

Then:

$$I = gE - jbE \tag{105}$$

Equation (105) states that the current consists of two components, one in phase and the other in quadrature with the voltage. The corresponding vector diagram is shown in Fig. 43. The minus sign before the quadrature component indicates that the voltage leads the current. That this is the same relation as indicated by equation (101) is evident; for, if the current lags behind the voltage, necessarily the voltage leads the current.

The factor $\frac{r}{r^2 + x^2}$ is called the *conductance* of the circuit and is represented by the symbol $g$.

The factor $\frac{x}{r^2 + x^2}$ is called the *susceptance* of the circuit and is represented by the symbol $b$.

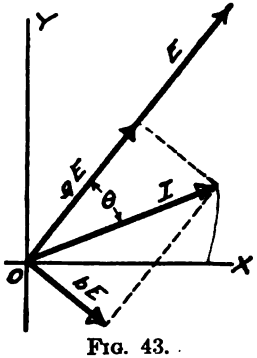



FIG. 43.

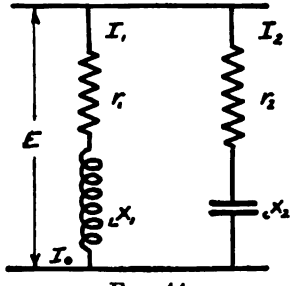



FIG. 44.

Let

$$y = g - jb; \text{ or } Y = \sqrt{g^2 + b^2} \tag{106}$$

Then $y$ or $Y$ is the admittance of the circuit.
From (105) and (106):

$$\dot{I} = y\dot{E}, \text{ or } I = YE \tag{107}$$

Combining this with (98) in Chap. IV:

$$Y = \frac{1}{Z}, \text{ or } Z = \frac{1}{Y} \tag{108}$$

The practical unit for admittance, conductance and susceptance is the reciprocal ohm, called the *mho*.

For an illustration take two circuits as represented in Fig. 44.
In the first circuit: $r_1 = 12$ ohms and ${}_ex_1 = 10$ ohms.
In the second circuit: $r_2 = 8$ ohms, and ${}_ex_2 = 15$ ohms.

Let the voltage for both circuits be 120 volts.

$$g_1 = \frac{r_1}{r_1^2 + {}_\iota x_1^2} = \frac{12}{12^2 + 10^2} = 0.0492 \text{ mhos.}$$

$$b_1 = \frac{{}_\iota x_1}{r_1^2 + {}_\iota x_1^2} = \frac{10}{12^2 + 10^2} = 0.0409 \text{ mhos.}$$

$$I_1 = g_1 E - jb_1 E = 0.0492E - j0.0409E \text{ amperes.} \quad (109)$$

$$g_2 = \frac{r_2}{r_2^2 + {}_\sigma x_2^2} = \frac{8}{8^2 + 15^2} = 0.0276 \text{ mhos.}$$

$$b_2 = \frac{{}_\sigma x_2}{r_2^2 + {}_\sigma x_2^2} = \frac{15}{8^2 + 15^2} = 0.0519 \text{ mhos.}$$

$$\dot{I}_2 = g_2 \dot{E} + jb_2 \dot{E} = 0.0276\dot{E} + j0.0519\dot{E} \text{ amperes.} \quad (110)$$

The total current is the vector sum of the component currents as shown graphically in Fig. 45 and analytically by equation (111).

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = 0.0492\dot{E} - j0.0409\dot{E} + 0.0276\dot{E} + j0.0519\dot{E} = (0.0768 + j0.011)\dot{E} = (g_0 + jb_0)\dot{E} \quad (111)$$

$$I = E\sqrt{(g_0^2 + b_0^2)} = 0.0843\ E = 10.12 \text{ amperes.} \quad (112)$$

**Ohm's Law.**—From the discussion in this and the previous chapter it is evident that Ohm's law may be expressed either by (113) or (114).

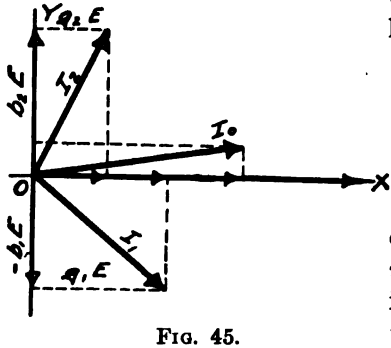



FIG. 45.

$$\dot{E} = z\dot{I} \quad (113)$$

$$\dot{I} = y\dot{E} \quad (114)$$

Therefore "the joint impedance of a number of *series-connected* impedances if expressed in complex quantities, is equal to the sum of the individual impedances; the joint admittance of a number of *parallel-connected* admittances, if expressed in complex quantities, is equal to the sum of the individual admittances. In diagrammatic representation, combination by the parallelogram law takes the place of addition of the complex quantities."

**Kirchoff's Laws.**—For direct currents Kirchoff's laws state:

(*a*) The sum of all the electromotive forces in a circuit equals zero. $\Sigma E = 0$.

(*b*) The sum of all the currents flowing to a point is zero. $\Sigma I = 0$.

In alternating currents the phase relations as well as the magnitude of the component parts must be included. Any voltage wave can be resolved into a component in phase with the current and a second component in quadrature with the current. In combining several voltages these components must be taken separately as neither can have any effect on the magnitude of the other. Likewise the currents can be resolved into one component in phase with the voltage and a second component in quadrature with the voltage. For the same reason neither of these components can have any effect on the magnitude of the other, and therefore must be taken separately when several currents are combined.

Kirchoff's laws, therefore, reëstablished for alternating currents are:

(*c*) The sum of all the real components and the sum of all the quadrature components of the electromotive forces in a circuit equal zero.

$$\Sigma \dot{E} = 0, \text{ or } \Sigma_r \dot{E} = 0 \text{ and } \Sigma_x \dot{E} = 0 \qquad (117)$$

(*d*) The sum of the real components and the sum of the quadrature components of currents flowing to a point equal zero.

$$\Sigma \dot{I} = 0, \text{ or } \Sigma_g \dot{I} = 0 \text{ and } \Sigma_b \dot{I} = 0 \qquad (118)$$

## PROBLEMS

**1.** Given a circuit having a constant reactance of 0.5 ohms in series with a resistance that can be varied from 0 to 3 ohms. Plot curves for impedance, admittance, conductance and susceptance as ordinates and with resistance as abscissæ.

**2.** Given a circuit having a constant resistance of 0.5 ohms in series with a reactance that can be varied from +2 ohms (inductive) to −2 ohms (condensive). Plot curves for impedance, admittance, conductance and susceptance as ordinates and with reactance as abscissæ.

**3.** A hot wire galvanometer having a resistance of 4.3 ohms is shunted with a 0.005 m.f. condenser. What percentage of the total current will pass through the meter at frequencies of 3,000; 30,000; 300,000; and 3,000,000 cycles?

For Experiments see page 528.

## CHAPTER VI

## SOLUTION OF CIRCUITS

Having in the preceding chapters shown that Ohm's and Kirchoff's laws, when expressed in their complete form, apply to alternating currents, we are in position to calculate alternating-current circuit networks containing resistance, inductive reactance and condensive reactance in as concise a manner as for direct currents. The symbols and expressions for the circuit constants and the equations for Ohm's and Kirchoff's laws as applied to alternating currents are grouped for ready reference.

$$\text{Resistance} = r = \frac{g}{g^2 + b^2} = \frac{g}{g^2 + ({}_c b - {}_\iota b)^2} \tag{121}$$

$$\text{Reactance} = x = {}_\iota x - {}_c x = \frac{b}{g^2 + b^2} = \frac{{}_c b - {}_\iota b}{g^2 + ({}_\iota b - {}_c b)^2} \tag{122}$$

$$\text{Impedance} = z = r \pm jx = r + j({}_\iota x - {}_c x) = \frac{1}{y} \tag{123}$$

$$\text{Impedance} = Z = \sqrt{r^2 + x^2} = \sqrt{r^2 + ({}_\iota x - {}_c x)^2} = \frac{1}{Y} \tag{124}$$

$$\text{Conductance} = g = \frac{r}{r^2 + x^2} = \frac{r}{r^2 + ({}_\iota x - {}_c x)^2} \tag{125}$$

$$\text{Susceptance} = b = \frac{x}{r^2 + x^2} = \frac{{}_\iota x - {}_c x}{r^2 + ({}_\iota x - {}_c x)^2} \tag{126}$$

$$\text{Admittance} = y = g \mp jb = g - j({}_\iota b - {}_c b) = \frac{1}{z} \tag{127}$$

$$\text{Admittance} = Y = \sqrt{g^2 + b^2} = \sqrt{g^2 + ({}_\iota b - {}_c b)^2} = \frac{1}{Z} \tag{128}$$

Ohm's Law in notation indicating both magnitude and phase relations:

$$\dot{E} = z\dot{I} = (r + jx)\dot{I} = [r + j({}_\iota x - {}_c x)]\dot{I} \tag{129}$$

$$\dot{I} = y\dot{E} = (g - jb)\dot{E} = [g - j({}_\iota b - {}_c b)]\dot{E} \tag{130}$$

For absolute values without directly indicating phase relations:

$$E = ZI = I\sqrt{r^2 + x^2} = I\sqrt{r^2 + ({}_\iota x - {}_c x)^2} \tag{131}$$

$$I = YE = E\sqrt{g^2 + b^2} = E\sqrt{g^2 + ({}_\iota b - {}_c b)^2} \tag{132}$$

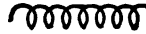
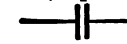
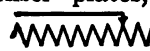
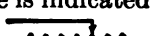
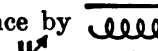
Kirchoff's law in notation indicating both magnitude and phase relations:

$$\Sigma\dot{E} = 0 \text{ or } \Sigma_r E = 0 \text{ and } \Sigma j_x E = 0 \qquad (133)$$

$$\Sigma\dot{I} = 0 \text{ or } \Sigma_g I = 0 \text{ and } \Sigma j_b I = 0 \qquad (134)$$

From the above equations it is evident that the field of constant direct currents is merely the special case of alternating currents when $f = 0$. The energy stored in both the magnetic and the dielectric fields under this condition remains constant, and hence only the electric circuit need be considered in direct-current problems.

It is obviously impossible to discuss all of the infinite variety of combinations which can be imagined or which may exist in a network of circuits; therefore, only some of the simpler and more common types will be considered.

**Symbols.**—In the circuit diagrams certain symbols are used to represent the quantities and the arrangement of the instruments in the circuits. Thus resistance is indicated by a saw-tooth line , inductance or inductive reactance by a coil, , and condensance (capacity) or condensive reactance by condenser plates, . A variable resistance is indicated by , a variable inductive reactance by , and a variable condensive reactance by . The symbol, , indicates a complete cycle. The notation used in the text may readily be kept in mind by referring to the corresponding circuit and vector diagrams. The subscripts in the lower right-hand corner give the position in the circuit while the subscripts in the lower left-hand corner indicate the nature of the circuit. Thus ${}_{i}x_2$ indicates inductive reactance in circuit No. 2; and ${}_{c}x_1$ indicates condensive reactance in circuit No. 1. Similarly ${}_{r}E_3$ indicates the voltage consumed by the resistance $r$ in circuit No. 3; and ${}_{c}I_2$ the condensance current flowing in circuit No. 2. The dots above $\dot{E}$ and $\dot{I}$ indicate a complex or vector nature of the voltage and current, while the same letters without the dot stand for the absolute or numerical values.

## I. SERIES CIRCUITS

### A. Resistance and Inductance.

**Problem 1.**—Circuit diagram, Fig. 46. Vector diagram, Fig. 47.

Given: $r = 4.8$ ohms; $L = 0.02$ henry; $I = 10$ amperes; $f = 60$ cycles.

Find: ${}_rE$, ${}_lE$, $\dot{E}_0$, $E_0$, $\theta_0$, $\cos\theta_0$.

${}_rE = rI = 48$ volts $= E_{ab}$.

${}_lE = {}_lxI = 2\pi fLI = 75.4$ volts $= E_{bd}$.

$\dot{E}_0 = \dot{I}(r + j_l x) = {}_r\dot{E} + j_l\dot{E} = 48 + j75.4$ volts $= \dot{E}_{ad}$.

$E_0 = \sqrt{{}_rE^2 + {}_lE^2} = \sqrt{48^2 + 75.4^2} = 89.4$ volts $= E_{ad}$.

$\theta_0 = \tan^{-1}\frac{{}_lx}{r} = 57°\ 31'$; $\cos\theta_0 = 0.537$.

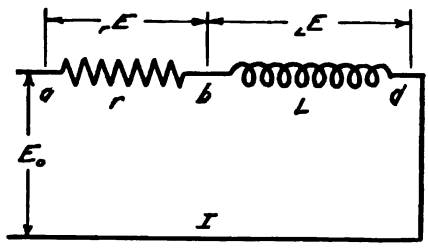



FIG. 46.

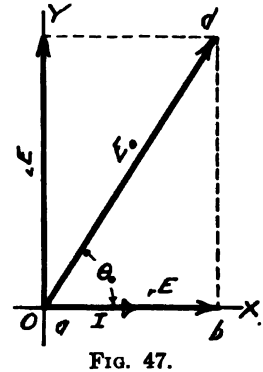



FIG. 47.

**Problem 2.**—Given the same circuit as in Fig. 46 with $r$, $L$ and $I$ the same as in problem 1, but $f = 25$ cycles.

Find: ${}_rE$, ${}_lE$, $\dot{E}_0$, $E_0$, $\theta_0$ and $\cos\theta_0$.

Draw the corresponding vector diagram.

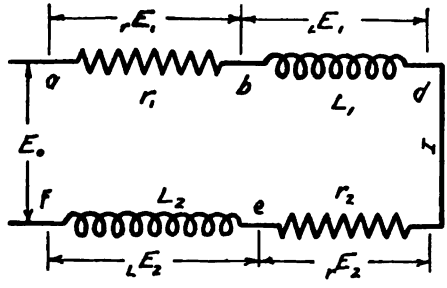



FIG. 48.

**Problem 3.**—Circuit diagram, Fig. 48. Vector diagram, Fig. 49.

Given: $r_1 = 8.2$ and $r_2 = 2.7$ ohms; $L_1 = 0.01$ and $L_2 = 0.03$ henry; $f = 25$ cycles; $I = 10$ amperes.

Find: ${}_rE_1$, ${}_\iota E_1$, ${}_rE_2$, ${}_\iota E_2$, ${}_rE$, ${}_\iota E$, $\dot{E}_0$, $E_0$, $\theta_1$, $\theta_2$, $\theta_0$, cos $\theta_0$.

${}_rE_1 = r_1 I = 82.0$ volts $= E_{ab}$.

${}_\iota E_1 = {}_\iota x_1 I = 15.7$ volts $= E_{bd}$.

$E_1 = I\sqrt{r_1^2 + {}_\iota x_1^2} = 83.5$ volts $= E_{ad}$.

${}_rE_2 = r_2 I = 27.0$ volts $= E_{de}$.

${}_\iota E_2 = {}_\iota x_2 I = 47.1$ volts $= E_{ef}$.

$E_2 = I\sqrt{r_2^2 + {}_\iota x_2^2} = 54.2$ volts $= \dot{E}_{df}$.

${}_rE = {}_rE_1 + {}_rE_2 = 109.0$ volts.

${}_\iota E = {}_\iota E_1 + {}_\iota E_2 = 62.8$ volts.

$\dot{E}_0 = {}_rE + j_\iota E = 109.0 + j62.8$ volts $= \dot{E}_{af}$.

$E_0 = \sqrt{{}_rE^2 + {}_\iota E^2} = 125.7$ volts $= E_{af}$.

$\theta_1 = \tan^{-1}\frac{{}_\iota x_1}{r_1} = 10°\ 50'$. $\theta_2 = \tan^{-1}\frac{{}_\iota x_2}{r_2} = 60°\ 10'$.

$\theta_0 = \tan^{-1}\frac{{}_\iota x_1 + {}_\iota x_2}{r_1 + r_2} = 30°$; cos $\theta_0 = 0.866$.

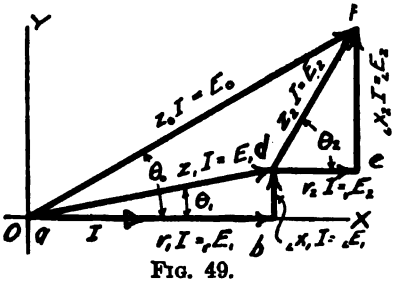



FIG. 49.

**Problem 4.**—Same circuit and constants as in problem 3 except the frequency is 60 cycles per sec.

Draw vector diagram and find ${}_rE_1$, ${}_\iota E_1$, ${}_rE_2$, ${}_\iota E_2$, $E_1$, $E_2$, ${}_rE$, ${}_\iota E$, $\dot{E}_0$, $E_0$, $\theta_1$, $\theta_2$, $\theta_0$, and cos $\theta_0$.

## B. Resistance and Condensance.

**Problem 5.**—Circuit diagram, Fig. 50. Vector diagram, Fig. 51.

Given: $r = 6.3$ ohms; $C = 440$ microfarads; $I = 10$ amperes; $f = 50$ cycles.

Find: ${}_rE$, ${}_cE$, $\dot{E}_0$, $E_0$, $\theta_0$ and cos $\theta_0$.

${}_rE = rI = 63.0$ volts $= E_{ab}$.

${}_cE = {}_cxI = 72.4$ volts $= E_{bd}$.

$\dot{E}_0 = \dot{I}(r - j_c x) = {}_r\dot{E} - j_c\dot{E} = 63.0 - j72.4$ volts $= \dot{E}_{ad}$.

$E_0 = I\sqrt{r^2 + {}_cx^2} = \sqrt{{}_rE^2 + {}_cE^2} = 95.9$ volts $= E_{ad}$.

$\theta = \tan^{-1}\frac{7.24}{6.3} = 49°$; cos $\theta = 0.656$.

**Problem 6.**—Given the same circuit and constants as in problem 5 except $f = 60$ cycles.

Find: ${}_rE$, ${}_cE$, $\dot{E}_0$, $E_0$, $\theta_0$ and $\cos\theta_0$.

Draw the vector diagram.

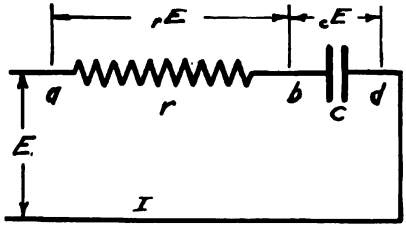



FIG. 50.

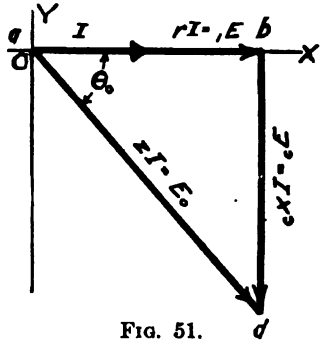



FIG. 51.

**Problem 7.**—Circuit diagram, Fig. 52. Vector diagram, Fig. 53.

$r_1 = 8.0$ and $r_2 = 3.5$ ohms; $C_1 = 800$ and $C_2 = 250$ microfarads; $I = 10$ amperes; $f = 60$ cycles.

Find: ${}_rE_1$, ${}_cE_1$, ${}_rE_2$, ${}_cE_2$, ${}_rE$, ${}_cE$, $E_1$, $E_2$, $\dot{E}_0$, $E_0$, $\theta_1$, $\theta_2$, and $\theta_0$.

${}_rE_1 = r_1 I = 80.0$ volts $= E_{ab}$.

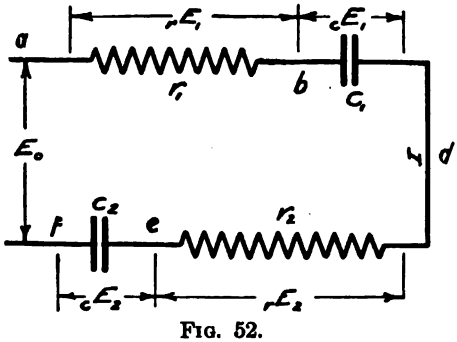



FIG. 52.

${}_rE_2 = r_2 I = 35.0$ volts $= E_{de}$.

${}_cE_1 = {}_cx_1 I = \dfrac{I}{2\pi f C_1} = 33.2$ volts $= E_{bd}$.

${}_cE_2 = {}_cx_2 I = 106.1$ volts $= E_{ef}$.

${}_rE = {}_rE_1 + {}_rE_2 = 115.0$ volts.

${}_cE = {}_cE_1 + {}_cE_2 = 139.3$ volts.

$\dot{E}_1 = I(r_1 - j_c x_1) = {}_rE_1 - j_c E_1 = 80 - j33.2$ volts $= \dot{E}_{ad}$.

$\dot{E}_2 = I(r_2 - j_c x_2) = {}_rE_2 - j_c E_2 = 35 - j106.1$ volts $= E_{df}$.

$\dot{E}_0 = \dot{I}[(r_1 + r_2) - j({}_cx_1 + {}_cx_2)] = 115 - j139.3$ volts $= \dot{E}_{af}$.

$E_0 = \sqrt{{}_rE^2 + {}_cE^2} = 180.6$ volts $= E_{af}$.

$\theta_1 = \tan^{-1}\frac{{}_cx_1}{r_1} = 22° 33'$.

$\theta_2 = \tan^{-1}\frac{{}_cx_2}{r_2} = 71° 45'$.

$\theta_0 = \tan^{-1}\frac{{}_cx_1 + {}_cx_2}{r_1 + r_2} = 50° 26'$.

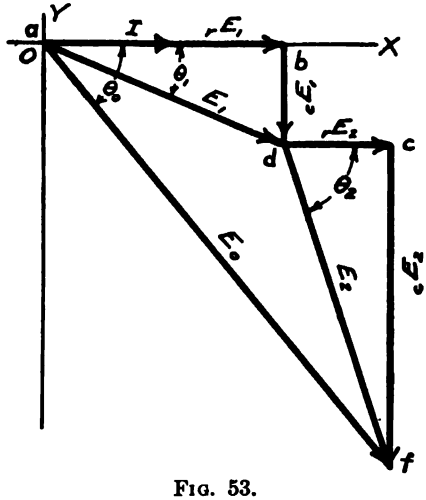



FIG. 53.

**Problem 8.**—Given the same circuit and constants as in problem 7 except $f = 25$ cycles.

Find: ${}_rE_1$, ${}_rE_2$, ${}_cE_1$, ${}_cE_2$, ${}_rE$, ${}_cE$, $\dot{E}_1$, $\dot{E}_2$, $\dot{E}_0$, $E_0$, $\theta_1$, $\theta_2$ and $\theta_0$.

Draw the corresponding vector diagram.

### C. Resistance, Inductance and Condensance.

**Problem 9.**—Circuit diagram, Fig. 54. Vector diagram, Fig. 55.

Given: $r = 4.2$ ohms; $L = 0.03$ henry; $C = 450$ microfarads; $I = 10$ amperes; $f = 50$ cycles.

Find: ${}_rE$, ${}_lE$, ${}_cE$, $\dot{E}_0$, $E_0$ and $\theta_0$.

${}_rE = rI = 42.0$ volts.

${}_lE = 2\pi fLI = {}_lxI = 94.2$ volts.

${}_cE = \frac{I}{2\pi fC} = {}_cxI = 70.7$ volts.

$\dot{E}_0 = \dot{I}[r + j({}_lx - {}_cx)] = {}_r\dot{E} + j({}_l\dot{E} - {}_c\dot{E}) = 42.0 + j23.5$ volts.

$E_0 = I\sqrt{r^2 + ({}_lx - {}_cx)^2} = 48.1$ volts.

$$\theta_0 = \tan^{-1}\frac{x}{r} = \tan^{-1}\frac{{}_{\iota}x - {}_{c}x}{r} = 29^\circ\ 15'.$$

**Problem 10.**—Given the same circuit and constants as in problem 9 except the frequency is 25 cycles per sec.

Find: ${}_{r}E$, ${}_{\iota}E$, ${}_{c}E$, $\dot{E}_0$, $E_0$ and $\theta_0$. Draw vector diagram.

**Problem 11.**—Given the same circuit and constants as in problem 9 except $f = 40$ cycles.

Find: ${}_{r}E$, ${}_{\iota}E$, ${}_{c}E$, $\dot{E}_0$, $E_0$ and $\theta_0$. Draw vector diagram.

In a series circuit all the voltage absorbed by resistance is in phase with the current. Hence the total resistance voltage is the product of the arithmetical sum of all the resistances by

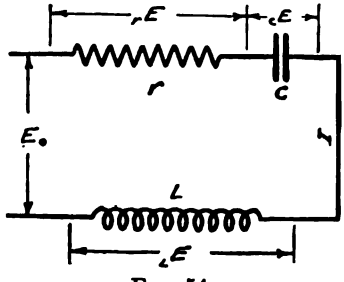



FIG. 54.

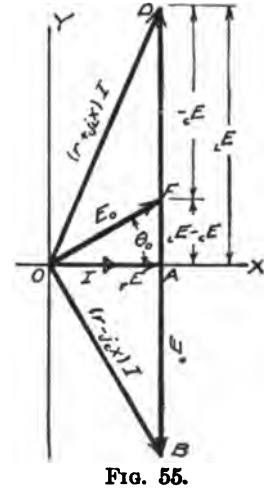



FIG. 55.

the current. This will be true whether the resistance is distributed over the circuit or concentrated at one or more points. Since the condensive reactance is of opposite sign to the inductive reactance, the sum of all the reactances in the circuit is equal to the algebraic sum of the separate inductive and condensive reactances. Thus in a series circuit as shown in Fig. 56 the total resistance voltage is found by multiplying the sum of all the resistances by the current. Likewise the total voltage taken in quadrature with the current is found by taking the product of the current into the algebraic sum of all the reactances.

**Problem 12.**—Circuit diagram, Fig. 56. Vector diagram, Fig. 57.

Given: $r_1 = 5.0$; ${}_{\iota}x_1 = 6.6$; ${}_{c}x_1 = 12.0$; $r_2 = 2.5$; ${}_{c}x_2 = 5.5$;

$_1x_2 = 10.0$; $_2x_3 = 7.4$; $r_3 = 8.8$—all in ohms; $I = 10$ amperes; $f = 60$ cycles.

Find: $\dot{E}_0$, $E_0$, and $\theta_0$.

$$\dot{E}_0 = \dot{I}\{(r_1 + r_2 + r_3) + j[(_1x_1 + _1x_2) - (_2x_1 + _2x_2 + _2x_3)]\} = 10\,[16.3 + j(16.6 - 24.9)] = 163 - j83 \text{ volts.}$$

Hence the voltage in phase with the current is 163 volts and the quadrature component is 83 volts, with the current leading the voltage.

$$E_0 = \sqrt{163^2 + 83^2} = 182.7 \text{ volts.}$$

$$\theta = \tan^{-1}\frac{83}{163} = 26° 56'.$$

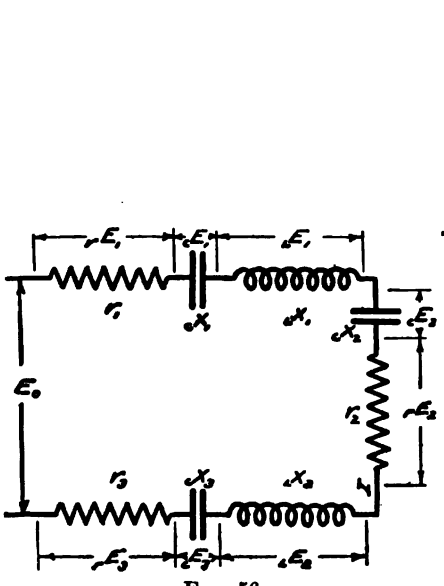


FIG. 56.

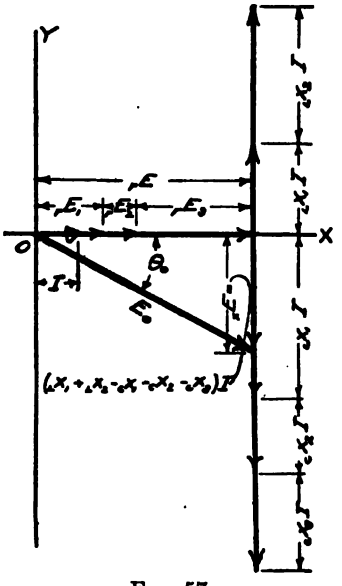


FIG. 57.

**Problem 13.**—Given the same circuit and the same resistance, inductance and condensance as in problem 12. $f = 50$ cycles.

Find: $\dot{E}_0$, $E_0$ and $\theta_0$. Draw vector diagram.

**Problem 14.**—Given the same circuit and the same inductance, condensance and resistance as in problem 12. $f = 25$ cycles.

Find: $\dot{E}_0$, $E_0$ and $\theta_0$. Draw vector diagram.

**D. Resonance.**—In any series circuit having a given inductance and condensance the relative value of the inductive

and condensive reactance depends on the frequency of the current, ${}_{L}x = 2\pi fL$; ${}_{C}x = \frac{1}{2\pi fC}$.

Since ${}_{L}x$ is directly and ${}_{C}x$ inversely proportional to $f$, a frequency may be found at which ${}_{L}x = {}_{C}x$ or $2\pi fL = \frac{1}{2\pi fC}$, and hence for resonance,

$$f = \frac{1}{2\pi\sqrt{LC}}. \qquad (135)$$

For the frequency at which ${}_{L}x = {}_{C}x$ the circuit is in resonance and the resistance alone opposes the flow of current. Hence under resonance conditions $E = rI$, since $x = {}_{L}x - {}_{C}x = 0$.

It should be kept clearly in mind that a circuit must contain both inductance and condensance in order to give resonance.

**Problem 15.**—Circuit diagram, Fig. 58.

Let $r = 0.5$ ohm; $L = 0.05$ henry; $C = 400$ microfarads; $E_0 = 100$ volts.

(*a*) Find the frequency for resonance.

(*b*) Under resonance conditions, find $I$, ${}_{L}E$, ${}_{C}E$, ${}_{r}E$, and draw vector diagram.

(*c*) Plot curves using frequencies from 0 to 100 as abscissæ and the corresponding values of ${}_{L}x$, ${}_{C}x$, $x$, $z$, $I$, ${}_{L}E$, ${}_{C}E$, and ${}_{r}E$ as ordinates.

It is seen that this method of solving circuits may readily be applied to all possible combinations of resistance, inductive reactance and condensive reactance in series. The order of the various parts is of no consequence in determining the relations of the current and the total voltage. Since the current is the same throughout it can to best advantage be selected as the reference vector. Then all the resistance will absorb voltage in phase with the current. Hence the total voltage absorbed by the several resistances in the circuit must be equal to the product of the sum of all the resistances into the current. The vector for the resistance voltage is in phase with the current vector.

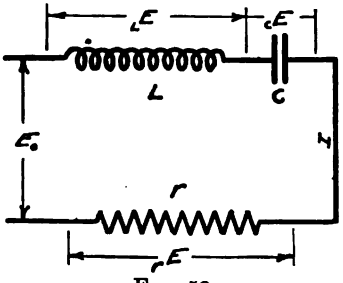


FIG. 58.

Likewise the total reactance is the algebraic sum of all the reactances in the circuit. Inductive reactance is considered as positive and condensive reactance as negative. The total voltage absorbed by all the reactances is equal to the product of the algebraic sum of all the reactances in the circuit by the

current. The reactance voltage is in time quadrature with the current and hence the corresponding vector will be in quadrature with the current vector.

The total voltage consumed by both the resistance and the reactance is the square root of the sum of the squares of the resistance and the reactance voltages. These relations are expressed by equations (136) and (137). Let the sum of all the resistances in the circuit be $r_1 + r_2 + r_3 + \text{etc.} = R$. Let the sum of all the inductive reactances in the circuit be ${}_\iota x_1 + {}_\iota x_2 + {}_\iota x_3 + \text{etc.} = {}_\iota X$; and the sum of all the condensive reactances be ${}_\circ x_1 + {}_\circ x_2 + {}_\circ x_3 + \text{etc.} = {}_\circ X$.

$$\dot{E}_0 = [R + j({}_\iota X - {}_\circ X)]\dot{I}_0 \qquad (136)$$

$$E_0 = I_0 \sqrt{R^2 + ({}_\iota X - {}_\circ X)^2} = ZI_0 \qquad (137)$$

By referring to the vector diagram certain simple relations between the total voltage and the voltages of the parts of the circuit may be noted.

(*a*) If non-inductive resistances are connected in series, the sum of the several voltages is equal to the total voltage. Likewise if condensers are connected in series with short pieces of wire so as to make the resistance negligible, the sum of the several voltages is equal to the total voltage. Commercial inductive reactances have more or less resistance but if the resistance is relatively small the voltage will be almost entirely absorbed by the inductive reactance.

(*b*) If the *time constant*, that is, the ratio between the inductance and the resistance, is the same in two coils connected in series, then the total voltage is equal to the sum of the two voltages across the coils. By drawing a vector diagram it becomes evident that the impedance drops for the two coils lie in the same straight line.

(*c*) When resistance, inductive reactance and condensive reactance are connected in series the sum of the separate voltages is always larger than the total voltage. Since the inductive and condensive reactances give opposing effects the total voltage may be less than the voltage consumed by either the inductive or the condensive reactances.

## II. PARALLEL CIRCUITS

The constants of parallel circuits are often given in terms of resistance, inductance and condensance, or resistance, inductive

and condensive reactance and impedance of each of the parallel circuits. In order to find the total current, the relative phase positions as well as the magnitudes must be taken into consideration. By converting the constants given in ohms to the corresponding values in mhos, that is, into conductance and susceptance, the components of the current in phase with the voltage and the part in quadrature may be separated. The total current can then be found in the same manner as the total voltage is found in series circuits.

**A. Resistance and Inductance.**

**Problem 16.**—Circuit diagram, Fig. 59. Vector diagram, Fig. 60.

Given: $g_1 = 0.22$; $b_1 = 0.00$; $g_2 = 0.00$; and $_{\iota}b_2 = 0.12$ mho; $E_0 = 100$ volts.

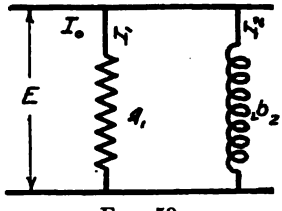



FIG. 59.

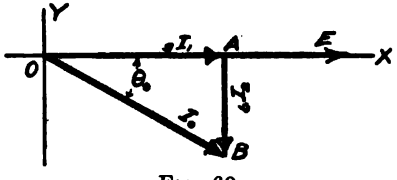



FIG. 60.

Find: $\dot{I}_1, \dot{I}_2, \dot{I}_0, I_0, \theta_0$.

$\dot{I}_1 = g_1\dot{E} - jb_1\dot{E} = 0.22E = 22.0$ amperes.

$\dot{I}_2 = g_2\dot{E} - j_{\iota}b_2\dot{E} = -j0.12E = -j12.0$ amperes.

$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = 22.0 - j12.0$ amperes.

$I_0 = \sqrt{I_1^2 + I_2^2} = 25.0$ amperes.

$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = -28° 35'$.

**Problem 17.**—Circuit diagram, Fig. 61. Vector diagram, Fig. 62.

$r_1 = 6.4$ ohms $_{\iota}x_1 = 4.8$ ohms.

$r_2 = 2.4$ ohms, $_{\iota}x_2 = 8.0$ ohms.

$E = 100$ volts, $f = 60$ cycles.

Find: $g_1, g_2, g_0, _{\iota}b_1, _{\iota}b_2, b_0, y_0, \dot{I}_1, \dot{I}_2, _{g}\dot{I}, _{b}\dot{I}, \dot{I}_0, I_0, \theta_0$.

$$g_1 = \frac{r_1}{r_1^2 + {}_{\iota}x_1^2} = 0.10 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_{\iota}x_2^2} = 0.0344 \text{ mho.}$$

$$_cb_1 = \frac{_cx_1}{r_1^2 + {_cx_1}^2} = 0.0750 \text{ mho.}$$

$$_cb_2 = \frac{_cx_2}{r_2^2 + {_cx_2}^2} = 0.1148 \text{ mho}$$

$g_0 = g_1 + g_2 = 0.1344$ mho.
$b_0 = {_cb_1} + {_cb_2} = 0.1898$ mho.
$y_0 = g_0 - jb_0 = 0.1344 - j0.1898$ mho.
$\dot{I}_1 = (g_1 - j_cb_1)\dot{E} = (0.10 - j0.075)\dot{E}$ amperes.
$\dot{I}_2 = (g_2 - j_cb_2)\dot{E} = (0.0344 - j0.1148)\dot{E}$ amperes.
$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 = (g_0 - jb_0)\dot{E} = (0.1344 - j0.1898)\dot{E}$ amperes.
${_gI} = g_0E = 13.44$ amperes.

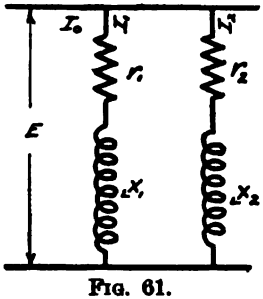



FIG. 61.

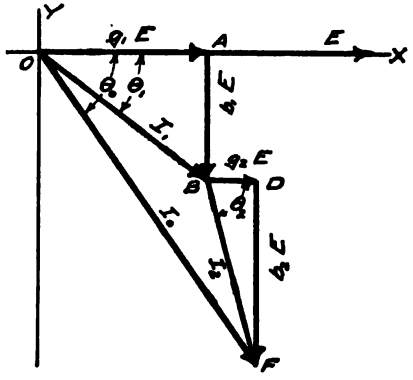



FIG. 62.

$${_bI} = b_0\dot{E} = 18.98 \text{ amperes.}$$

$$I_0 = \sqrt{{_gI^2} + {_bI^2}} = E\sqrt{g_0^2 + b_0^2} = 23.25 \text{ amperes.}$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 54° 40'.$$

**Problems 18.**—Given the same circuit as in problem 17 and let $E = 100$ volts and $f = 25$ cycles.

Find: $g_0$, $b_0$, $y_0$, ${_g\dot{I}}$, ${_b\dot{I}}$, $\dot{I}_0$, $I_0$, and $\theta_0$. Draw the vector diagram.

**B. Resistance and Condensance.**

**Problem 19.**—Circuit diagram, Fig. 63. Vector diagram, Fig. 64.

Given: $g_1 = 0.32$; ${_cb_1} = 0.00$; $g_2 = 0.00$; ${_cb_2} = 0.21$ mho; $E = 100$ volts.

Find: $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_0$, $I_0$, and $\theta_0$.

$\dot{I}_1 = (g_1 + j_cb_1)\dot{E} = 0.32E$ amperes.
$\dot{I}_2 = (g_2 + j_cb_2)\dot{E} = j0.21E$ amperes.

$$\dot{I}_0 = \dot{I}_1 + j\dot{I}_2 = (0.32 + j0.21)\dot{E} \text{ amperes.}$$

$$I_0 = E\sqrt{0.32^2 + 0.21^2} = 38.25 \text{ amperes.}$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 33^\circ\ 17'.$$

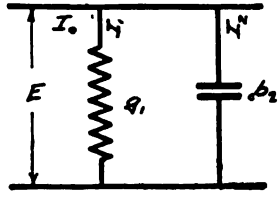



FIG. 63.

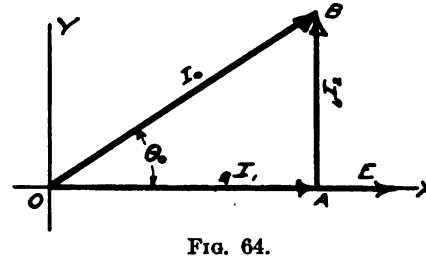



FIG. 64.

**Problem 20.**—Circuit diagram, Fig. 65. Vector diagram, Fig. 66.

Given: $r_1 = 9.0$; $r_2 = 12.0$; $_cx_1 = 15.0$; $_cx_2 = 10.0$ ohms; $E = 100$ volts; $f = 25$ cycles.

Find: $g_1$, $g_2$, $g_0$, $_cb_1$, $_cb_2$, $b_0$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, $_g\dot{I}$, $_b\dot{I}$, $\dot{I}_0$, $I_0$, $\theta_1$, $\theta_2$, and $\theta_0$.

$$g_1 = \frac{r_1}{r_1^2 + {}_cx_1^2} = 0.0294 \text{ mho.}$$

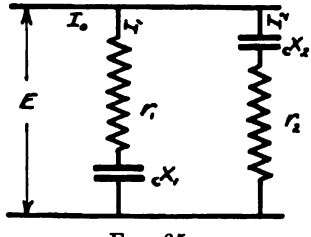



FIG. 65.

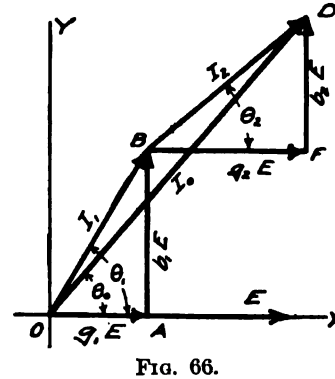



FIG. 66.

$$_cb_1 = \frac{{}_cx_1}{r_1^2 + {}_cx_1^2} = 0.0490 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_cx_2^2} = 0.0492 \text{ mho.}$$

$$_cb_2 = \frac{{}_cx_2}{r_2^2 + {}_cx_2^2} = 0.0410 \text{ mho.}$$

$g_0 = g_1 + g_2 = 0.0786$ mho.

$b_0 = {}_cb_1 + {}_cb_2 = 0.0900$ mho.

$y_0 = g_0 + jb_0 = 0.0786 + j0.0900$ mho.

$\dot{I}_1 = (g_1 + j_cb_1)\dot{E} = (0.0294 + j0.0490)\dot{E}$ amperes.

$\dot{I}_2 = (g_2 + j_cb_2)\dot{E} = (0.0492 + j0.0410)\dot{E}$ amperes.

${}_g\dot{I} = g_0\dot{E} = 0.0786E$ amperes.

${}_b\dot{I} = jb_0\dot{E} = j0.0900E$ amperes.

$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = E\sqrt{g_0^2 + b_0^2} = 11.95$ amperes.

$\theta_1 = \tan^{-1}\frac{{}_cb_1}{g_1} = 59°\ 1'.$

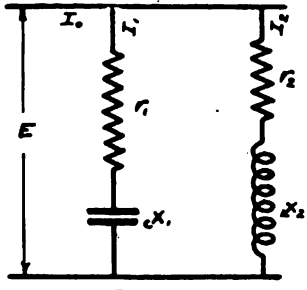



FIG. 67.

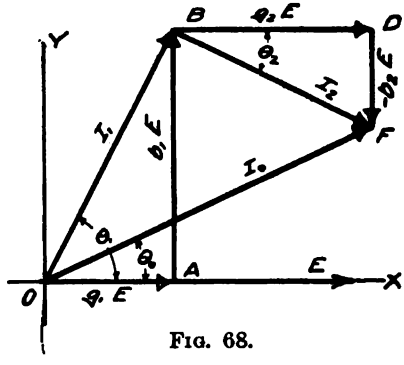



FIG. 68.

$\theta_2 = \tan^{-1}\frac{{}_cb_2}{g_2} = 39°\ 52'.$

$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 48°\ 52'.$

**Problem 21.**—Given the same circuit as in problem 20 with $E = 100$ volts and $f = 60$ cycles.

Find: $g_1, g_2, g_0, {}_cb_1, {}_cb_2, b_0, y_0, \dot{I}_1, \dot{I}_2, {}_g\dot{I}, {}_b\dot{I}, \dot{I}_0, I_0, \theta_1, \theta_2, \theta_0$. Draw the vector diagram.

### C. Resistance, Inductance and Condensance.

**Problem 22.**—Circuit diagram, Fig. 67. Vector diagram, Fig. 68.

Given: $r_1 = 3.4$; $r_2 = 8.5$; ${}_cx_1 = 6.6$; ${}_lx_2 = 4.1$ ohms; $E = 100$ volts; $f = 50$ cycles.

Find: $g_0, b_0, y_0, \dot{I}_1, \dot{I}_2, {}_g\dot{I}, {}_b\dot{I}, \dot{I}_0, I_0, \theta_1, \theta_2$ and $\theta_0$.

$$g_0 = g_1 + g_2 = \frac{r_1}{r_1^2 + {}_cx_1^2} + \frac{r_2}{r_2^2 + {}_lx_2^2} = 0.062 + 0.096 = 0.158$$

mho.

$$b_0 = {}_cb_1 - {}_lb_2 = \frac{{}_cx_1}{r_1^2 + {}_cx_1^2} - \frac{{}_lx_2}{r_2^2 + {}_lx_2^2}$$

$$= 0.120 - 0.046 = 0.074 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.158 + j0.074 \text{ mho.}$$

$$\dot{I}_1 = (g_1 + j_cb_1)\dot{E} = (0.062 + j0.120)\dot{E} \text{ amperes.}$$

$$\dot{I}_2 = (g_2 - j_lb_2)\dot{E} = (0.096 - j0.046)\dot{E} \text{ amperes.}$$

$${}_g\dot{I} = g_0\dot{E} = 0.158\dot{E} = 15.8 \text{ amperes.}$$

$${}_b\dot{I} = jb_0\dot{E} = j0.074\dot{E} = j7.4 \text{ amperes.}$$

$$\dot{I}_0 = (g_0 + jb_0)\dot{E} = (0.158 + j0.074)\dot{E} \text{ amperes.}$$

$$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = (g_0^2 + b_0^2)^{1/2}E = 17.5 \text{ amperes.}$$

$$\theta_1 = \tan^{-1}\frac{{}_cb_1}{g_1} = 62^\circ\ 40'; \quad \theta_2 = \tan^{-1}\frac{{}_lb_2}{g_2} = -25^\circ\ 39';$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 25^\circ\ 8'.$$

**Problem 23.**—Given the same circuit as in problem 22, with $E = 100$ volts and $f = 40$ cycles.

Find: $g_0$, $b_0$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, ${}_b\dot{I}$, ${}_g\dot{I}$, $\dot{I}_0$, $I_0$, $\theta_1$, $\theta_2$, and $\theta_0$. Draw the vector diagram.

**Problem 24.**—Circuit diagram, Fig. 67. Vector diagram, similar to Fig. 68.

Given: $r_1 = 0.1$; $r_2 = 0.3$; ${}_cx_1 = 8.4$; ${}_lx_2 = 5.4$ ohms. $E = 100$ volts; $f = 60$ cycles.

Find: $g_0$, $b_0$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, ${}_g\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, $I_0$, $\theta_1$, $\theta_2$, $\theta_0$.

$$g_0 = g_1 + g_2 = \frac{r_1}{r_1^2 + {}_cx_1^2} + \frac{r_2}{r_2^2 + {}_lx_2^2} = 0.0014 + 0.0102 =$$

$$0.0116 \text{ mho.}$$

$$b_0 = {}_cb_1 - {}_lb_2 = \frac{{}_cx_1}{r_1^2 + {}_cx_1^2} - \frac{{}_lx_2}{r_2^2 + {}_lx_2^2} = 0.1190 - 0.1844 =$$

$$-0.0654 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.0116 - j0.0654 \text{ mho.}$$

$$\dot{I}_1 = (g_1 + j_cb_1)\dot{E} = (0.0014 + j0.1190)\dot{E} \text{ amperes.}$$

$$\dot{I}_2 = (g_2 - j_lb_2)\dot{E} = (0.0102 - j0.1844)\dot{E} \text{ amperes.}$$

$${}_g\dot{I} = g_0\dot{E} = 0.0116\dot{E} = 1.16 \text{ amperes.}$$

$${}_b\dot{I} = jb_0\dot{E} = -j0.0654\dot{E} = j6.54 \text{ amperes.}$$

$$\dot{I}_0 = (g_0 + jb_0)\dot{E} = (0.0116 - j0.0654)\dot{E} \text{ amperes.}$$

$$I_0 = \sqrt{{}_gI^2 + {}_bI^2} = (g_0^2 + b_0^2)^{1/2}E = 6.64 \text{ amperes.}$$

$$\theta_1 = \tan^{-1}\frac{{}_cb_1}{g_1} = 89^\circ\ 19'; \quad \theta_2 = \tan^{-1}\frac{{}_lb_2}{g_2} = -86^\circ\ 35'.$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = -79^\circ\ 45'.$$

**Problem 25.**—Given the same circuit as in problem 24 with $E = 100$ volts and $f = 25$ cycles.

Find: $g_0$, $b_c$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, ${}_c\dot{I}$, ${}_b\dot{I}$, $\dot{I}_0$, $I_0$, $\theta_1$, $\theta_2$, $\theta_0$. Draw vector diagrams.

**Problem 26.**—Circuit diagram, Fig. 69. Vector diagram, Fig. 70.

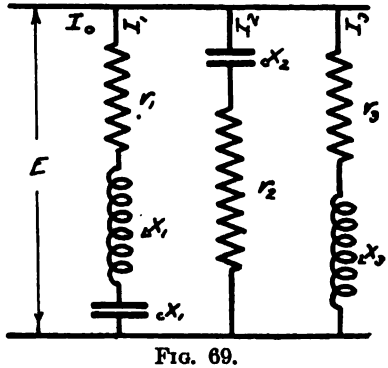



FIG. 69.

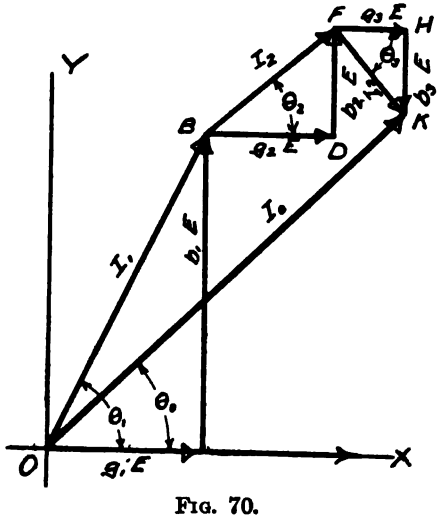



FIG. 70.

Given: $r_1 = 1.5$; $r_2 = 5.6$; $r_3 = 7.0$; ${}_bx_1 = 3.4$; ${}_cx_1 = 6.5$; ${}_cx_2 = 4.7$; ${}_bx_3 = 8.2$ ohms; $E = 100$ volts; $f = 60$ cycles.

Find: $g_0$, $b_0$, $y_0$, $\dot{I}_0$, $I_0$, $\theta_0$.

$$g_0 = g_1 + g_2 + g_3 = \frac{r_1}{r_1^2 + ({}_bx_1 - {}_cx_1)^2} + \frac{r_2}{r_2^2 + {}_cx_2^2} + \frac{r_3}{r_3^2 + {}_bx_3^2}$$

$$= 0.1264 + 0.1045 + 0.0601 = 0.2910 \text{ mho.}$$

$$b_0 = b_1 + b_2 + b_3 = \frac{-({}_{\iota}x_1 - {}_{c}x_1)}{r_1^2 + ({}_{\iota}x_1 - {}_{c}x_1)^2} + \frac{{}_{c}x_2}{r_2^2 + {}_{c}x_2^2} - \frac{{}_{\iota}x_3}{r_3^2 + {}_{\iota}x_3^2}$$

$$= 0.260 + 0.088 - 0.0705 = 0.2775 \text{ mho.}$$

$$y_0 = g_0 + jb_0 = 0.2910 + j0.2775 \text{ mho.}$$

$$\dot{I}_0 = y_0\dot{E} = (0.2910 + j0.2775)\dot{E} \text{ amperes.}$$

$$I_0 = Y_0E = (g_0^2 + b_0^2)^{\frac{1}{2}}E = 40.2 \text{ amperes.}$$

$$\theta_0 = \tan^{-1}\frac{b_0}{g_0} = 43° 38'.$$

**Problem 27.**—In the same circuit as in problem 26, let the applied voltage be 100 volts and the frequency 25 cycles.

Find: $g_0$, $b_0$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$, ${}_{c}I$, ${}_{b}I$, $\dot{I}_0$, $I_0$, $\theta_0$, $\theta_1$, $\theta_2$, $\theta_3$. Draw the vector diagram.

**D. Resonance.**[1]—In series circuits, Fig. 58, under resonance conditions the voltage across either the inductance or the condensance may be greater than the total voltage impressed on the circuit. Similarly in parallel circuits the current in either circuit as $I_1$ or $I_2$ in Fig. 67, may be greater than their resultant $I_0$. To produce a marked difference, $r_1$ and $r_2$ must be small in comparison to ${}_{\iota}x_1$ and ${}_{c}x_2$, and hence $g_1$ and $g_2$ must be small in comparison to ${}_{\iota}b_1$ and ${}_{c}b_2$. The system is in resonance at the frequency for which the total admittance, $y_1 + y_2$, is a minimum. For values of $r_1$ and $r_2$ for which ${}_{\iota}g_1$ and ${}_{c}g_2$ are negligibly small in comparison to ${}_{\iota}b_1$ and ${}_{c}b_2$ resonance exists when ${}_{\iota}b_1 + {}_{c}b_2 = 0$ or for,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{138}$$

the same frequency as for the corresponding series circuit.

**Problem 28.**—Find the frequency for resonance in circuit similar to Fig. 67 with $r_1 = r_2 = 0.1$ ohm; ${}_{\iota}x_1 = 17.2$ ohm; ${}_{c}x_2 = 12.7$ ohm; $f = 60$ cycles. Under resonance conditions find: $g_0$, $b_0$, $y_0$, $\dot{I}_1$, $\dot{I}_2$, ${}_{c}I$, ${}_{b}I$, $\dot{I}_0$, $I_0$, $\theta_1$, $\theta_2$ and $\theta_0$. Draw vector diagram.

**Problem 29.**—For the circuit used in problem 28, plot curves in rectangular coördinates with frequencies from 20 to 80 as abscissæ and the corresponding values for $I_0$, $I_1$ and $I_2$ as ordinates.

[1] See Bull. No. 74, U. S. Bureau of Standards.

**E. Graphical Method for Parallel Circuits.**

**Problem 30.**—Given the same circuit as in problem 20.

The symbolic method is often tedious and the solution may be gained more easily graphically. Thus in problem 20, let the voltage be represented by the line $OX$ in Fig. 71. From $O$ draw the line $OA$ at an angle of $\theta^\circ_1$, above $OX$. $\theta_1 = \tan^{-1}\frac{\omega x_1}{r_1}$ $= 59^\circ\ 1'$. Then $OA$ is equal to $r_1 I_1$. $XA$ is equal to $\omega x_1 I_1$, and $\theta_1$ is the angle of lead. The current in this branch, $I_1$, is equal to $\frac{OA}{r_1} = \frac{AX}{X_1} = 5.7 = OA'$, and laid off from $O$ along $OA$. The current in the second branch, $I_2$, is found in a

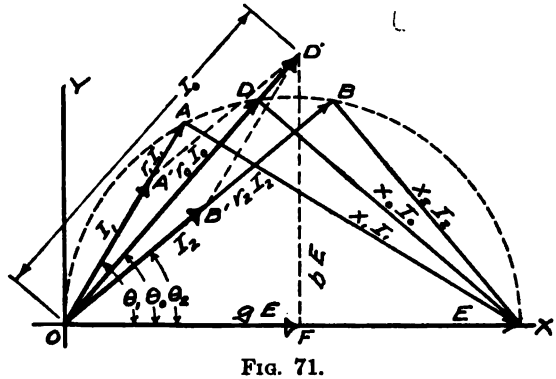



FIG. 71.

similar manner. From $O$ lay off the line $OB$ at an angle $\theta^\circ_2$, above $OX$. $\theta_2 = \tan^{-1}\frac{\omega x_2}{r_2} = 39^\circ\ 52'$. The current, $I_2$, is equal to $\frac{OB}{r_2} = 6.2 = OB'$ and laid off along $OB$.

The total current, $I_0$, is the vector sum of $I_1$ and $I_2$ and is represented in the diagram by the line $OD'$. The voltage consumed by the equivalent resistance of the two circuits in parallel is therefore represented by the line $OD$ and the voltage taken by the equivalent reactance of the two parallel circuits by the line $XD$.

The angle $XOD$ is the angle of lead for the total current and $= \theta_0 = 48^\circ\ 52'$.

By dropping a perpendicular line from $D'$ to $OX$, the current $I_0$ is separated into its components in parallel and in quadrature with the voltage. Thus $OF$ is equal to $_g I_0$, and $FD'$ to $_b I_0$.

**Problem 31.**—Solve problem 21 graphically.

**Problem 32.**—Solve problems 22, 23, 26 and 27 graphically.

**Problem 33.**—Find the frequency for resonance in problems 9, 12, 20 and 24.

## III. SERIES AND PARALLEL CIRCUITS

By the symbolic method as explained for series and parallel circuits any network of circuits can be solved. Whenever two or more circuits are in parallel the constants are expressed in terms of conductance and susceptance. By adding all the conductances the total conductance is obtained; and the algebraic sum of all the susceptances gives the total susceptance. Thus

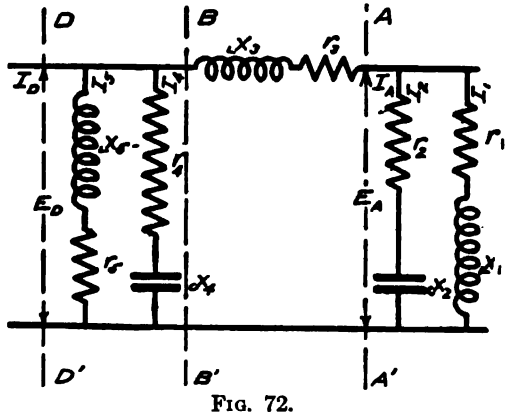



FIG. 72.

equivalent conductance, susceptance and admittance of the parallel parts may be obtained. In order to combine this equivalent circuit with quantities in series, the constants must be transformed into equivalent resistance and reactance. An example will make the method clear.

**Problem 34.**—Given the circuit diagram shown in Fig. 72, beginning at the right-hand end, that is, farthest away from the generator, there are two circuits in parallel, up to the dotted line $AA'$, with the constants:

$$r_1 = 3.0; {}_{\cdot}x_1 = 4.0; r_2 = 6.2 \text{ and } {}_{\cdot}x_2 = 3.3 \text{ ohms.}$$

To combine parallel circuits the constants must be expressed in terms of conductance and susceptance.

$$g_1 = \frac{r_1}{r_1^2 + {}_{\cdot}x_1^2} = 0.120 \text{ mho.}$$

$$_{L}b_1 = \frac{{}_{L}x_1}{r_1^2 + {}_{L}x_1^2} = 0.160 \text{ mho.}$$

$$g_2 = \frac{r_2}{r_2^2 + {}_{C}x_2^2} = 0.125 \text{ mho.}$$

$$_{C}b_2 = \frac{{}_{C}x_2}{r_2^2 + {}_{C}x_2^2} = 0.067 \text{ mho.}$$

$$g_A = g_1 + g_2 = 0.245 \text{ mho.}$$
$$b_A = {}_{C}b_2 - {}_{L}b_1 = 0.093 \text{ mho.}$$
$$y_A = g_A - jb_A = 0.245 - j0.093 \text{ mho.}$$

The equivalent resistance, $AA'$:

$$r_A = \frac{g_A}{g_A^2 + b_A^2} = 3.58 \text{ ohms.}$$

The equivalent reactance:

$$x_A = \frac{b_A}{g_A^2 + b_A^2} = 1.35 \text{ ohms.}$$

The equivalent impedance.

$$z_A = \frac{1}{g_A - jb_A} = \frac{g_A}{g_A^2 + b_A^2} + j\frac{b_A}{g_A^2 + b_A^2} = r_A + jx_A$$
$$= 3.58 + j1.35 \text{ ohms.}$$

Having derived expressions for the equivalent resistance and reactance of the two circuits on the right of $AA'$, this can now be combined with the part between $AA'$ and $BB'$. Let the value of the constants for this part be:

$$r_3 = 2.0 \text{ and } {}_{L}x_3 = 3.5 \text{ ohms.}$$

Therefore at the dotted line $BB'$ the combined resistance and reactance for circuits (1), (2) and (3) are

$$r_B = r_A + r_3 = 5.58 \text{ ohms.}$$
$$x_B = x_A + x_3 = 4.85 \text{ ohms.}$$

The next step will be the combination of the two parallel circuits (4) and (5) between $BB'$ and $DD'$ with the equivalent circuit just obtained from circuits (1), (2) and (3).

$$_{C}x_4 = 2.1 \text{ ohms.}$$
$$r_4 = 2.5 \text{ ohms.}$$
$$r_5 = 1.8 \text{ ohms.}$$
$$_{L}x_5 = 3.2 \text{ ohms.}$$

$z_B = r_B + jx_B = 5.58 + j4.85$ ohms.
$z_4 = r_4 - j_c x_4 = 2.5 - j2.1$ ohms.
$z_5 = r_5 + j_L x_5 = 1.8 + j3.2$ ohms.

This will give the desired solution for both magnitude and phase relations of the voltage and current at $DD'$. In order to combine these parallel circuits, the resistance and reactance of each circuit must be changed into conductance and susceptance.

$$g_B = \frac{r_B}{r_B^2 + x_B^2} = 0.1018 \text{ mho.}$$

$$b_B = \frac{x_B}{r_B^2 + x_B^2} = 0.0884 \text{ mho.}$$

$$g_4 = \frac{r_4}{r_4^2 + {}_c x_4^2} = 0.2345 \text{ mho.}$$

$$b_4 = \frac{{}_c x_4}{r_4^2 + {}_c x_4^2} = 0.1970 \text{ mho.}$$

$$g_5 = \frac{r_5}{r_5^2 + {}_L x_5^2} = 0.1335 \text{ mho.}$$

$$b_5 = \frac{{}_L x_5}{r_5^2 + {}_L x_5^2} = 0.2377 \text{ mho.}$$

The total conductance at $DD'$ is the sum of the conductances in the three parallel circuits.

$$g_D = g_B + g_4 + g_5 = 0.1018 + 0.2345 + 0.1335 = 0.470 \text{ mho.}$$

The total susceptance is equal to the algebraic sum of the susceptances in the three parallel circuits.

$$b_D = b_B - b_4 + b_5 = 0.088 - 0.197 + 0.237 = 0.129 \text{ mho.}$$

Hence the total admittance of the circuit at $DD'$:

$$y_D = g_D - jb_D = 0.470 - j0.129 \text{ mho.}$$
$$Y_D = \sqrt{g_D^2 + b_D^2} = 0.488 \text{ mho}$$
$$\theta_D = \tan^{-1} \frac{b_D}{g_D} = 15° 20'.$$

Having determined the equivalent conductance and susceptance for the parallel circuits and the resistance and reactance for the series circuits, the value of the currents in any part of the circuit may be found for any assumed voltage; or conversely, if the current is measured in any part of the circuit, voltage and current may be calculated for any part of the network.

**Problem 35.**—A graphical solution of problem 34 is given in Fig. 73. Along $OX$ lay off any convenient length $OB$ to represent the voltage ($E_1 = 100$ volts) at the line $AA'$, Fig. 72. With $OB$ as a diameter describe a circle. Draw the line $r_1I_1$ through $O$, making an angle of $\theta^\circ_1$ below the line $OX$. $\theta_1 = \tan^{-1}\frac{x_1}{r_1} =$

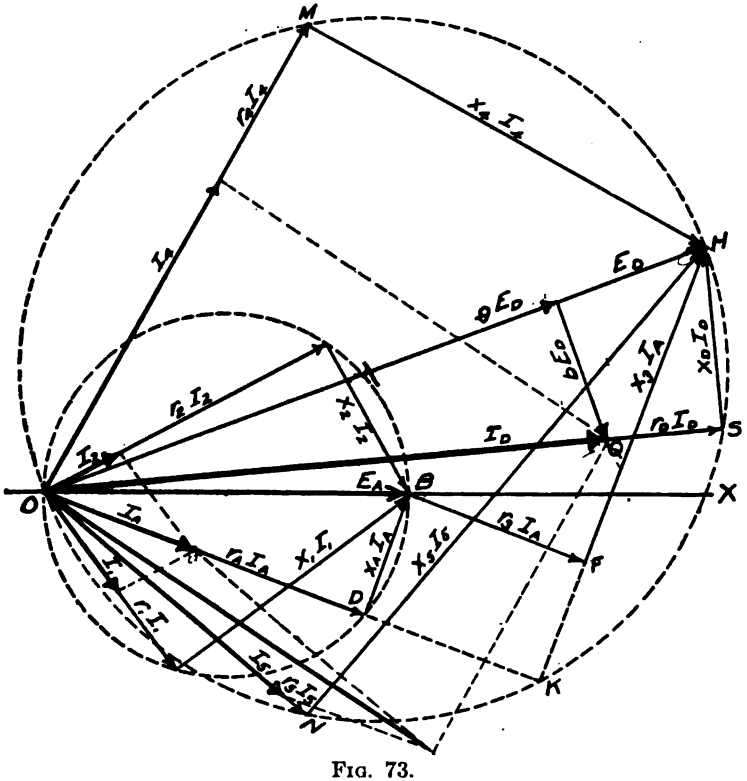



FIG. 73.

53° 8′. The voltage consumed by the reactance $x_1$ is therefore represented by the line $x_1I_1$. The current $I_1$ lies along the line $r_1I_1$ and is equal to 20 amp. Similarly, lay off the line $r_2I_2$ through the origin, at an angle of $\theta^\circ_2$ above the line $OX$. $\theta_2 = \tan^{-1}\frac{x_2}{r_2} = 28°$. The voltage consumed by the reactance $x_2$ is therefore represented by the line $x_2I_2$. The current $I_2$ lies along $r_2I_2$ and is equal to 14.2 amp. Combining $I_1$ and $I_2$ vectorially, we have $I_A$ and the line continued to the circum-

ference of the circle $OD$ gives $r_A I_A$, the voltage consumed by the equivalent resistance $r_A$ of the two circuits. The line $x_A I_A$ represents the magnitude and phase position of the voltage consumed by the equivalent reactance $x_A$ of the two circuits. The voltage triangle $ODB$ and the current vector $I_A$ therefore represent the magnitude and phase relations of a circuit equivalent to the two parallel circuits on the right of the dotted line $AA'$.

The second step will be to combine this equivalent circuit with $r_3$ and ${}_z x_3$ in series.

From $B$ draw the line $BF$ parallel to $OD(r_A I_A)$ and equal to $r_3 I_A$. From $F$ draw the line $FH$ parallel to $DB(x_A I_A)$ and equal to $x_3 I_A$. Draw the line $OH$ and continue $OD$ until it intersects the line $FH$ at $K$. Then the voltage triangle $OKH$ and the current vector $I_A$ represent the magnitude and phase relations of the current and voltage at the dotted line $BB'$. The third step is the combination of the equivalent circuit with the parallel circuits (4) and (5); all three having the same voltage impressed, $OH$.

Upon $OH$ as a diameter draw a circle. Through the origin, $O$, draw $OM$ at an angle of $\theta^\circ_4$ above the diameter $OH$. Then $OM$ is equal to $r_4 I_4$. $\theta_4 = \tan^{-1}\frac{x_4}{r_4} = 40^\circ\, 2'$. Also the line $HM$ represents in magnitude and phase position $x_4 I_4$. The current $I_4$ lies along $r_4 I_4$ and is equal to $\frac{OM}{r_4} = 59.2$ amp.

Similarly draw the line $ON$ through $O$, at $\theta^\circ_5$ below the voltage vector $OH$. Then $ON$ is equal to $r_5 I_5$ and $HN$ is equal to $x_5 I_5$; $\theta_5 = \tan^{-1}\frac{x_5}{r_5} = 60^\circ\, 39'$. The current vector $I_5$ lies along the line $r_5 I_5$ and is equal to $\frac{ON}{r_5} = 52.2$ amp. Combining the currents $\dot{I}_A$, $\dot{I}_4$ and $\dot{I}_5$ vectorially, we have the line $OQ$ representing in magnitude and phase position the total current with reference to the vector $OH$, the vector for the total voltage. Extending the line $OQ$ until it intersects the circle at $S$ we have the voltage triangle, $OSH$, in which $OS = r_D I_D$, $OH = z_D I_D$ and $HS = x_D I_D$.

The equivalent circuit for the whole network in Fig. 72 is therefore represented by the current vector $I_0$ and the voltage triangle $OSH$.

**Problem 36.**—Analytical solution of the network in problem 34 for double the frequency.

**Problem 37.**—Graphical solution of problem 36.

**Problem 38.**—Analytical solution of network in problem 34 for half the frequency.

**Problem 39.**—Graphical solution of problem 38.

## IV. VARIABLE RESISTANCE OR REACTANCE. CURRENT LOCI

**A. Constant Resistance and Variable Reactance.**—Let a constant voltage be impressed upon a circuit having a constant

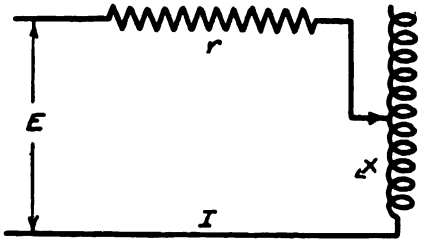



FIG. 74.

resistance and a reactance that can be varied as shown in the circuit diagram, Fig. 74. The corresponding vector diagram and current locus is shown in Fig. 75. Let the line *OA* along

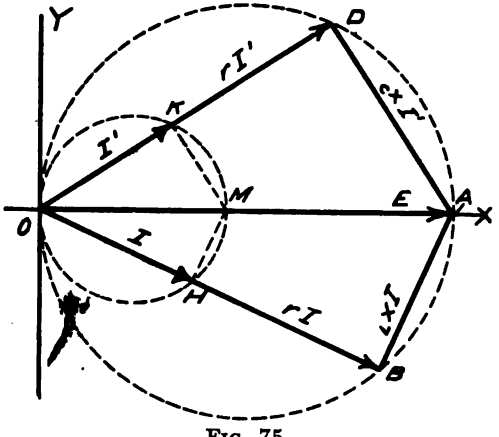



FIG. 75.

the *X*-axis represent the impressed voltage, *E*; and the line *OM* the current when only the resistance, *r*, is in the circuit. On *OA* and *OM* as diameters describe circles. With an inductive reactance *x* in series with the constant resistance, the

voltages consumed by the resistance and reactance respectively are represented by the lines $OB$ and $BA$, and the current by $OH$. By varying the reactance from zero to infinity the point $B$ describes the semicircle $ABO$. The point $H$ describes a similar arc $MHO$ since $OAB$ and $OMH$ are similar triangles.

If the inductive reactance be replaced by a condensive reactance ${}_{c}x$, then the lines $OD$ and $DA$ represent the resistance and reactance drops, and $OK$ the current $I'$. If the condensive reactance be varied from zero to infinity, the point $D$ will describe the semicircle $ADO$. Likewise the point $K$ will describe the semicircle $MKO$ since the triangles $OAD$ and $OMK$ are similar. The circle $OHMK$ is therefore the locus of the current in a circuit having constant resistance, variable reactance and constant voltage impressed on the terminals.

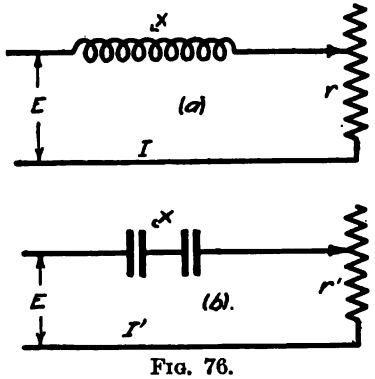



FIG. 76.

**B. Constant Reactance and Variable Resistance.**—The circuit diagrams for inductive or condensive reactances are shown in ($a$) and ($b$), Fig. 76. Let the impressed voltage, $E$, be constant.

In Fig. 77 lay off $OA$ to represent $E$ and let $OF$ represent the current flowing through ${}_{l}x$ with no resistance in series. Let ${}_{l}x$ in ($a$) be equal to ${}_{c}x$ in ($b$), and hence $OM$ equal to $OF$ will represent the current flowing through $x$ in ($b$) when the resistance in series is equal to zero.

Upon $OA$ as a diameter describe the circle $OBAD$, and upon $OM$ and $OF$ describe semicircles as shown in Fig. 77.

With a resistance $r$ in series with ${}_{l}x$ the lines $OB$ and $AB$ represent the voltages consumed by $r$ and ${}_{l}x$ respectively. The current $I$ is represented by the line $OH$ since the triangles $OAB$ and $FOH$ are similar. By varying the resistance from zero to

infinity the point $B$ describes the semicircle $OBA$ and the point $H$ the semicircle $FHO$. With a resistance $r'$ in series with the condensive reactance $_cx$ in ($b$), the voltage consumed by the resistance is represented by the vector $OD$, and that consumed by the reactance, by the line $DA$. The current $I'$ is represented by the vector $OK$ since the triangles $MKO$ and $ODA$ are similar. By varying the resistance in series with $_cx$ in ($b$) from zero to infinity the point $D$ follows the semicircle $ODA$, and the point $K$ the semicircle $MKO$. The locus of the current is therefore the semicircle $FHO$ for the inductive reactance $_ix$ and $MKO$ for the condensive reactance $_cx$.

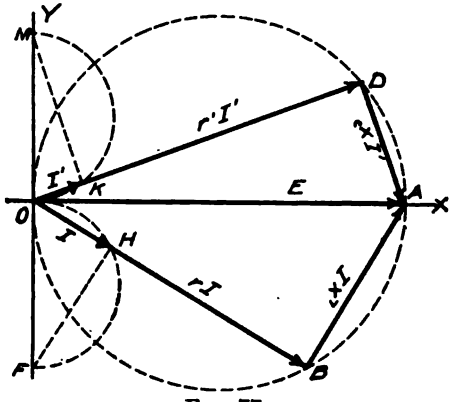



FIG. 77.

## V. RESISTANCE IN SERIES WITH A CIRCUIT

The circuit diagram in Fig. 78 is typical of many simple commercial circuits; a receiver circuit having variable load and power factor, in series with a resistance. Let the generator voltage, $E_0$, the series resistance, $r_2$, and the receiver impedance $z_1$, be constant, but the receiver resistance, $r_1$, and reactance, $x_1$, be variable.

$$z_1 = r_1 + jx_1;\ Z_1^2 = r_1^2 + x_1^2 \tag{136}$$

$$z_0 = r_1 + r_2 + jx_1$$

$$Z_0^2 = (r_1 + r_2)^2 + x_1^2 \tag{140}$$

Hence the current is:

$$I = \frac{E_0}{z_0} = \frac{E_0}{r_1 + r_2 + jx_1} = \frac{E_0(r_1 + r_2 - jx_1)}{(r_1 + r_2)^2 + x_1^2} \tag{141}$$

The voltage at the receiver circuit:

$$\dot{E}_1 = \dot{I}z_1 = \frac{E_0\,(r_1 + jx_1)}{r_1 + r_2 + jx_1} = \frac{E_0(r_1 + jx_1)(r_1 + r_2 - jx_1)}{(r_1 + r_2)^2 + x_1^2}$$

$$= \frac{E_0(z_1^2 + r_1r_2 + jr_2x_1)}{z_1^2 + 2r_1r_2 + r_2^2} \tag{142}$$

In absolute values:

$$I = \frac{E_0}{\sqrt{(r_1 + r_2)^2 + x_1^2}} = \frac{E_0}{\sqrt{Z_1^2 + 2r_1r_2 + r_2^2}} \tag{143}$$

$$E_1 = \frac{E_0\sqrt{r_1^2 + x_1^2}}{\sqrt{(r_1 + r_2)^2 + x_1^2}} = \frac{E_0Z_1}{\sqrt{Z_1^2 + 2r_1r_2 + r_2^2}} \tag{144}$$

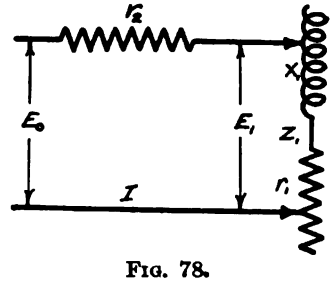



FIG. 78.

Current and voltage phase difference at the receiver circuit terminals:

$$\theta_1 = \tan^{-1}\frac{x_1}{r_1} \tag{145}$$

and at the generator:

$$\theta_0 = \tan^{-1}\frac{x_1}{r_1 + r_2} \tag{146}$$

In Fig. 79 is shown the vector diagram for the values:
$E_0 = 100$ volts; $r_2 = 0.5$ ohm; $r_1 = 0.8$ ohm;
$x_1 = 0.6$ ohm; and hence $z_1 = 1$.
The current vector is taken along the $X$-axis.

The voltage vector $E_0$ is at an angle $\theta_0 = \tan^{-1}\frac{x_1}{r_1 + r_2}$ from the current and drawn to any convenient scale. The vector for the receiver voltage, $E_1$, is at an angle of $\theta_1 = \tan^{-1}\frac{x_1}{r_1}$ from the current, and the magnitude may be calculated from the given data. The voltages consumed by the resistance, $r_1I$ and

$r_2I$ are in phase with the current while the voltage taken by the reactance $x_1I$ is in quadrature with the current.

By keeping $E_0$, $r_2$ and $z_1$ constant, but varying $r_1$ from zero to unity, and $x_1$ from 1 ohm inductive reactance to zero, the locus of the voltage $E_0$ described by the point $A$ will be $MAQ$ a quadrant of a circle. If $x_1$ be changed from zero to 1 ohm condensive reactance with simultaneous variation in $r_1$ so as to keep $z_1$ constant, the locus of the voltage will be the semicircle $MAQF$. The corresponding positions of the point $B$ give the locus of the receiver voltage $E_1$ in the curve $HBNK$. The curve in the two quadrants is symmetrical and hence the value of the receiver voltage will be the same for any two equal inductive and condensive reactances. With the constant assumed ($Z_1 = 1$) the magnitude of the current in amperes is in each case numerically equal to $E_1$ in volts.

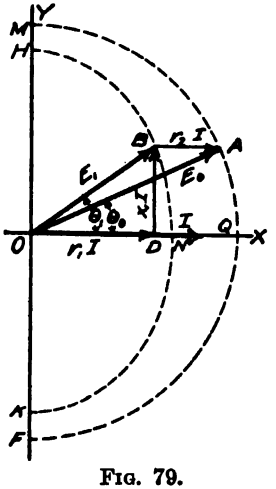



FIG. 79.

**Problem 40.**—Given $E_0 = 100$ volts; $r_2 = 0.8$ ohm; $Z_1 = 1$ ohm.

Let $x_1$ vary from $+1$ to $-1$ ohm. Plot in rectangular coördinates the values of $E_1$ and $I$ as ordinates with $x_1$ as abscissæ.

## VI. REACTANCE IN SERIES WITH A CIRCUIT

In a circuit having reactance instead of resistance in series, the relations of the current and receiver voltage to the constant generator voltage are derived in the same manner. The circuit diagram Fig. 80 shows an inductive reactance in series with an inductive receiver circuit. To make the discussion general the reactance in series may be taken either as inductive or as condensive and the impedance in the receiver circuit may consist of resistance and either inductive or condensive reactance. The receiver circuit impedance:

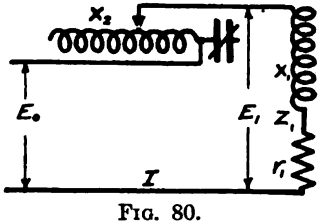



FIG. 80.

$$z_1 = r_1 + jx_1.$$

The total impedance:

$$z_0 = z_1 + jx_2 = r_1 + j(x_1 + x_2) \tag{147}$$

The current is:

$$\dot{I} = \frac{\dot{E}_0}{z_0} = \frac{\dot{E}_0}{r_1 + j(x_1 + x_2)} \tag{148}$$

In absolute values:

$$I = \frac{E_0}{\sqrt{r_1^2 + (x_1 + x_2)^2}} = \frac{E_0}{\sqrt{z_1^2 + 2x_1 x_2 + x_2^2}} \tag{149}$$

The receiver voltage:

$$\dot{E}_1 = \dot{I}z_1 = \dot{E}_0 \frac{r_1 + jx_1}{r_1 + j(x_1 + x_2)} \tag{150}$$

$$E_1 = E_0 \frac{\sqrt{r_1^2 + x_1^2}}{\sqrt{r_1^2 + (x_1 + x_2)^2}} = \frac{E_0 z_1}{\sqrt{z_1^2 + 2x_1 x_2 + x_2^2}} \tag{151}$$

$$\theta_1 = \tan^{-1} \frac{x_1}{r_1} \tag{152}$$

$$\theta_0 = \tan^{-1} \frac{x_1 + x_2}{r_1} \tag{153}$$

Since the reactances $x_1$ and $x_2$ may be either inductive or condensive, four combinations are possible:

($a$) $x_1$ inductive, $x_2$ inductive,
($b$) $x_1$ inductive, $x_2$ condensive,
($c$) $x_1$ condensive, $x_2$ inductive,
($d$) $x_1$ condensive, $x_2$ condensive.

From equations (149) and (151) it is evident that the numerical value depends on the square of the algebraic sum of $x_1$ and $x_2$. If both are inductive or both condensive the algebraic sum is the same as the arithmetical sum; but if one is inductive and the other condensive the algebraic sum is equal to the arithmetical difference of the two reactances. Hence for numerical or absolute values ($a$) and ($d$) are alike, and ($b$) and ($c$) will give similar results. The phase position of the current and voltage is, however, affected by the positive or negative sign before the reactance and these relations may be determined by equations (148), (149), (152) and (153).

If $x_1$ is inductive, varying $x_2$ from 1 ohm inductive to 1 ohm condensive reactance, with $E_0$ constant, it is seen from

equation (149) that the current will be a maximum when $x_1 - x_2 = 0$. Hence for maximum current $x_2$ must be inductive if $x_1$ is condensive and *vice versa*.

**Problem 41.**—Given $E_0 = 100$ volts; $z_1 = 1$ ohm; $x_1 = 0.4$ ohm. Circuit diagram as in Fig. 80. Plot in rectangular coördinates the values for $E_1$ and $I$ with $x_2$ varying from $+1$ to $-1$ ohm as abscissæ.

**Problem 42.**—Given $E_0 = 100$ volts: $z = 1$ ohm; $x_1 = 0.7$ ohm with circuit diagram as in Fig. 80. Plot curves for $E_1$ and $I$ with $x_2$ as abscissæ as in problem 41.

## VII. IMPEDANCE IN SERIES WITH A CIRCUIT

Regulation of the receiver voltage might be obtained by means of a reactance placed in series as discussed in the preceding paragraph. However, reactance coils always dissipate

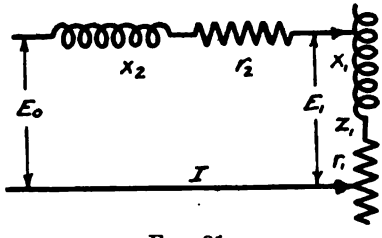



FIG. 81.

energy due to the ohmic resistance of the wire and the hysteresis in the iron, and hence an impedance is inserted instead of the desired reactance, as shown in Fig. 81.

$$z_1 = r_1 + jx_1;\ z_2 = r_2 + jx_2 \tag{154}$$

$$Z_1 = \sqrt{r_1^2 + x_1^2};\ Z_2 = \sqrt{r_2^2 + x_2^2} \tag{155}$$

The total impedance:

$$z_0 = r_1 + r_2 + j(x_1 + x_2) \tag{156}$$

$$Z_0 = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2} \tag{157}$$

The current:

$$\dot{I} = \frac{\dot{E}_0}{z_0} = \frac{\dot{E}_0}{(r_1 + r_2) + j(x_1 + x_2)} = \dot{E}_0 \frac{(r_1 + r_2) - j(x_1 + x_2)}{(r_1 + r_2)^2 + (x_1 + x_2)^2} \tag{158}$$

In absolute values:

$$I = \frac{E_0}{Z_0} = \frac{E_0}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}$$

$$= \frac{E_0}{\sqrt{(Z_1^2 + Z_2^2 + 2(r_1 r_2 + x_1 x_2)}} \tag{159}$$

The receiver voltage:

$$\dot{E}_1 = z_1 \dot{I} = \frac{\dot{E}_0 (r_1 + jx_1)}{r_1 + r_2 + j(x_1 + x_2)} \tag{160}$$

$$E_1 = Z_1 I = \frac{E_0 \sqrt{r_1^2 + x_1^2}}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}$$

$$= \frac{E_0 Z_1}{\sqrt{Z_1^2 + Z_2^2 + 2(r_1 r_2 + x_1 x_2)}} \tag{161}$$

The phase difference at the receiver circuit:

$$\theta_1 = \tan^{-1} \frac{x_1}{r_1} \tag{162}$$

The phase difference at the generator:

$$\theta_0 = \tan^{-1} \frac{x_1 + x_2}{r_1 + r_2} \tag{163}$$

As in the discussion of resistance and reactance in series let $E_0$, $r_2$, $x_2$, and $z_1$ be constant, but $x_1$ and $r_1$ be variable.

With resistance in series, $E_1$ is a minimum when $x_1 = 0$; and with the circuit having reactance in series the voltage at the receiver circuit is a minimum when $r_1 = 0$ and the reactances $x_1$ and $x_2$ are either both inductive or both condensive. With impedance in series, the receiver voltage is a minimum when $\theta_1$ is equal to and of the same sign as $\theta_2$, or $\theta_0$; that is, when $\frac{x_1}{r_1} = \frac{x_2}{r_2}$.

In equation (161) $E_1$ is a minimum when the denominator is a maximum. Since $Z_1$, $Z_2$, $r_2$ and $x_2$ are constants, the denominator will be a maximum when $r_1 r_2 + x_1 x_2$ is a maximum, or with a single variable $r_1$ when

$$\frac{d}{dr_1}(r_1 r_2 + x_2 \sqrt{z_1^2 - r_1^2}) = 0 \tag{164}$$

$$r_2 + \tfrac{1}{2}\left(\frac{x_2}{\sqrt{z_1^2 - r_1^2}}\right)(-2r_1) = 0 \tag{165}$$

or,

$$\frac{x_1}{r_1} = \frac{x_2}{r_2}$$

This topic is discussed more fully under Regulation in Chap. XXII.

**Problem 44.**—$E_0 = 100$ volts; $x_2 = 1$ ohm; $r_2 = 1$ ohm, with circuit diagram as in Fig. 81. $x_1 = kr_1$, varying from 0 to 5 ohms. Draw the loci of the current and voltages $E_1$ and $E_2$ for values of $k = 1$ and $k = 5$ when referred to $E_0$ as a reference vector.

## VIII. COMMERCIAL CIRCUITS

In the preceding problems the resistance and reactance of each elemental circuit have been given and methods for finding the resultant equivalent impedance or admittance have been explained. In commercial systems the circuits generally consist of complex networks, in which it may be difficult or impracticable to measure the resistance and reactance of each elemental division. The total values are found directly by taking voltmeter, ammeter and wattmeter readings, and from these data, the constants of the circuits may be calculated. As explained in Chap. VII the wattmeter measures directly the power expended, which may be expressed as $rI^2$.

$$r = \frac{\text{(wattmeter reading in watts)}}{\text{(ammeter reading in amperes)}^2} \text{ ohms} \qquad (166)$$

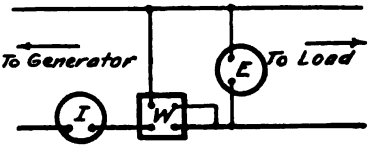



FIG. 82.

With $E$, $I$ and $r$ known, the numerical values of $x$, $z$, $g$, $b$, $y$ and $\theta$ may be calculated. Thus, by the simple expedient of taking voltmeter, ammeter and wattmeter readings the equivalent resistance, reactance and impedance may be obtained for circuits of any complexity. All that part of the system further away from the generator than the point at which the readings are taken will be included in the calculated values.

**Problem 45.**—Given a circuit as shown in Fig. 82. $E = 224$ volts; $I = 15.2$ amp.; $W = 2{,}126$ watts; inductive load.

Find: $r$, $x$, $z$, $g$, $b$, $y$, $Z$, $Y$ and $\theta$.

Draw the impedance triangle. Draw the vector diagram.

**Problem 46.**—Same data as for problem 45 except the reactance is condensive.

Find: $r$, $x$, $z$, $g$, $b$, $y$, $Z$, $Y$ and $\theta$.

Draw the impedance triangle. Draw the vector diagram.

For Experiments see page 529.

## CHAPTER VII

## ELECTRIC POWER, POWER FACTOR AND REACTIVE POWER

In direct currents the power expended in a circuit, measured in watts, is equal to the product of the volts by the amperes. In alternating currents the power in the circuit is at any instant equal to the product of the instantaneous current and voltage. In circuits having resistance only, the energy delivered to a circuit

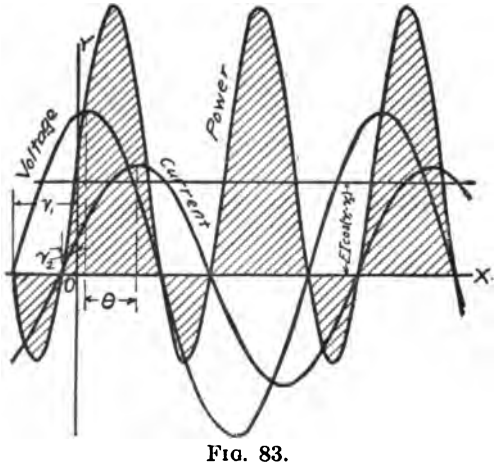



FIG. 83.

is immediately changed into heat, or light, and can not return to the generator; *i.e.*, the process is not reversible. Therefore, the average power delivered to a non-inductive load is the sum of the instantaneous powers over a cycle divided by the time for the cycle. In circuits having magnetic or dielectric induction, part of the energy is stored in the magnetic or dielectric (electrostatic) fields in such forms while the current or voltage is increasing, that, when the current or voltage decreases, the energy is returned to the electric circuit; that is, the process is automatically reversible. Consequently, only part of the energy flowing in an alternating-current system having inductance or condens-

ance is left in the circuit. For sine waves, as explained in Chap. I, energy is stored in the magnetic field while the current increases, and this energy returns to the circuit when the current decreases. Likewise energy is stored in the dielectric field while the voltage increases, and returns in the next quarter cycle of the voltage wave. The instantaneous value of the power in the circuit is at all times equal to the product of the corresponding instantaneous current and voltage. But the direction of energy flow for sine voltage and current waves will reverse four times in each cycle.

This is shown graphically in Fig. 83, for a current lagging $\theta°$ behind the voltage wave, and in Fig. 84 for a current leading the

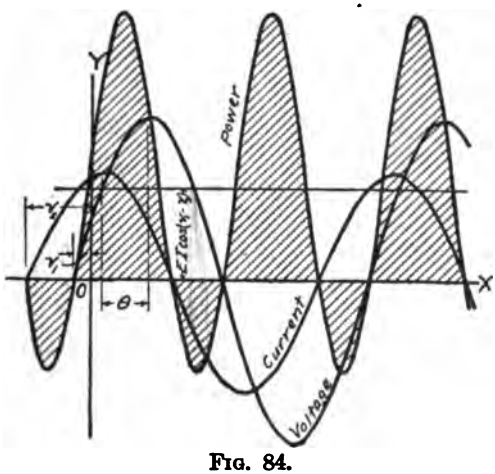



FIG. 84.

voltage by $\theta°$. The shaded areas indicate energy; the positive areas, the energy delivered by the generator; and the areas below the $X$-axis, the energy returned to the generator. The difference of these areas for one cycle is the total energy left in the circuit during one cycle. This quantity divided by the time for one cycle gives the average power delivered to the circuit.

The instantaneous value of the power, $p = ei$ (170)

For sine waves:

$$e = {}^{\prime\prime}E \sin (\omega t + \gamma_1) \tag{171}$$

$$i = {}^{\prime\prime}I \sin (\omega t + \gamma_2) \tag{172}$$

$$p = {}^{\prime\prime}E\,{}^{\prime\prime}I \sin (\omega t + \gamma_1) \sin (\omega t + \gamma_2) \tag{173}$$

From trigonometry:

$$\sin x \sin y = \tfrac{1}{2}\cos(x - y) - \tfrac{1}{2}\cos(x + y)$$

Let: $x = \omega t + \gamma_1$, and $y = \omega t + \gamma_2$

$$p = EI[\cos(\gamma_1 - \gamma_2) - \cos(2\omega t + \gamma_1 + \gamma_2)] \qquad (174)$$

From this it is seen that the instantaneous value of the power is also a sine function of the time, of double the frequency of the current or voltage, and with its reference axis displaced by $EI\cos(\gamma_1 - \gamma_2)$ from the $X$-axis.

The average value of the power during a complete cycle is:

$$P = \frac{1}{T}\int_o^T eidt \qquad (175)$$

$$= \frac{\omega}{2\pi} EI \int_o^{\frac{2\pi}{\omega}} [\cos(\gamma_1 - \gamma_2) - \cos(2\omega t + \gamma_1 + \gamma_2)]dt \qquad (176)$$

$$= EI\cos(\gamma_1 - \gamma_2) = EI\cos\theta \qquad (177)$$

Let $\theta = \gamma_1 - \gamma_2$ = phase angle between the current $I$ and voltage $E$.

The product $EI$ is called the apparent power and is often referred to as *volt-amperes* or *kilovolt-amperes* (kv.a.). Generators are usually rated in kv.a. as the heating depends on the current in the conductors and not on the power transmitted.

The factor $\cos\theta$ determines what part of the power in the system is delivered to the load. For this reason it is called the *Power Factor*. Thus multiplying the apparent power by the power factor gives the *true power* or the *power delivered* to the circuit. In a non-reactive circuit, Fig. 85, the current and voltage are in phase, and therefore $\gamma_1 = \gamma_2$ and $\theta = 0$. Hence, $\cos\theta = 1$, or the circuit has *Unity Power Factor*. Power factors less than unity are usually expressed in per cent. Thus 85 per cent. power factor means that $\cos\theta = 0.85$; and hence that the kw. = 85 per cent. of the kv.a. = 0.85 kv.a.

The shaded areas in Figs. 83 and 84 show how the energy flows to and fro when the current lags or leads the voltage. Also that the amount of power actually delivered to the circuit is proportional to the cosine of the angle of lead or lag. Evidently the more nearly $\cos\theta$ approaches unity the less energy will be surging to and fro in the line. When the current and voltage

are in phase, Fig. 85, $\cos \theta = 1$; and in this case, *and this case only*, the power is at no instant negative and hence all the energy

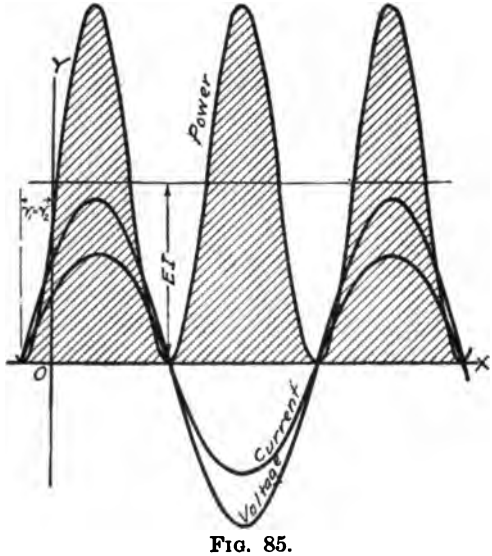



FIG. 85.

is delivered to the load, although it is transmitted from the generator in pulsations.

The greatest amount of surging will occur when the phase-angle difference of the current and voltage is 90°, for then

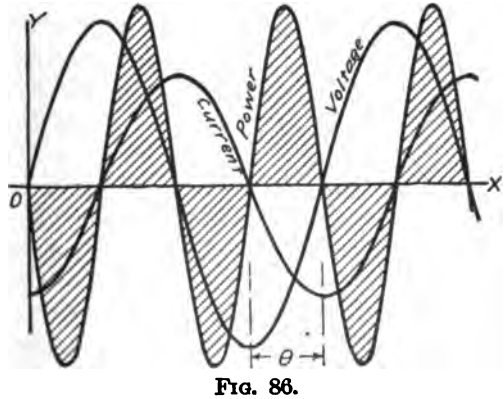



FIG. 86.

$\cos \theta = 0$ and hence all the power is reactive, or returns to the generator.

Under these conditions, Figs. 86 and 87, all of the energy surges to and fro and no actual transmission of power occurs. In this case the area above the $X$-axis of the power curve equals the area below the axis; or an amount of energy returns to the generator equal to that supplied by the generator to the line. While it is of first importance to determine the value of the power

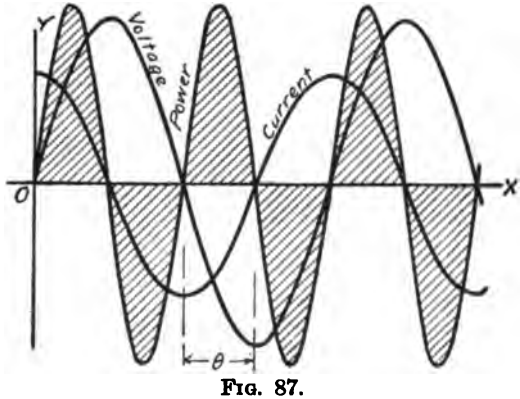



FIG. 87.

delivered in a circuit, it is often desirable to measure the reactive energy, or the part that surges to and fro in the line. A vector diagram of a current, $I$, leading a voltage, $E$, by $\theta°$ is shown in Fig. 88. If the current be divided into two components, in phase and in quadrature with the voltage, we have the vectors $OB = {}_{o}I = I \cos \theta$; $OD = {}_{b}I = I \sin \theta$. It is evident that the product of the voltage with the component in phase with the voltage will always be positive and hence gives the real or effective power. The component, $I \sin \theta$, is in quadrature with the voltage and hence during each cycle it will, half the time, be on the same side of the $X$-axis as the voltage vector and half the time on the opposite side. Hence the product $EI \sin \theta$ will be positive during half the cycle and negative during the other half. The average, therefore, is zero.

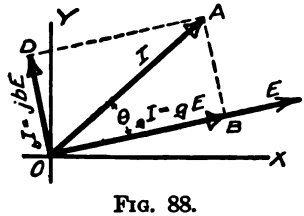



FIG. 88.

The two expressions give the quantitative values of the power in the circuit. The *real* or *effective power*, that is, the power

delivered to the circuit by the generator and which does not return to the generator is:

$$P = EI \cos \theta \tag{178}$$

The reactive power, or the power that surges to and fro between the generator and the line is:

$$_{r}P = EI \sin \theta \tag{179}$$

In making a graphical representation in Fig. 88 the current was divided into two components in phase and in quadrature with the voltage. The same result is gained by dividing the voltage into two components, in phase and in quadrature with the cur-

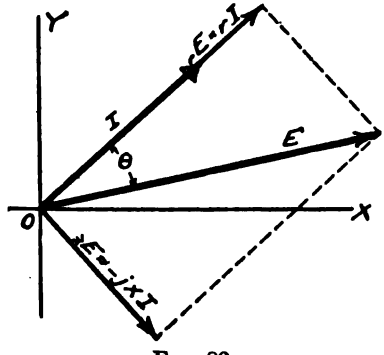



FIG. 89.

rent as shown in Fig. 89. For constant-voltage systems the current would naturally be considered as consisting of components in phase and in quadrature with the voltage as in Fig. 88. For similar reasons in constant-current circuits the voltage may be considered as made up of components in phase and in quadrature with the current, Fig. 89. Since by far the larger part of alternating-current distribution is by the constant-potential system it is customary to take the voltage as the reference vector and to speak of the *power component of the current*, the *active* or *energy current*, $I \cos \theta$; and of the *wattless, reactive* or *quadrature current*, $I \sin \theta$. For constant-current circuits the terms power component of the voltage and the quadrature component of the voltage would be appropriate. The term "wattless" is a misnomer as both the active and the quadrature components have power and therefore are not without watts.

The energy transfer under unity power-factor conditions at the

generator is readily shown by a power diagram. For illustration, take a system with two parallel circuits as shown in the circuit and vector diagrams in Fig. 90. In parallel circuits the voltage is the common factor and hence the currents should be divided into components in phase and in quadrature with the voltage.

$$\dot{I}_1 = g_1\dot{E} + j_c b_1\dot{E} \text{ and } \dot{I}_2 = g_2\dot{E} - j_l b_2\dot{E}$$

In rectangular coördinates with time as abscissæ, we have given in Fig. 91 the voltage, current and power curves for the several components of the currents $I_1$ and $I_2$ and the voltage $E$.

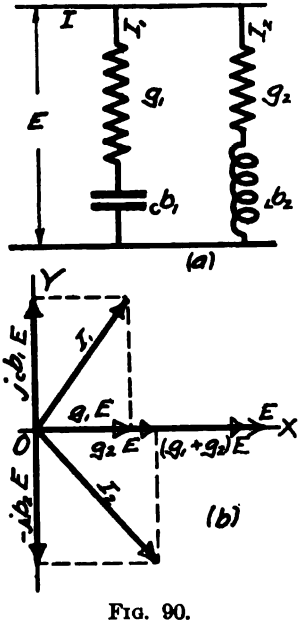



Fig. 90.

In (*a*) curves for $E$, $g_1E$ and the resultant power $g_1E^2$ or $P_1$.

In (*b*) curves for $E$, $g_2E$ and the resultant power $g_2E^2$ or $P_2$.

In (*c*) curves for $E$, ${}_cb_1E$ and the resultant power (reactive) ${}_cb_1E^2$ or ${}_rP_1$.

In (*d*) curves for $E$, $-{}_lb_2E$ and the resultant power (reactive) $-{}_lb_2E^2$ or ${}_rP_2$.

The power curves in both (*a*) and (*b*) are always positive; that is, the energy transmitted is absorbed by the resistance and changed into heat. In both (*c*) and (*d*) the successive half cycles of the power wave are of opposite sign and this shows that the flow of energy reverses, or the power goes in one direction while the wave is positive and in the opposite direction during the negative half of the wave. It is of importance to note that when the power ordinate is positive in (*c*), the corresponding ordinate in (*d*) is negative, and *vice versa*. The diagram therefore shows graphically the energy transfer from the condensance in circuit 1 to the inductance in circuit 2, or in the reverse direction, for the successive half cycles of the power wave. Under resonance conditions ${}_cb_1 = {}_lb_2$, or the corresponding instantaneous power ordinates in (*c*) and (*d*) are always equal.

This means that the energy stored dielectrically in circuit 1 when the voltage is a maximum is transferred and stored mag-

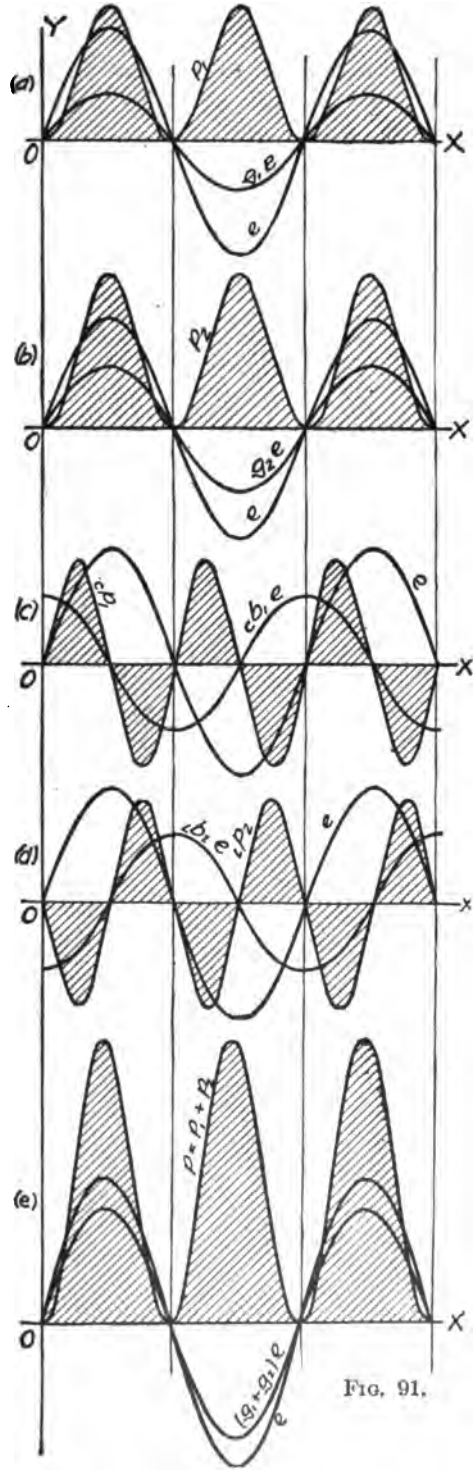

Y
(a)
$p_1$
0
X
$q_1 e$
e
(b)
$p_2$
0
X
$q_2 e$
e
(c)
$cp_2$
$cb_1 e$
e
0
X
(d)
$cb_1 e$
$cp_2$
e
0
X
(e)
$p = p_1 + p_2$
0
X
$(q_1 + q_2) e$
e

FIG. 91.

netically in circuit 2 while the line voltage decreases to its zero value, or in one-fourth cycle. During the next quarter cycle of the voltage wave the energy in the magnetic field of circuit 2 is returned and stored in the dielectric field of circuit 1.

In the mains coming from the generator the power transmitted is the sum of (*a*) and (*b*) as shown in (*e*). The current $I = g_1E + g_2E$ and is in phase with $E$. At the generator the power factor is unity.

## PROBLEMS

**1.** A single-phase induction motor delivers 8.7 hp. to the belt. Efficiency of motor, 91.5 per cent.; impressed voltage 240 volts; power factor, 87.5 per cent.

(*a*) Find the kv.a., the kw. or $P$, the reactive power $_zP$, and the current, $I$.

(*b*) Draw the vector diagram showing $E$, $I$, $\theta$, $gE$, $bE$, $rI$ and $xI$.

(*c*) In rectangular coördinates draw curves for $e$, $i$, $ei$, $ge$, $be$, $p$, $_zp$ and $p_0$.

**2.** Given a single-phase circuit (equivalent) having two motors operating in parallel, as shown in Fig. 92.

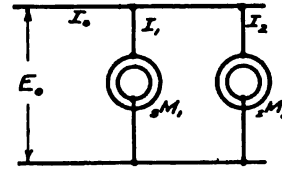



FIG. 92.

$_zM_2$ = an induction motor taking 7.0 kw. at 85 per cent. power factor (current lagging).

$_zM_4$ = an overexcited synchronous motor taking 8.2 kw. at 94 per cent. power factor (current leading).

$E_0 = 240$ volts; $f = 60$ cycles.

(*a*) Find $I_1$, $I_2$, $I_0$, $\theta_1$, $\theta_2$, $\theta_0$, $g_1E_0$, $b_1E_0$, $g_2E_0$, $b_2E_0$, $g_0E_0$ and $b_0E_0$.

(*b*) Draw vector diagram showing the quantities called for in (*a*).

**3.** Given the same data as for Problem 2:

(*a*) Find $P_1$, $_zP_1$, $P_2$, $_zP_2$, $P_0$, $_zP_0$, $(\text{kv.a.})_1$, $(\text{kv.a.})_2$, $(\text{kv.a.})_0$.

(*b*) Draw curves in rectangular coördinates (assuming simple sine wave forms) for $e_0$, $i_1$, $e_1i_1$, $g_1e_0$, $b_1e_0$, $p_1$, $_zp_1$, $i_2$, $e_2i_2$, $g_2e_0$, $b_2e_0$, $p_2$, $_zp_2$, $i_0$, $e_0i_0$, $g_0e_0$, $b_0e_0$, $p_0$ and $_zp_0$.

For Experiments see page 530.

## CHAPTER VIII

## INSTRUMENTS

To obtain quantitative data in alternating-current circuits, instruments of many types are in commercial use. While studying alternating-current phenomena it is essential that the principles upon which the instruments operate should be clearly understood. Instruments of the electrodynamometer type are simplest in principle and probably can be used to best advantage when studying the fundamental principles of alternating currents.

The most important instrument for gaining a clear insight into the nature of alternating currents is the oscillograph. Two types are in common use in this country, the Duddell and the three-element oscillograph made by the General Electric Co.

(*a*) **Duddell Oscillograph.**[1]—This instrument consists essentially of a modified moving-coil galvanometer combined with a rotating or vibrating mirror, a moving photographic film, or a falling photographic plate. The galvanometer portion often referred to as the oscillograph is shown diagrammatically in Fig. 93.

"In a narrow gap between the poles $N$, $S$, of a powerful magnet are stretched two parallel conductors, $s$, $s'$, formed by bending a thin strip of phosphor bronze back on itself over an ivory pulley, $P$. A spiral spring attached to this pulley serves to keep a uniform tension on the strips, and a guide piece, $G$, limits the length of the vibrating portion to the part actually in the magnetic field. A small mirror, $M$, bridges across the two strips as shown in the figure. The effect of passing a current through such a 'vibrator' is to cause one of the strips to advance while the other recedes, and the mirror is then turned about a vertical axis."

"The whole of the 'vibrator,' as this part of the instrument is called, is immersed in an oil-bath, the object of the oil being to damp the movement of the strips, and make the instrument dead-beat."

"The beam of light, coming from an arc lamp, and reflected from the mirror, $M$, is received on a photographic film; with constant strength of the magnetic field the instantaneous value of the current is proportional to the linear displacement of the spot of light focused on the

[1] DUDDELL, London *Electrician*, XXXIV, 636.

film. With alternating currents the spot of light oscillates to and fro as the current varies and with the film stationary would thus trace a straight line. To obtain an image of the wave form, the photographic film is moved in a direction at right angles to the direction of the movement of the spot of light. Or a second mirror can be interposed in the path of the beam of light, and this mirror caused to vibrate or rotate so as to impart to the beam of light a uniform motion about an axis at right angles to the zero position of the beam and also is in the initial plane of vibration. The spot of light will then trace out on a

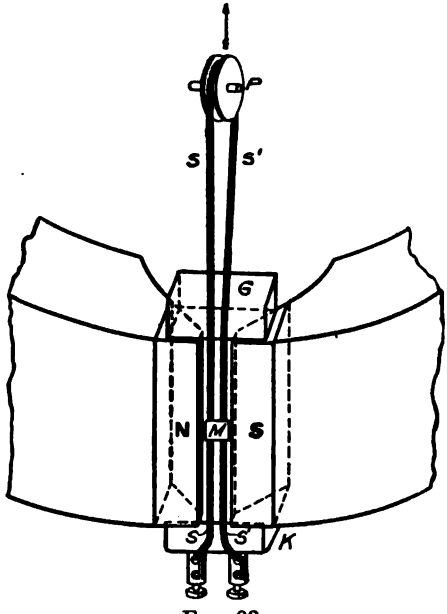



FIG. 93.

stationary screen or plate the time curve of the variation of the current flowing in the vibrator. If the vibrations are periodic, as in alternating currents, then the mirrors can be synchronized and the spot of light caused to trace out the wave form over and over again."

With two vibrators in the same field, arranged so as to throw the beam of light on the same photographic film, the instantaneous values of the current and the voltage may be obtained simultaneously. A circuit diagram of a two-element oscillograph for reading simultaneously the instantaneous values of both the current and the voltage in a low-tension circuit is shown in Fig. 94.

In vibrator No. 1 the magnitude of the current flowing, and hence the amplitude of the vibration may be regulated by adjusting the resistance in $R_1$ and $R_2$. Curve No. 1 traced on the film records the instantaneous value of the current.

Likewise for vibrator No. 2 the current flowing can be regulated by adjusting the resistance in $R_3$ and its instantaneous values

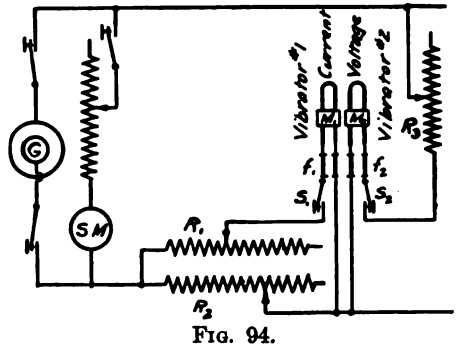



FIG. 94.

will be proportional to the instantaneous value of the line voltage. Hence the curve No. 2 traced at the film will record the instantaneous values of the voltage. Thus on the same film records of the wave form of both the current and voltage are obtained. The instrument is calibrated by direct currents so that the amplitude of the alternating current and voltage waves may be measured in amperes and volts.

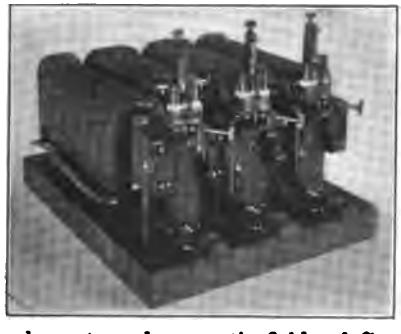


FIG. 95.—Moving elements and magnetic fields of General Electric Co.'s three element oscillograph.

(*b*) **G. E. Three-element Oscillograph.**[1]—The oscillograph made by the General Electric Co. operates on the same principle as the

[1] ROBINSON, A. I. E. E. *Trans.* XXIV, 185.

Duddell instrument. The design differs chiefly in the mechanical arrangement and the relative size of the several parts, and provides a convenient and reliable instrument both for laboratory and commercial work. The arrangement of the electro-magnets and the three vibrators is shown in Fig. 95, and a vertical cross-section of the vibrating element in Fig. 96. The three vibrators, Fig. 95, are independent of each other and can readily be insulated for ordinary pressures. The vibrating strips and mirrors are of silver. The vibrating element can be turned around a vertical axis passing through the center of the mirror by the screw $Q$. The containing cell for the whole vibrating element is also movable around a horizontal axis, passing through the center of the mirror by means of the screw, $S$. These two adjustments make it possible to bring the image to any desired place on the photographic film or vibrating mirror. In Fig.

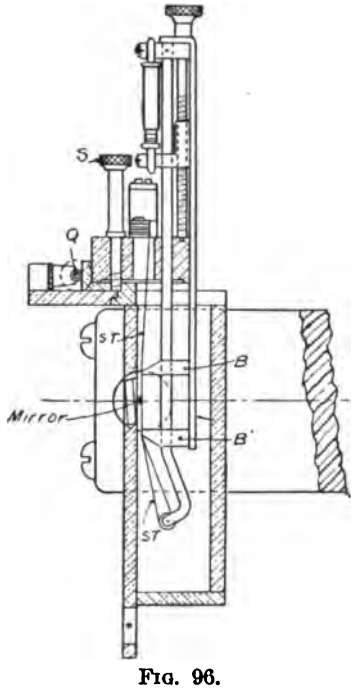



FIG. 96.

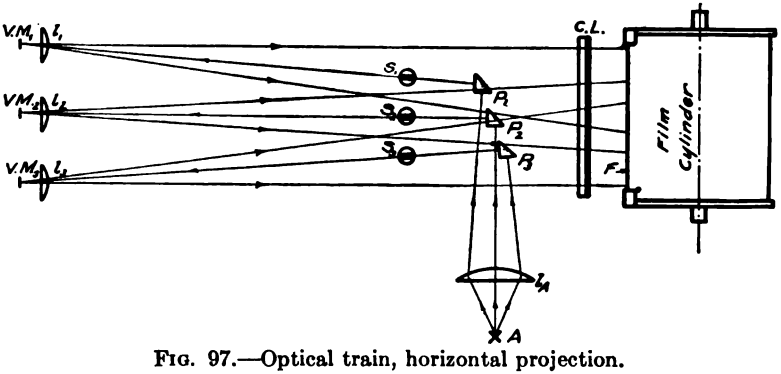



FIG. 97.—Optical train, horizontal projection.

95 $TT'$ are the terminals carrying current to the vibrating strips, marked $ST$ in Fig. 96. The mirror with that portion of the loop

that vibrates lies between the supports *BB'*. The size of the mirror is about 20 by 10 mils and the vibrating element has a natural period of one five-thousandth of a second. By immersion in oil the instrument is made dead-beat.

The optical train for photographic work is shown in horizontal projection in Fig. 97; and the vertical projection in Fig. 98. The arc lamp is at *A* and the directions of the arrows indicate the paths of the beams of light. $P_1$, $P_2$, $P_3$ are right-angled prism mirrors; $S_1$, $S_2$, $S_3$ adjustable slits; $l_A$, $l_1$, $l_2$, $l_3$ condensing

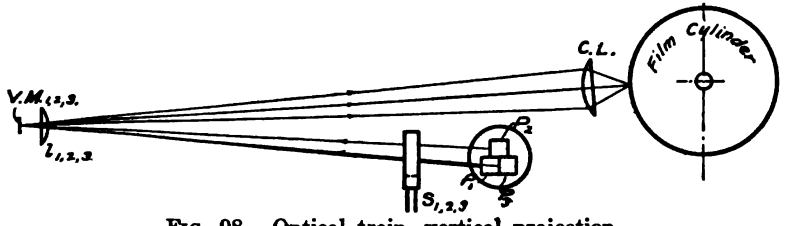



FIG. 98.—Optical train, vertical projection.

lenses; $VM_1$, $VM_2$, $VM_3$, the vibrating mirrors; *CL*, a cylindrical lens focusing the light rays on the photographic film on the revolving cylinder.

(*c*) **Voltmeters, Electrodynamometer Type.**—In direct-current voltmeters a movable coil is pivoted in the field of a strong permanent magnet; a current passing through the coil reacts on the field and produces a torque proportional to the strength of the current. If an alternating current passes through this instrument, the reversal of the current in the successive half cycles also produces a simultaneous reversal of the torque and hence tends to turn the pointer in both directions. In an oscillograph with a very light moving element the torque reversal produces an oscillation proportional to the currents flowing, but in an ordinary indicating ammeter or voltmeter the moving element is too heavy and as the resultant torque for two successive half waves is nil, the pointer does not move. In order to produce a torque in the same direction for both halves of an alternating-voltage wave it is necessary to reverse the direction of the field simultaneously with the reversal of the voltage.

In the electrodynamometer type, of which the Weston voltmeter, model 18, is a good example, the magnetic field is produced by a current from the same circuit as supplies the movable ele-

ment. This is indicated diagrammatically in Fig. 99. The current in the voltmeter passes first through a stationary coil, $S$, producing a magnetic field, then through in the movable element, $M$, and finally through a non-inductive resistance, $R$. Since the magnetic field and the current in the movable coil reverse direction simultaneously, the resultant torques for the successive half cycles are in the same direction. As in direct-current instruments, this torque is opposed by two hair springs, and an air vane eliminates undesirable vibrations or oscillations.

The torque varies directly as the product of the current, $i$, in the movable element and the strength of field, $\phi$.

$$\text{Torque} \propto i\phi \tag{185}$$

Since the field coil is in series with the movable element, the magnetic field also varies as the current.

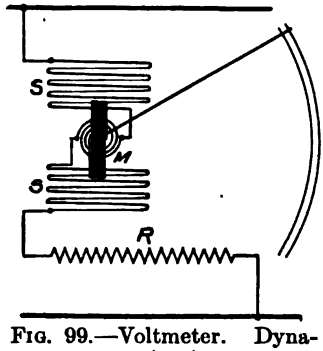



FIG. 99.—Voltmeter. Dynamometer type.

$$\phi \propto i \tag{186}$$

Hence:

$$\text{Torque} \propto i^2 \tag{187}$$

The average torque for a complete cycle is therefore proportional to the average square of the current. By calibrating the scale so as to give the square root of the average torque, the alternating-current voltmeter indicates directly the *effective values*, or the *root mean square* of the instantaneous values.

The voltmeter circuit has of necessity some inductance since the interaction of the current in the movable element upon the field of the stationary coil is the source of the required torque. Hence, the current passing through the instrument is:

$$i = \frac{e}{Z} = \frac{e}{\sqrt{r^2 + {}_Lx^2}} = \frac{e}{\sqrt{r^2 + (2\pi fL)^2}} \tag{188}$$

and therefore the indications of the instrument will be affected by the frequency of the impressed voltage. If the instrument be calibrated for one frequency, it will not indicate correctly the line voltage for any other frequency. Fortunately the resistance can be made very large relative to the reactance for ordinary frequencies, so the error introduced by the inductance is negligibly small even for a wide range in frequency.

(*d*) **Wattmeters, Electrodynamometer Type.**—In a wattmeter the magnetic field is produced by the current flowing through a stationary coil connected in series with the power circuit. In the movable element a small current, proportional to the voltage across the circuit measured, is flowing, thus providing a torque at any instant proportional to the product of the current and the voltage, or the instantaneous watts passing through the instrument.

$$\text{Torque} \propto ei \tag{189}$$

The electrodynamometer type of wattmeter, of which the Weston model 16, and the General Electric Co's. type $P_3$ are good examples, may be used on either direct or alternating currents. If the instrument be calibrated with direct currents

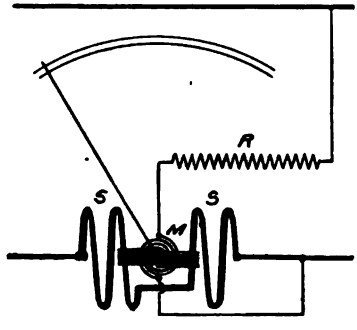



FIG. 100.—Wattmeter. Dynamometer type.

it would indicate the *effective* or *r.m.s.* values when used on alternating-current circuits. From the circuit diagram in Fig. 100 it is seen that both the stationary and movable coils introduce inductance into the circuit and therefore the time-phase relation of the current and voltage waves is not the same in the instrument as in the circuit measured. As any variation in the time-phase relation between the current and voltage alters the power factor this introduces an error. Fortunately the inductance is small and hence the reactance for commercial frequencies, so this source of error is of little importance in ordinary measurements on electric power circuits.

For Experiments see page 531.

## CHAPTER IX

## VOLTAGE AND CURRENT IN SINGLE-PHASE AND POLYPHASE SYSTEMS

(*a*) **Single-phase Systems.**—Any alternator with a single circuit in the armature has the same current, at any instant, in all the conductors. In distinction from the polyphase alternators having several circuits with currents of different values appearing simultaneously, the single circuit winding is said to produce single-phase currents. The fundamental plan of the wiring diagram for a single-phase alternator is shown in Fig. 101. The parts of the conductors passing under the north and south poles are so placed as to produce voltage in the same direction at any position. In the diagram the direction of the current is indicated by dots and crosses and the direction of the rotation of the field by an arrow. Part *B* shows the wiring diagram on a plane instead of on the cylindrical surface of the armature. The diagram shows the fundamental arrangement necessary to produce a single-phase current.

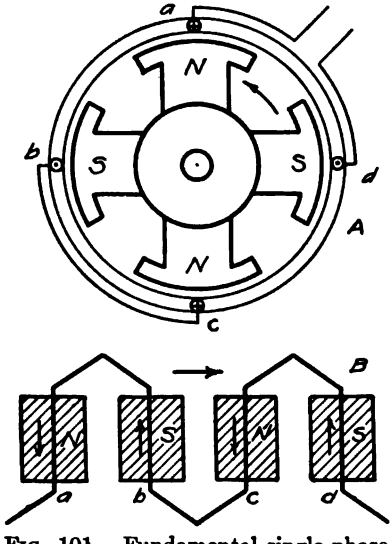



FIG. 101.—Fundamental single-phase winding diagram.

In order to produce the required voltage in commercial machines several layers of the simple winding are connected in series, often giving a complex appearance, but the principles involved are essentially the same as for the fundamental arrangement shown in Fig. 101. One or more single-phase alternators connected to a distribution network of any complexity form a single-phase system. The circuits discussed in the preceding

chapters are all single-phase and the vector diagrams at the generators give the current- and voltage-phase relations of single-phase alternators supplying the power.

(*b*) **Electrical Degree.**—The voltage and current waves from alternators complete one cycle in $2\pi$ or 360°. Hence for one revolution of the generator there are produced as many complete cycles of the voltage and current waves as the alternator has pairs of poles. For a four-pole machine half a revolution of the rotating spider produces a complete voltage wave of 360°. The distance of two-pole pitches in all cases produces a complete voltage wave and for convenience this is called 360 electrical degrees.

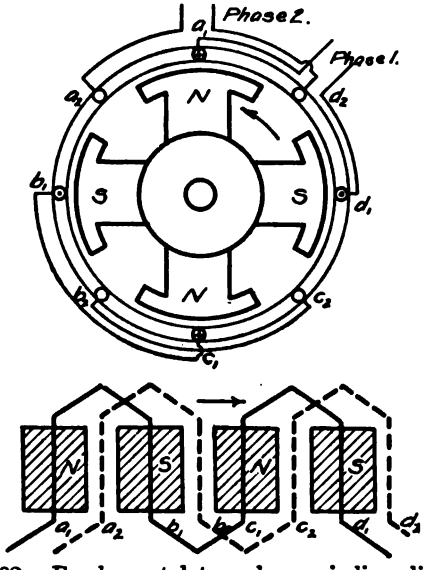



FIG. 102.—Fundamental two-phase winding diagram.

(*c*) **Frequency.**—Since a complete wave is produced while the conductor passes over one pair of poles, the number of cycles per revolution is $\frac{p}{2}$ when $p$ is the number of poles.

$$\text{Hence the cycles per second} = \frac{p}{2} \times \frac{\text{r.p.m.}}{60} = f \qquad (191)$$

The American Institute of Electrical Engineers has adopted 60 and 25 cycles per second as standard frequencies.

(*d*) **Two-phase Systems.**—In a two-phase or quarter-phase alternator, two separate single-phase circuits are placed on the

same armature, 90 electrical degrees apart as illustrated in Fig. 102. The generator has four terminals and in a four-wire distribution the two circuits are kept separate. Since the two circuits are alike and differ only in position on the armature, the resulting voltage waves are of the same shape but differ by 90° or a quarter phase in time position. The voltage waves for the two phases are shown in Fig. 103. If the generator be connected to a two-phase circuit with a balanced inductive load the current in each phase will lag behind the voltage as shown by the vector diagram in Fig. 104.

To save copper in the transmission line, two of the four terminals are connected together in the two-phase, three-wire system.

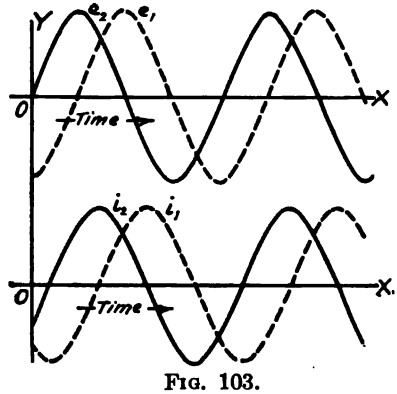



FIG. 103.

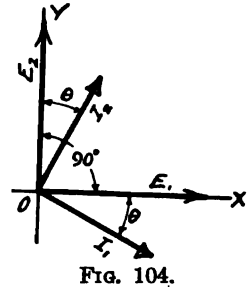



FIG. 104.

The circuit diagram and the corresponding voltage and current vector diagrams are shown in Fig. 105. Since the two circuits are alike and pass through the same magnetic field at the same speed, the effective voltages are equal.

$$E_1 = E_2 \tag{192}$$

The resultant voltage between the outside mains $A$ and $B$ is the vector resultant of $E_1$ and $E_2$.

$$\dot{E}_3 = \dot{E}_1 + j\dot{E}_2 \text{ or}$$
$$E_3 = \sqrt{2}E_1 = \sqrt{2}E_2 \tag{193}$$

If the two receiving circuits are alike, the current flowing in the main $A$ is equal to the current in main $B$, or $I_1 = I_2$. The current in the common return is the vector sum of $I_1$ and $I_2$.

$$\dot{I}_3 = \dot{I}_1 + j\dot{I}_2 \text{ or } I_3 = \sqrt{2}I_1 \tag{194}$$

Let (*b*) in Fig. 105 represent the current and voltage vector diagram corresponding to the connections in (*a*). If instead of connecting terminals *a* and *b* together for the common return, terminal *a* be connected to $b_1$ the voltage vector $E_2$ would be reversed. The vector diagram for the currents and voltages would then be as shown in (*c*), Fig. 105. Comparing the two

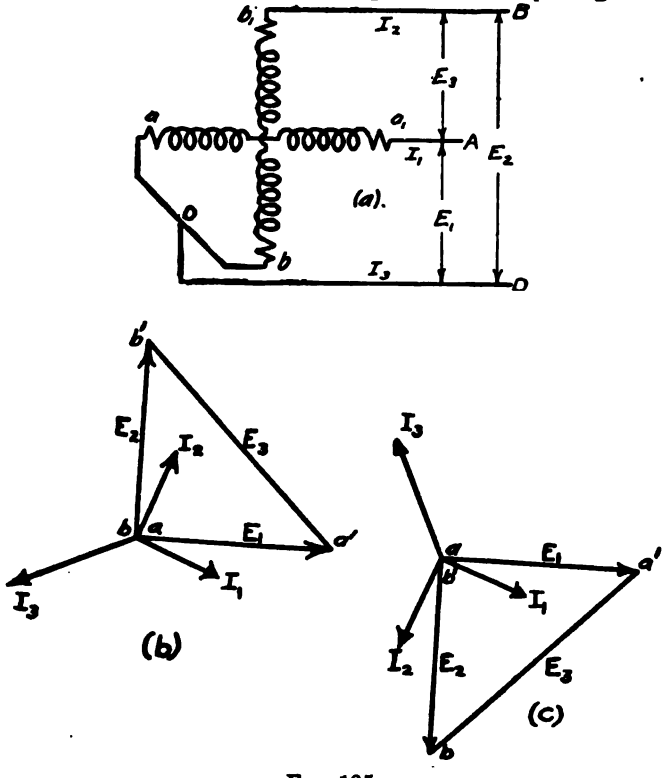



FIG. 105.

vector diagrams it is seen that the corresponding currents and voltages are of the same magnitude but differ in time phase by 90°.

(*e*) **Three-phase Systems.**—The fundamental wiring diagram is shown in Fig. 106. A three-phase winding consists of three single-phase windings spaced 120 electrical degrees, so that the voltages generated in them successively reach their maximum 120° apart. The sine voltage and current waves for the three phases are shown in Fig. 107, and the vector diagram of the voltages and currents for a balanced three-phase load, having

the same resistance and condensive reactance in each phase, is shown in Fig. 108.

(*f*) **The Δ and Y Connections.**—As seen from the circuit diagram in Fig. 106 the three-phase winding requires six leads or

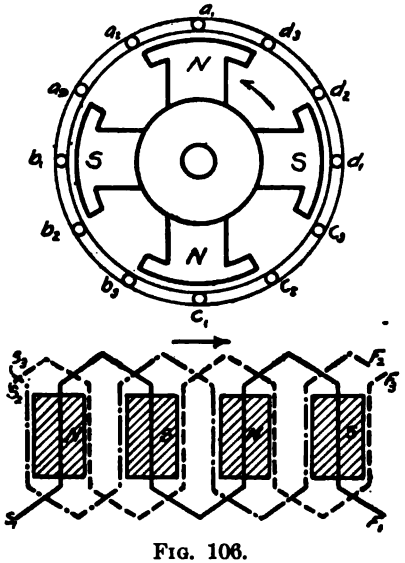



FIG. 106.

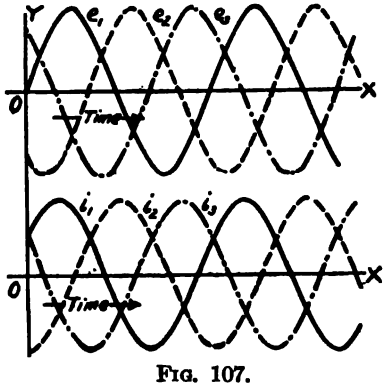



FIG. 107.

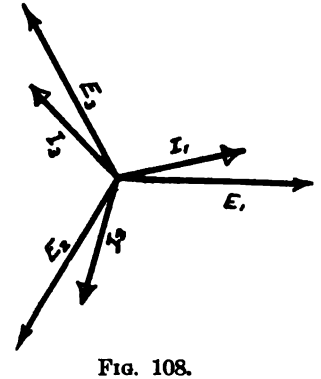


FIG. 108.

terminals, two for each phase. It is usual to combine these terminals by pairs so that only three leads are brought out from the alternator to the distribution system. This connection between the three phases is made in two ways. If the three phases be connected as shown by the winding diagram in Fig. 109 and

the corresponding circuit diagram in Fig. 110, it is called a Delta connection, and is represented by the letter Δ. If the three circuits be connected as shown by the winding diagram in Fig. 111 and the corresponding circuit diagram in Fig. 112, it is called a Star or Wye connection and is represented by the letter *Y*.

With the windings in the armature forming a closed circuit as in the Δ connection it appears likely that circulating currents would flow in the armature. By examining the three voltage

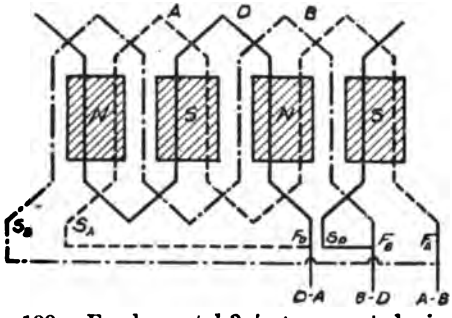



FIG. 109.—Fundamental 3-$\phi$, Δ-connected winding.

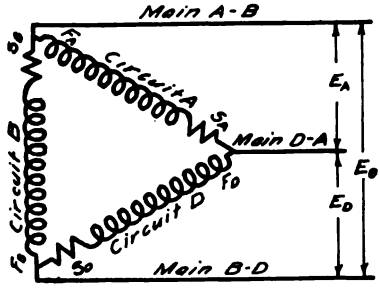



FIG. 110.

waves in Fig. 107 and the vectors in Fig. 108 it is seen that the sum of the voltages is at every instant equal to zero. It should be noted that while this holds true for the three fundamental sine waves, it is not the case for waves having third or ninth harmonics. The conditions which cause circulating currents are explained in Chap. XXIII, while the discussion in this chapter deals only with simple sine waves. By comparing the three current waves in Fig. 107 and the corresponding vectors in Fig. 108 it is seen that the sum of the three currents is at any instant equal to zero, and hence, the star connection will not change the flow of the currents in either phase. As neither the voltage nor

the current in either phase is affected by making the Δ or *Y* connection, the same diagram expresses the phase relations of the *voltages and the currents in the several phases* whether the circuit is connected in delta or in star.

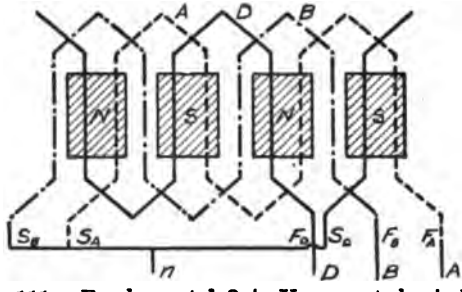



FIG. 111.—Fundamental 3-$\phi$, Y-connected winding.

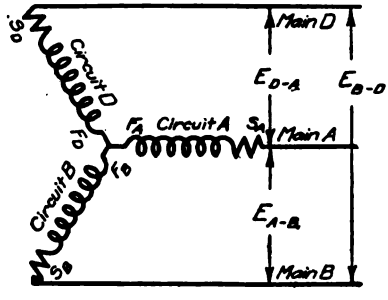



FIG. 112.

(*g*) **Delta Connection. Voltage and Current.**—Although in a Δ connection the current in each circuit, that is, between $S_A$ and $F_A$, $S_B$ and $F_B$, $S_D$ and $F_D$ in Fig. 110, is the same as when the circuits were separate, the currents in the mains *AB*, *BD* and *DA* are formed by combining the currents in the corresponding circuits. Thus if the currents in the three circuits be $I_A$, $I_B$ and $I_D$ the current in each main is the vector difference of the currents in the corresponding circuits. This can be seen by inserting a vibrator of an oscillograph in each phase as indicated in Fig. 113. In order that the vibrators in the three circuits shall trace the three currents in the proper sequence, the terminals must be connected in the same direction in the three circuits. Thus in circuit *A*, $\alpha$ is nearest $S_A$ with $\lambda$ next to $F_A$; in circuit *B*, $\alpha$ is

nearest $S_B$ and $\lambda$ next to $F_B$; and in circuit $D$, $\alpha$ is nearest to $S_D$ and $\lambda$ next to $F_D$. If then the vibrator be inserted in main $DA$ with $\lambda$ nearest to the junction as in Fig. 113, then the component of the current coming from circuit $D$ flows in the same direction through the vibrator as it does in the first position. But the component of the current from circuit $A$ passes through the vibrator in the reverse direction. Hence the current in the main $DA$ is the *vector difference* of the currents in circuits $A$ and $D$.

$$\dot{I}_{AB} = \dot{I}_A - \dot{I}_B \tag{195}$$

Similarly:

$$\dot{I}_{BD} = \dot{I}_B - \dot{I}_D \tag{196}$$

$$\dot{I}_{DA} = \dot{I}_D - \dot{I}_A \tag{197}$$

The corresponding vector diagram with the rotation in the counter-clockwise direction is shown in Fig. 114. For the $\Delta$ connec-

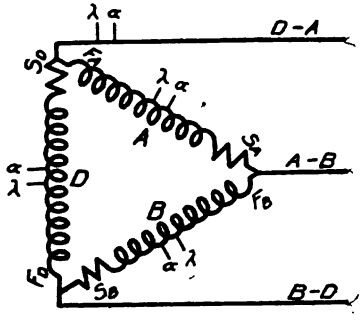



FIG. 113.

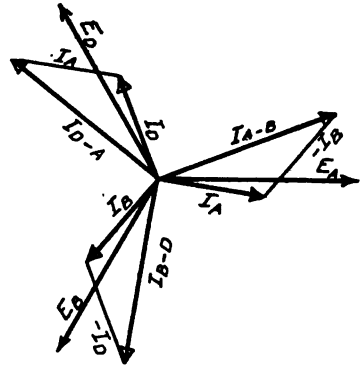



FIG. 114.

tion the voltage between the mains is the same as for each circuit and is represented by the same voltage vectors as if the circuits were separate. In equations (195) to (197) the currents in the mains are expressed in vector notation. To find the absolute value the phase position, as well as the magnitude of the component vectors, must be known. Let the three-phase load be balanced so that both in magnitude and phase relation the currents and voltages in the three phases are alike. Or stated in the form of equations:

$$E_A = E_B = E_D \tag{198}$$

$$I_A = I_B = I_D \tag{199}$$

$$\theta_A = \theta_B = \theta_D \tag{200}$$

Then the triangle *OFH*, Fig. 114, is isosceles and the angle *OFH* equals 120°. Hence *OH* equals $\sqrt{3}$ times *OF*, or the current in the main is equal to $\sqrt{3}$ times the current in a circuit.

$$I_{AB} = \sqrt{3}I_A = 1.73I_A \tag{201}$$

Similarly:

$$I_{BD} = \sqrt{3}I_B \text{ and } I_{DA} = \sqrt{3}I_D.$$

(*h*) **Star Connection. Voltage and Current.**—In the star connection the same current that flows in each circuit must pass into the corresponding main, as only one circuit is connected to each main. The voltage between each pair of mains is, however, the resultant of voltages in the two circuits in series between the

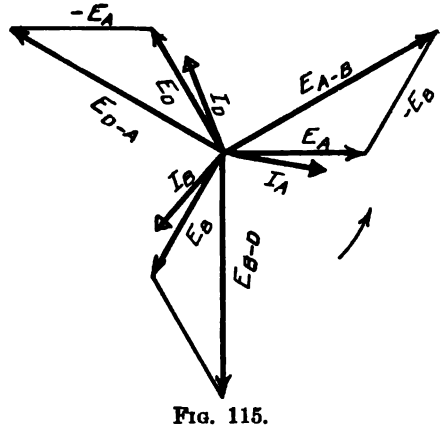



FIG. 115.

mains. By making connections to oscillographic vibrators similar to those explained for the currents in the preceding paragraph, it is readily seen that the voltage between each pair of mains is the *vector difference* of the voltages in the corresponding circuits. Hence for balanced loads the voltage and current relations are as shown in Fig. 115. Expressed in the form of equations:

$$I_A = I_B = I_D \tag{202}$$

$$E_A = E_B = E_D \tag{203}$$

$$\dot{E}_{AB} = \dot{E}_A - \dot{E}_B\,;\, \dot{E}_{BD} = \dot{E}_B - \dot{E}_D\,;\, \dot{E}_{DA} = \dot{E}_D - \dot{E}_A \tag{204}$$

The triangle *OFH* in Fig. 115 is isosceles and the angle *OFH* is equal to 120°.

Hence:

$$E_{AB} = \sqrt{3}E_A = 1.73E_A \tag{205}$$

Similarly:

$$E_{BD} = \sqrt{3}E_B \text{ and } E_{DA} = \sqrt{3}E_D \tag{206}$$

(*i*) **Equivalent or Topographic Vector Diagrams.**—To show directly the time-phase relations of the several voltages and currents in the three-phase systems, all the vectors are drawn through the origin, as in Figs. 114 and 115. It is often convenient to draw the vectors so as to indicate their position in the circuit diagram, forming triangles or polygons in which the direction and length of the sides represent the vector values. Thus in Fig. 115 the voltages between the mains may be moved into parallel positions so as to form the triangle shown in Fig. 116.

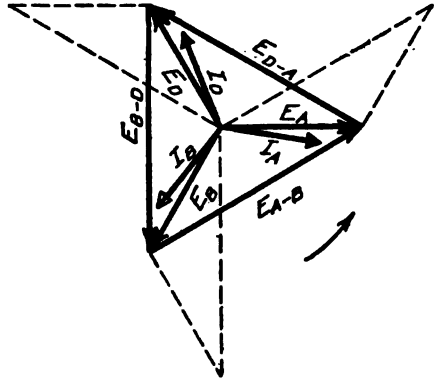



FIG. 116.

The sides of the triangle represent the voltages between the mains while the lines joining the vertices to the center give the voltages in the circuits. The simplicity of this form of vector diagram is of considerable importance when dealing with more complex systems, as may be seen from Figs. 301, 302 and 307 to 313 inclusive.

(*j*) **Sequence of Phases.**—In the laboratory or in commercial work it is often necessary to determine the time sequence of the several phases in a polyphase system. Thus in the three-phase system indicated by the terminals *A*, *B* and *D* in Fig. 117 the sequence of the phases may be either *AB*, *BD*, *DA* or *AD*, *BA*, *DB*.

A simple arrangement for determining the direction of rotation

is shown in Fig. 117. A resistance, $r$, in series with a condensance, $_{c}x$, is connected across one phase, $AD$.

In Fig. 118 let the vector $E_{A-D}$ represent the voltage in the phase $AD$. For convenience let $r$ and $_{c}x$ be approximately equal. The current therefore leads $E_{A-D}$ by about 45° and $rI$ and $xI$ complete the triangle with a right angle at $Q$. With the position of $E_{A-D}$ assumed, the vectors for $E_{B-A}$ and $E_{D-B}$ must be represented either by the full lines $BA$, $DB$ or the broken lines $B'A$, $DB'$. It is apparent from the diagram that the voltage between $Q$ and terminal $B$ is proportional to either the length of the line $QB$ or $QB'$. Hence, if the voltmeter reads less than the phase voltage the sequence is indicated by the full lines in Fig. 118, while if the reading is greater than the phase voltage rotation is in the reverse direction.

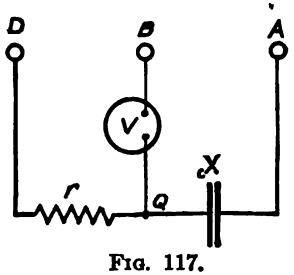



Fig. 117.

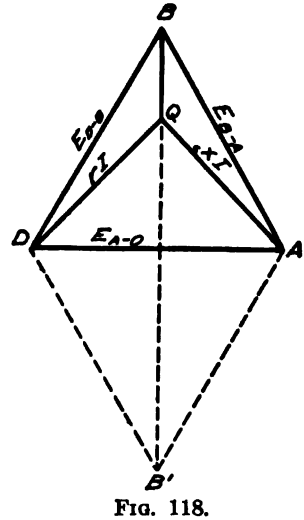



Fig. 118.

(*k*) **Commercial Polyphase Systems.**—Besides the two-phase and three-phase systems, four-phase, six-phase and twelve-phase systems are in commercial use, although to a comparatively limited extent. For machines like the rotary converter a higher efficiency may be obtained in six or twelve-phase designs than for the corresponding two- or three-phase types. See Chap. XVI.

## PROBLEMS

1. Given a balanced two-phase, three-wire system (circuit diagram in Fig. 105). Voltage in each phase = 120 volts; current in lines $A$ or $B$ = 50 amp.; $\theta$ = 30° (current leading). $A$ leads $B$.

(*a*) Find the current in the common wire, $D$.

(*b*) Find the voltage between the outside wires, $A$ and $B$.

(*c*) Draw the vector diagram.

(*d*) Draw curves for the voltage and currents in rectangular coördinates.

**2.** Given a balanced, three-phase, delta-connected system (circuit diagram in Fig. 110). Voltage in each phase = 220 volts; current in each circuit = 35 amp.; $\theta = 40°$ (current lagging).

(*a*) Find the current in each main.

(*b*) Draw the vector diagram showing the voltages and currents in the mains and circuits for the three phases.

(*c*) Draw in rectangular coördinates the corresponding curves for the voltages and currents.

**3.** Given a balanced, three-phase, star-connected system (circuit diagram in Fig. 112). Voltage in each circuit = 125 volts; current in each main = 40 amp.; $\theta = 20°$ (current leading).

(*a*) Find the voltages between the mains.

(*b*) Draw the vector diagram showing the currents and voltages in the circuits and mains for the three phases.

(*c*) Draw curves in rectangular coördinates for the currents and voltages.

**4.** Given a balanced, three-phase, delta-connected system. Current in each main = 60 amp.; voltage between mains = 220 volts; $\theta = 30°$ (current lagging). Phase sequence *ABD*.

(*a*) Find the current in each circuit.

(*b*) Draw vector diagram for the currents and voltages.

**5.** Given a balanced, three-phase, star-connected system. Current in each main = 40 amp.; voltage between mains = 120 volts; $\theta = 70°$ (current leading). Phase sequence *ABD*.

(*a*) Find the voltage in each circuit.

(*b*) Draw vector diagram showing the currents and voltages.

**6.** An unsymmetrical star-connected three-phase load has the following line voltages:

$$\dot{E}_{1-2} = 100;$$
$$\dot{E}_{2-3} = -45 + j90;$$
and $$\dot{E}_{3-1} = -55 - j90.$$

The phase impedances are:

$$Z_1 = 10;$$
$$Z_2 = 2 + j7;$$
$$Z_3 = 2 - j9 \text{ ohms.}$$

Solve for vector values of phase voltages and line currents. Give a complete topographic diagram of all voltages. Show the current vectors on the same plot.

For Experiments see page 532.

## CHAPTER X

## POWER IN BALANCED TWO-PHASE AND THREE-PHASE SYSTEMS

The electric power in single-phase systems was discussed in Chap. VII. It was shown that the expression for the instantaneous power for sine waves is:

$$\begin{aligned} ei &= {}^{M}E{}^{M}I \sin(\omega t + \gamma_1) \sin(\omega t + \gamma_2) \\ &= EI[\cos(\gamma_1 - \gamma_2) - \cos(2\omega t + \gamma_1 + \gamma_2)] \end{aligned} \tag{210}$$

The equation shows that the power curve is also a sine wave but pulsates with double the frequency of the voltage or current. It was also proved that the average power for single-phase circuits with sine-wave current and voltage is:

$$P = EI \cos\theta \tag{211}$$

where $\theta$ is the difference in phase of the current and voltage.

Similarly the reactive power traveling to and fro in the circuit is:

$$_{r}P = EI \sin\theta \tag{212}$$

(*a*) **Two-phase Circuit.**—A two-phase circuit consists of two single-phase circuits differing by 90 electrical degrees in space position and hence producing voltage and current waves 90° out of phase. Let the load be balanced and the current and voltage waves be simple harmonic.

For the first phase let the instantaneous values be:

$$i_1 = {}^{M}I \sin(\omega t) = \sqrt{2}I \sin(\omega t) \tag{213}$$

$$e_1 = {}^{M}E \sin(\omega t + \theta) = \sqrt{2}E \sin(\omega t + \theta) \tag{214}$$

$$p_1 = e_1 i_1 = 2EI \sin^2(\omega t)\cos\theta + 2EI \sin(\omega t)\cos(\omega t)\sin\theta \tag{215}$$

Since the second phase voltage is 90 electrical time degrees from the first, the equations for the simultaneous instantaneous values for the second phase are:

$$i_2 = {}^{M}I \cos(\omega t) = \sqrt{2}I \cos(\omega t) \tag{216}$$

$$e_2 = {}^{M}E \cos(\omega t + \theta) = \sqrt{2}E \cos(\omega t + \theta) \tag{217}$$

$$p_2 = e_2 i_2 = 2EI \cos^2(\omega t)\cos\theta - 2EI \cos(\omega t)\sin(\omega t)\sin\theta \tag{218}$$

The instantaneous power for both circuits is therefore:

$$p = p_1 + p_2 = 2EI[\sin^2(\omega t) + \cos^2(\omega t)] \cos\theta = 2EI \cos\theta \quad (219)$$

*The instantaneous power in a two-phase circuit is thus found to be equal to twice the average power of one of the circuits.*

If the instantaneous power be constant it must be equal to the average power. Hence for balanced two-phase circuit and simple harmonic current and voltage:

$$P = 2EI \cos\theta \quad (220)$$

Power is measured by wattmeters. In single-phase circuits the current from one of the mains passes through the wattmeter and the voltage is taken between the mains. A four-wire two-

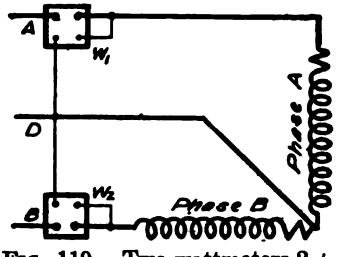



FIG. 119.—Two wattmeters 2-$\phi$, 3-wire, system.

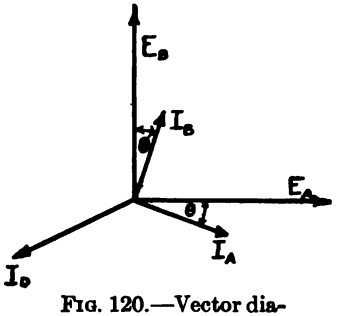



FIG. 120.—Vector diagram for Fig. 119.

phase circuit is merely two single-phase circuits and hence the power may be measured by one wattmeter in each circuit. In a three-wire, two-phase circuit the current connection on the wattmeter is made on the outside mains while the voltage is taken from the outside main to the common wire. The wiring diagram is shown in Fig. 119 where $W_1$ and $W_2$ represent the two wattmeters, and the corresponding vector diagram in Fig. 120.

The so-called *tangent method* is sometimes found convenient in determining the power factor in two-phase circuits. One wattmeter is used. The current coil is connected in the same way as for measuring power in single-phase circuits. With the voltage coil across the wires in the same circuit the wattmeter measures true watts.

$$p_1 = E_1 I_1 \cos\theta \quad (221)$$

Without changing the current connection transfer the voltage

coil to the second phase. The reading of the wattmeter is then:

$$p_2 = E_2 I_1 \sin \theta \tag{222}$$

Usually $E_1 = E_2$, and dividing the second reading by the first:

$$\frac{p_2}{p_1} = \tan \theta, \text{ or } \theta = \tan^{-1} \frac{p_2}{p_1} \tag{223}$$

Hence the power factor, $\cos \theta$, in a two-phase circuit can be found by the use of one wattmeter.

(*b*) **Three-phase Circuit, Δ Connection.**—Power in three-phase circuits is usually measured by two wattmeters connected as indicated in Fig. 121. In order to prove that the two wattmeters at any instant will measure the total power in the three circuits it is desirable to assume some directional notation. The arrows in both the circuits and the mains correspond, in a way, to the three-phase vector diagrams. It is immaterial which direction is assumed, only, if the current arrow in the circuit be taken counter-clockwise in one circuit, the arrows in the remaining two circuits must be in the same direction. Likewise the arrows for the currents in the mains may be taken from left to right as in the diagram, or in the reverse order, but the three arrows in the three mains must be in the same direction. It is evident that the arrangement of the arrows should be such that when the armature turns to a second position in which main $A$ occupies the place previously held by main $B$, the arrows in $A$ and in the adjoining circuits must then be the same as were those of main $B$ and the adjoining circuits. The arrows, therefore, represent directional conditions at any one position in space but not for the same time instant. Like the vector diagram, the arrows give the relative relation for all positions.

With the direction of the arrows as given in Fig. 121 the equations for the instantaneous values of the currents in the mains are:

$$i_A = i_1 - i_3 \tag{224}$$

$$i_B = i_2 - i_1 \tag{225}$$

$$i_D = i_3 - i_2 \tag{226}$$

The power indicated by the wattmeter is the product of the current in the main by the corresponding voltage.

For wattmeter $W_1$:

$$p_1 = e_2 i_B = e_2(i_2 - i_1) \tag{227}$$

For wattmeter $W_2$:

$$p_2 = (-e_3)i_A = -e_3(i_1 - i_2) \tag{228}$$

The instantaneous power in both wattmeters:

$$\begin{aligned} p &= p_1 + p_2 = e_2(i_2 - i_1) - e_3(i_1 - i_2) \\ &= e_2 i_2 + (-e_2 - e_3)i_1 + e_3 i_2 \end{aligned} \tag{229}$$

$$e_1 = -e_2 - e_3 \tag{230}$$

Hence,

$$p = e_1 i_1 + e_2 i_2 + e_3 i_3 \tag{231}$$

The product $e_1 i_1$ is the instantaneous power in circuit (1), $e_2 i_2$ in circuit (2) and $e_3 i_3$ in circuit (3). *Therefore the two wattmeters connected in this manner measure the total instantaneous power in the three circuits.*

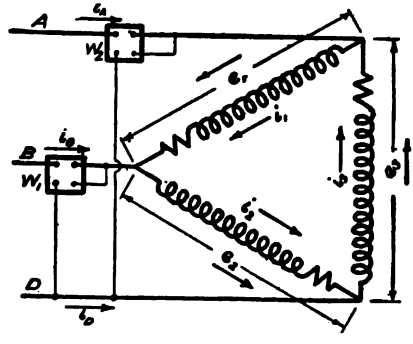



FIG. 121.—Two wattmeters, 3-$\phi$, $\Delta$ connection.

The value of the average power will be derived both analytically and by the aid of vector diagrams.

*First Method.*—For sine waves with a balanced inductive load causing a lagging current, let the voltage and current equations for phase (1) be:

$$e_1 = {}^{\prime\prime}E_1 \sin(\omega t) = \sqrt{2}E_1 \sin(\omega t) \tag{232}$$

$$i_1 = {}^{\prime\prime}I_1 \sin(\omega t - \theta) = \sqrt{2}I_1[\sin(\omega t)\cos\theta - \sin\theta\cos(\omega t)] \tag{233}$$

For phase (2):

$$\begin{aligned} e_2 &= {}^{\prime\prime}E_2 \sin(\omega t - 120°) \\ &= \sqrt{2}E_2[-\tfrac{1}{2}\sin(\omega t) - \frac{\sqrt{3}}{2}\cos(\omega t)] \end{aligned} \tag{234}$$

$$i_2 = {}^{\prime\prime}I_2 \sin(\omega t - 120° - \theta) = \sqrt{2}I_2[-\tfrac{1}{2}\sin(\omega t)\cos\theta - \frac{\sqrt{3}}{2}\cos(\omega t)\cos\theta + \tfrac{1}{2}\cos(\omega t)\sin\theta - \frac{\sqrt{3}}{2}\sin(\omega t)\sin\theta] \quad (235)$$

For phase (3):

$$e_3 = {}^{\prime\prime}E_3 \sin(\omega t - 240°)$$

$$= \sqrt{2}E_3[-\tfrac{1}{2}\sin(\omega t) + \frac{\sqrt{3}}{2}\cos(\omega t)] \quad (236)$$

$$i_3 = {}^{\prime\prime}I_3 \sin(\omega t - 240° - \theta)$$

$$= \sqrt{2}I_3[-\tfrac{1}{2}\sin(\omega i)\cos\theta + \frac{\sqrt{3}}{2}\cos(\omega t)\cos\theta + \tfrac{1}{2}\cos(\omega t)\sin\theta + \frac{\sqrt{3}}{2}\sin(\omega t)\sin\theta] \quad (237)$$

For a balanced load:

$$E_1 = E_2 = E_3 = E_c \text{ and } I_1 = I_2 = I_3 = I_c$$

where $E_c$ and $I_c$ denote the effective voltage and current, respectively, *in each circuit.*

The instantaneous power in the three circuits is the sum of the products of the respective instantaneous voltages and currents. Taking the sum of products $e_1i_1$, $e_2i_2$, and $e_3i_3$ in equations (232) to (237) and reducing to the simplest form gives equation (238).

$$p = e_1i_1 + e_2i_2 + e_3i_3 = 3E_cI_c\cos\theta \quad (238)$$

Equation (238) shows that the instantaneous power is a constant and hence equal to the average power.

$$p = P = 3E_cI_c\cos\theta \quad (239)$$

$$I = \sqrt{3}\,I_c \text{ and } E = E_c \quad (240)$$

Since the current in the mains, $I$, is equal to $\sqrt{3}$ times the current in the circuit, $I_c$, and the voltage between the mains, $E$, equals the voltage in the circuit, $E_c$, the power in a three-phase circuit expressed in terms of the current in the mains, the voltage between the mains and the power factor is:

$$P = \sqrt{3}EI\cos\theta \quad (241)$$

*Second Method.*—In Fig. 122 is shown the vector diagram corresponding to the circuit diagram in Fig. 121. In the circuits the currents lag $\theta°$ behind the respective voltages and are repre-

sented by the vectors $I_1$, $I_2$, and $I_3$. The currents in the mains are the vector differences of the circuit vectors and denoted by $I_A$, $I_B$ and $I_D$ in the diagram. The currents in the mains differ by 30° in phase position from the currents in the circuits and by the factor $\sqrt{3}$ in magnitude. The power indicated by wattmeter $W_1$, Figs. 121 and 122, is the product of the current in main $B$, the voltage between $B$ and $D$ and the cosine of the time-phase angle.

Power in wattmeter $W_1$:

$$P_1 = E_2 I_B \cos (30° - \theta) \tag{242}$$

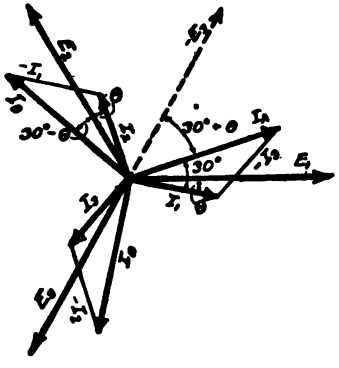


FIG. 122*a*.—Vector diagram, for Fig. 121.

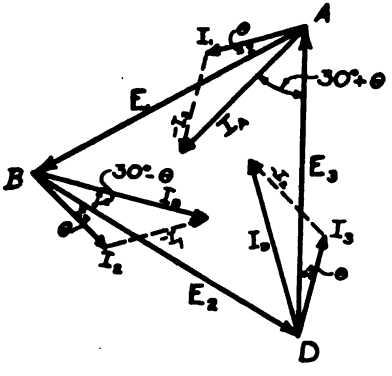



FIG. 122*b*.—Topographic vector diagram for Fig. 121.

Similarly the power in wattmeter $W_2$ is the product of the current in main $A$ by the voltage between mains $A$ and $D$ and the cosine of the time-phase angle. The time-phase angle between $I_A$ and $E_3$ is $(180° - 30° - \theta)$; and the cos $(180° - 30° - \theta) = - \cos (30° + \theta)$. The voltage connection, however, is made in the reverse order from that of $W_1$, and hence this reverses the sign. Therefore, the power flows from the generator when cos $(30° + \theta)$ is positive.

Power in wattmeter $W_2$:

$$P_2 = E_3 I_A \cos (30° + \theta) \tag{243}$$

In a balanced circuit

$$E_1 = E_2 = E_3 \text{ and } I_A = I_B = I_D$$

Hence the total power:

$$P = P_1 + P_2 = EI[\cos(30° - \theta) + \cos(30° + \theta)] \quad (244)$$

$$= 2EI \cos 30° \cos\theta = \sqrt{3}EI \cos\theta \quad (245)$$

From equations (242) and (243) the power factor may be obtained from the wattmeter readings.

$$P_1 + P_2 = 2EI \cos\theta \cos 30° \quad (246)$$

$$P_1 - P_2 = 2EI \sin\theta \sin 30° \quad (247)$$

$$\tan\theta = \sqrt{3}\frac{(P_1 - P_2)}{(P_1 + P_2)} \quad (248)$$

$$\theta = \tan^{-1}\sqrt{3}\frac{(P_1 - P_2)}{(P_1 + P_2)}$$

$\cos\theta$ = the power factor.

(*c*) **Three-phase, Y Connection.**—In Fig. 123 is shown a circuit diagram of a *Y*-connected three-phase circuit with direc-

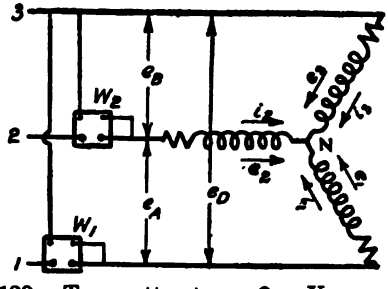



FIG. 123.—Two wattmeters. 3-$\phi$, Y connection.

tional arrows symmetrically arranged as in Fig. 121 for the Δ connection. The corresponding vector diagram is shown in Fig. 124. It is seen directly from Fig. 123 that the instantaneous power in wattmeter $W_1$ is:

$$p_1 = (-e_D)i_1 = (e_1 - e_3)i_1 \quad (249)$$

And for wattmeter $W_2$:

$$p_2 = e_B i_2 = (e_2 - e_3)i_2 \quad (250)$$

Hence the total instantaneous power in the wattmeters:

$$p = p_1 + p_2 = e_2 i_2 - e_2 i_2 + e_3 i_1 - e_3 i_1 \qquad (251)$$

But,

$$i_3 = -i_2 - i_1$$

and hence,

$$p = e_1 i_1 + e_2 i_2 + e_3 i_3 \qquad (252)$$

Thus the two wattmeters together indicate the total power in the three-phase *Y*-connected circuit at any instant.

The average power and the power factor in the *Y*-connected system are found in the same manner as for the Δ connection. Referring to the vector diagram in Fig. 124:

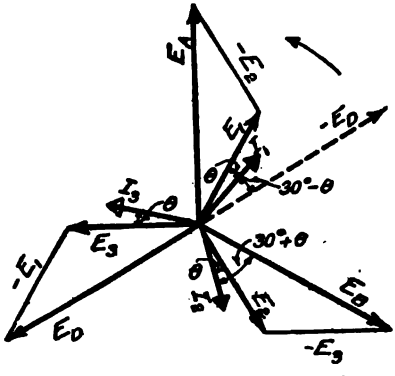



FIG. 124*a*.—Vector diagram for Fig. 123.

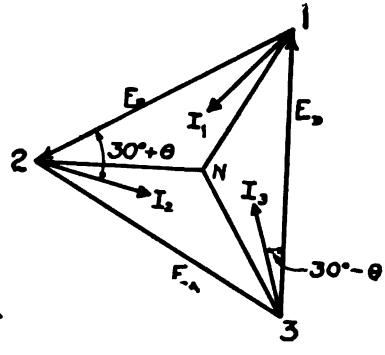



FIG. 124*b*.—Topographic vector diagram for Fig. 123.

The power in wattmeter $W_1$ is:

$$P_1 = E_D I_1 \cos (30° - \theta) \qquad (253)$$

For wattmeter $W_2$ the power is;

$$P_2 = E_B I_2 \cos (30° + \theta) \qquad (254)$$

With a balanced circuit:

$E_A = E_B = E_D = E$ and $I_1 = I_2 = I_3 = I$ in effective values.

The total power is therefore:

$$P = P_1 + P_2 = 2EI \cos \theta \cos 30° = \sqrt{3}EI \cos \theta \qquad (255)$$

In the same manner as for the delta-connection the power factor may be obtained from the two wattmeter readings. A curve showing the relation of the ratio of the readings to the power factor is given in Fig. 125.

Two important values for $\theta$ may be observed directly. *First,*

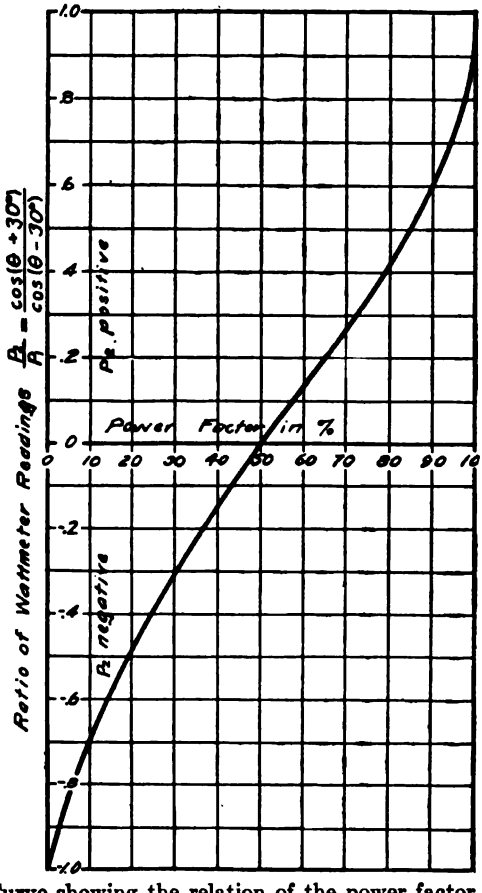



FIG. 125.—Curve showing the relation of the power factor to the ratio of wattmeter readings in balanced 3-$\phi$ circuits.

if the two wattmeters read alike, $P_1 = P_2$ and hence from equations (253) and (254) $\theta = 0$ and therefore $\cos \theta = 1$. *Second,* if one of the wattmeters, as $W_2$, reads zero, then $W_1$ indicates the total load and $\cos (30° + \theta) = 0$, or $\theta = 60°$. If $W_1$ has no reading and $W_2$ indicates the total power, then $\cos (30° - \theta) = 0$

and hence, $\theta = -60°$. If $\theta = 60°$ indicates a lagging current, then $\theta = -60°$ shows a similar leading current. If either of the wattmeters gives a negative reading, the value of $\theta$ is greater than 60° and the total power delivered is equal to the difference of the two wattmeter readings. It is, of course, necessary to know that both the current and voltage coils are properly connected for giving positive readings with $\theta$ less than 60° before concluding that a negative reading of the wattmeter indicates a power factor less than cos 60°. For power in circuits with distorted voltage and current waves, see Chap. XXIII, and for unbalanced load, Chap. XXV.

(*d*) **Reactive Power.**—If the power delivered and the power factor are known, the reactive power which travels to and fro in the circuit may be determined.

$$_xP = \sqrt{3}EI \sin \theta = P \tan \theta = \sqrt{3}(P_1 - P_2) \qquad (256)$$

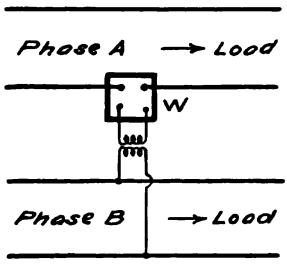



FIG. 126.

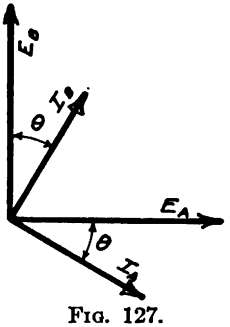



FIG. 127.

It is, however, sometimes more convenient to obtain the reactive power directly by means of a wattmeter. In a balanced two-phase system the reactive power may be measured by a wattmeter if the instrument is connected to the circuits as in Fig. 126. The potential transformer is inserted to protect the wattmeter from short-circuit between the current and voltage coils in the instrument. Since the current through the wattmeter comes from phase *A* and the voltage from phase *B*, it is evident from the corresponding vector diagram in Fig. 127 that the wattmeter reading is given by equation (257) and represents the reactive power in one phase.

$$E_A I_B \cos (90° - \theta) = E_A I_B \sin \theta = {}_xP_{(2)} \qquad (257)$$

$2_sP_{(2)}$ = reactive power in the two-phase circuit when $_sP_{(2)}$ is the wattmeter reading. Similarly the reactive power in a balanced

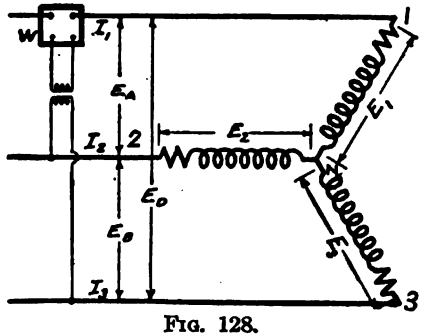



FIG. 128.

three-phase circuit is measured directly by a wattmeter connected as in Fig. 128. For a balanced load, either star or delta connected and for any power factor, the wattmeter indicates

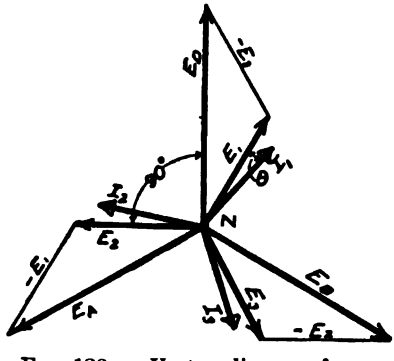



FIG. 129a.—Vector diagram for Fig. 128.

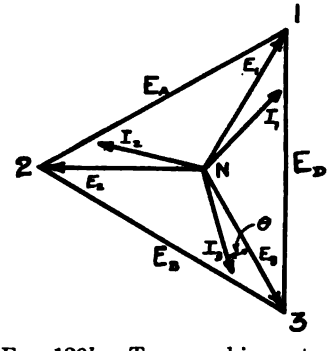



FIG. 129b.—Topographic vector diagram for Fig. 128.

the reactive power in the three-phase system. From Fig. 129*a* or Fig. 129*b*:

$$E_BI_1 \cos (90° - \theta) = E_BI_1 \sin \theta = {}_sP_{(3)} \qquad (258)$$

$\sqrt{3}_sP_{(3)} = \sqrt{3}E_BI_1 \sin \theta$ = total reactive power in a balanced 3-$\phi$ circuit when $_sP_{(3)}$ is the wattmeter reading.

## PROBLEMS

**1.** Given a three-phase, delta-connected, balanced system with an inductive load. Voltage (between mains), $E = 230$ volts; $P_2 = 3.8$ kw.; $P_1 = 8.4$ kw. Circuit diagram as in Fig. 121.

(*a*) Find the power factor.

(*b*) Find the current in the mains and the current in each circuit.

(*c*) Draw vector diagram showing the current and voltage in the wattmeter and the power-factor angle.

**2.** Given a three-phase, star-connected, balanced system with a condensive load. $E$ (between mains) = 230 volts; $P_1 = 10.5$ kw.; $P_2 = 4.8$ kw. Circuit diagram as in Fig. 123.

(*a*) Find the power factor.

(*b*) Find the current in the mains.

(*c*) Draw the corresponding vector diagram.

**3.** With the same system and load as in problem 1 let a wattmeter be connected as in Fig. 128.

(*a*) What is the reading of the wattmeter?

(*b*) Draw the vector diagram showing the current and voltage in the wattmeter.

**4.** With the same system and load as in problem 2, let a wattmeter be connected as in Fig. 128.

(*a*) Find the wattmeter reading.

(*b*) Draw the vector diagram showing the current and voltage in the wattmeter.

**5.** A three-phase induction motor takes 48.4 kw. at 87.5 per cent. power factor. The power is measured by two wattmeters, connected as in Fig. 121. What are the readings indicated by $W_1$ and $W_2$? If the voltage between mains is 448 volts, what current flows through each wattmeter?

**6.** A three-phase induction motor takes 28.5 kw. at 40 per cent. power factor while starting. Voltage between mains — 238 volts. Wattmeter connections as in Fig. 123.

(*a*) Find the readings of the two wattmeters.

(*b*) Find the currents in the mains.

(*c*) Draw vector diagram and indicate the currents and voltage passing through wattmeters $W_1$ and $W_2$.

**7.** In a three-phase balanced system the load consists of a three-phase induction motor taking 45.8 kw. at 73 per cent. power factor, 22 kw. of incandescent lamps and an over excited synchronous motor taking 72.5 kw. at 93 per cent. power factor. Voltage between mains — 120 volts. The total load is measured by two wattmeters, connected as in Fig. 121.

(*a*) Find the readings of $W_1$ and $W_2$.

(*b*) Draw vector diagrams for the three loads and for the total load.

(*c*) Find the reactive power for each of the three loads and for the total load.

For Experiments see page 534.

## CHAPTER XI

## MAGNETIC HYSTERESIS, EDDY CURRENTS, MUTUAL INDUCTION

When an alternating current flows in a conductor surrounded by air or other non-conducting material, energy is stored in the magnetic field surrounding the conductor, while the current increases, and returned to the electric circuit while the current decreases to its zero value. If, on the contrary, in the space within the range of the magnetic field of the current in the conductor, there be conducting material, and particularly with iron or other magnetic bodies, a part of the energy involved will be changed into heat. The laws for determining these losses will be discussed under magnetic hysteresis, eddy currents and mutual induction.

(*a*) **Magnetic Hysteresis.**—In Chap. II it was shown that the fundamental relation between magnetic flux, ${}^{M}\Phi$, and induced voltage, $E$, is expressed by equation (260).

$$E = \sqrt{2}\pi n f {}^{M}\Phi 10^{-8} \text{ volts} \tag{260}$$

$E$ is the effective volts (simple sine wave assumed) ${}^{M}\Phi$ the maximum number of lines of force, $n$ the number of turns in series, $f$ the frequency.

Let $A$ = cross-sectional area, and $\mathfrak{B}$ maximum magnetic flux density.

$$ {}^{M}\Phi = \mathfrak{B}A \tag{261}$$

$$E = \sqrt{2}\pi n f \mathfrak{B} A 10^{-8} \text{ volts} \tag{262}$$

Since the induced voltage varies with the rate of cutting lines of force, the voltage wave is displaced 90° from the corresponding flux wave, as shown in Fig. 130. If the conductor be surrounded by a material of constant permeability, like air, the magnetizing current is also a sine wave and in time phase with the flux. In an iron-clad circuit, as in Fig. 131, the same voltage of sine wave form produces decidedly different effects.

*First,* the reluctance of the iron is very low and hence a much stronger field is produced for a given magnetizing current.

*Second,* while the permeability of iron and steel is large as compared to air, it is not constant for all intensities of magnetization.

*Third,* the ratio between the magnetizing current and the magnetic flux is different for increasing and decreasing values of the current.

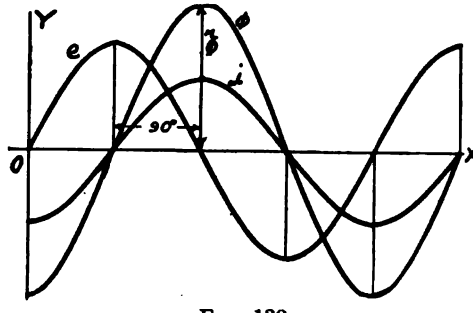



FIG. 130.

*Fourth,* these ratios also depend upon the maximum value of the flux density in each cycle.

Consider the electric circuit $AB$, Fig. 131, surrounding an iron core. The impressed voltage must at each instant be balanced by the induced voltage produced by the increasing or decreasing magnetic flux cutting across the several turns of the electric circuit (neglecting the small $ri$ drop in the conductor). Hence a magnetizing current will flow in the circuit just sufficient to produce the flux required to balance the impressed voltage. In the hysteresis loops, Fig. 132, is shown the relation between the magnetizing current, $_m i$, or magnetomotive force, $\mathfrak{IC}$, and the resulting magnetic flux, $^m\Phi$, or flux density, $\mathfrak{B}$. While the relation may be expressed by the equation, $\mathfrak{B} = \mu\mathfrak{IC}$, it must be kept in mind that the permeability is a variable affected by several conditions, as already noted.

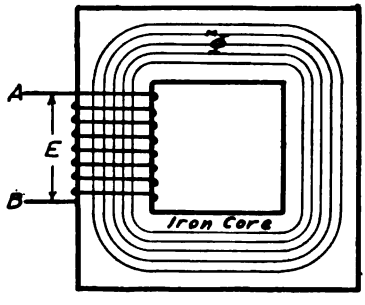



FIG. 131.

Plotting magnetic flux and magnetizing current in rectangular coördinates with time as abscissæ gives the shape of the current wave as in Fig. 133. With a given hysteresis loop and sine flux

wave the magnetizing current corresponding to any point on the flux wave is found by drawing the horizontal line *up*, Fig. 133, and taking the distance *us* from the hysteresis curve. By laying

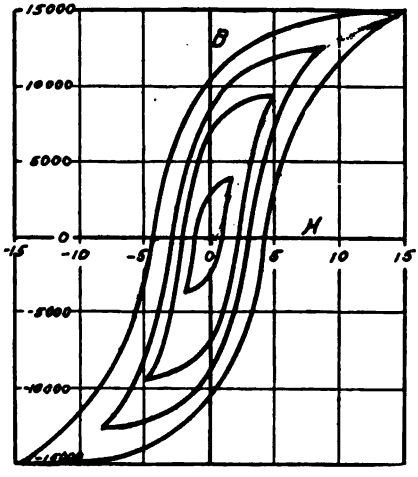



FIG. 132.

off on the ordinate *np* the distance *nq* equal to *us*, one point on the current curve is found. The maximum of the current curve will come at the same time as the maximum of the flux wave, but

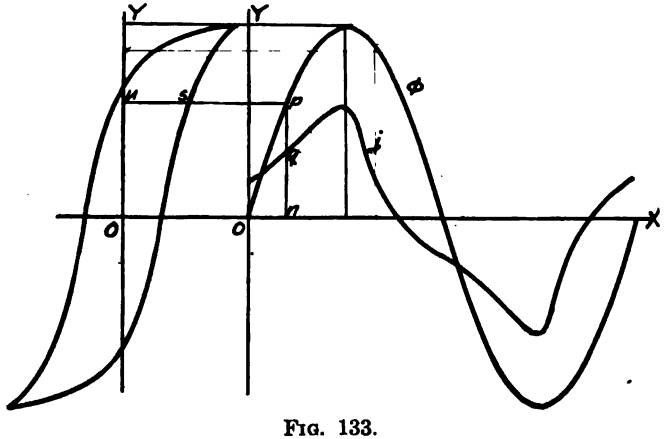



FIG. 133.

the current wave will not be sinusoidal in form. The wave consists of a third and smaller fifth harmonic superimposed on the fundamental sine wave. Unless special accuracy is required, the

magnetizing current may be considered equal to an equivalent sine wave as represented by the broken line in Fig. 134. The equivalent current wave has the same effective intensity and power as the actual magnetizing current, and unless the flux reaches the saturation point in the iron, the maxima of the two

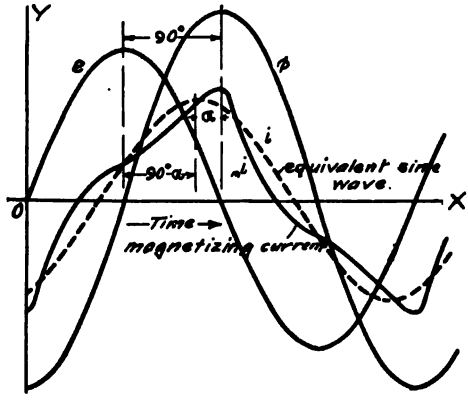



FIG. 134.

current waves are approximately the same. The equivalent sine wave reaches the maximum before the actual magnetizing current and hence before the maximum of the flux. This time-phase lead of the equivalent sine wave of the magnetizing current over the magnetic flux wave is called the *hysteretic angle of advance* and is represented by the letter $\alpha$. If the voltage, flux and equivalent current are sine waves their relations can be shown in a vector diagram as in Fig. 135. Placing the voltage along the $X$-axis, the corresponding vector for the magnetic flux will lie in the positive direction on the $Y$-axis. The vector for the equivalent magnetizing current will lead the flux by $\alpha$ or lag behind the voltage by $90° - \alpha$. The power consumed by the hysteresis will be the product of the current, voltage and the cosine of the phase angle.

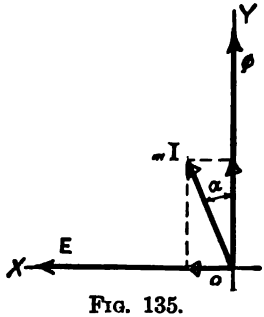



FIG. 135.

$$_hw = E_m I \cos (90° - \alpha) = E_m I \sin \alpha \qquad (263)$$

The exciting current can therefore be considered as consisting

of the components, ${}_{m}I \sin \alpha$ in phase with the voltage and called the *hysteretic* or *magnetic power component*, and ${}_{m}I \cos \alpha$, called the *magnetizing component* and which represents power stored in the magnetic field while the current increases and then returned to the circuit when the current decreases. Conversely, one may consider the voltage as consisting of two components: one, a power component, $E \sin \alpha$, in phase with the current, the *hysteretic power component;* and another, $E \cos \alpha$, in quadrature with the current and consisting of the voltage consumed by *self-induction.*

It is evident that the circuit producing the two components can be represented symbolically by an impedance, *i. e.*, a resistance and an inductive reactance:

$$z = r + j_{l}x = z \sin \alpha + jz \cos \alpha \qquad (264)$$

Similarly by an admittance or conductance and susceptance:

$$y = g - j_{l}b = y \sin \alpha - jy \cos \alpha \qquad (265)$$

In equations (264) and (265) representing the magnetizing circuit, the quantities, $z$, $r$, ${}_{l}x$ and $y$, $g$, ${}_{l}b$ are not constant as in circuits without iron but depend upon the maximum density of magnetic induction $\mathfrak{B}$. This introduces many complications in the analysis of circuits containing iron and necessitates approximations in calculations that otherwise could be made rigidly exact.

Dr. Steinmetz has proved that the energy loss due to hysteresis in iron and many kinds of steel is proportional to the 1.6th power of the maximum magnetic induction, $\mathfrak{B}$, and can be expressed by the formula:

$$ {}_{h}w = \eta \mathfrak{B}^{1.6} \qquad (266)$$

${}_{h}w$ = loss of energy in ergs, per cycle, per cubic centimeter.
$\mathfrak{B}$ = maximum magnetic induction, in lines of force per square centimeter.
$\eta$ = the coefficient of hysteresis.

The value of $\eta$ differs depending upon the material. Dr. Steinmetz found it to vary from 0.001 to 0.0055 in iron; from 0.0032 to 0.028 in cast steel; up to 0.08 for tungsten and manganese steels; while for silicon steel values even lower than 0.001 were obtained.

From (266) the loss of power for volume, $V$, and at frequency, $f$, is:

$$_hP = \eta f V \mathfrak{B}^{1.6} 10^{-7} \text{ watts} \qquad (267)$$

$$= \eta f V \left(\frac{^{w}\Phi}{A}\right)^{1.6} 10^{-7} \text{ watts.}$$

For certain grades of silicon steel (probably of a non-homogeneous nature) the hysteresis power loss is more closely approximated by using 1.7 instead of 1.6 as exponent for $\mathfrak{B}$.

(*b*) **Eddy Currents.**—The lines of force produced by an alternating current move outward radially from the conductor while the current is increasing and toward the conductor when the current decreases. While thus traversing the space surrounding the conductor carrying the current, the lines of force set up or induce electromotive forces in the conductor. If this voltage is induced in a conducting material like copper or iron, a local current will flow. The magnitude of this current will depend on the difference of potential induced in the conductor and the resistance in the path through which the current must flow. This induced current has also a magnetic field which in turn reacts upon the primary field. In solid masses of good conductors, like iron or copper, the voltage induced would be considerable and the resistance small. As a result large currents would flow in the mass taking curved paths much the same as eddies in a stream. In most electrical apparatus eddy currents are undesirable; and the design must be so made as to reduce the losses from eddy currents to a minimum. This is accomplished by laminating the iron or using iron wire as conductor of the magnetic flux. This method reduces the possible difference in potential between any two points and also increases the resistance of the path through which the eddy currents must flow. Eddy currents are true electric currents but flow in very short circuits. The induced voltage causing the eddy currents is proportional to the frequency and flux density

$$_eE \propto f\mathfrak{B}.$$

The resulting current is equal to the product of the voltage and conductivity of the circuit, hence:

$$_eI = \lambda_eE \propto \lambda f\mathfrak{B} \qquad (268)$$

The power is proportional to the product of the current and voltage:

$$_{e}P \propto {}_{e}I_{e}E \propto \lambda f^2 \mathfrak{B}^2 \tag{269}$$

From equation (262), $f\mathfrak{B}$ is proportional to $E$, hence:

$$_{e}P = kE^2\lambda \tag{270}$$

The loss of power by eddy currents is proportional to the square of the voltage and to the electric conductivity of the iron.

Like magnetic hysteresis, eddy currents produce a leading current with respect to the flux; or the eddy currents cause an *angle* of *advance*, $\beta$, and therefore the current has components both in phase and in quadrature with the voltage. The phase relations between the impressed voltage, the induced flux and the eddy current are shown by the vector diagram in Fig. 136. Expressing the angle in terms of the eddy-current conductance $g$ and the total admittance of the circuit $y$:

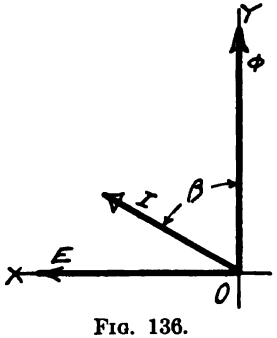


FIG. 136.

$$\sin \beta = \frac{g}{y} \tag{271}$$

From equation (270):

$$\frac{P}{E^2} = k\lambda = g \tag{272}$$

*The equivalent conductance due to the eddy currents in the iron is therefore a constant of the circuit, depending only upon the electric conductivity of the iron.*

To find the power loss from eddy currents let:

$\epsilon$ = eddy current coefficient
$V$ = the volume of the iron.

Dividing the expression for power in equation (269) by the frequency, we have the energy per cycle per cubic centimeter.

$$_{e}w = \epsilon\lambda f^2 \mathfrak{B}^2 \text{ ergs} \tag{273}$$

Hence total power:

$$_{e}P = \epsilon\lambda V f^2 \mathfrak{B}^2 10^{-7} \text{ watts} \tag{274}$$

As the quality of the iron is included in the conductivity factor,

$\lambda$, the coefficient of eddy currents, $\epsilon$, depends only upon the shape of the parts in the magnetic circuit. In commercial apparatus thin plates or sheets and wires are used, and the coefficient depends upon the thickness of the sheets, or upon the diameter of the wire.

1. *Thin Plates or Laminated Iron.*—Let the thickness of the iron be very small compared to the length of the sheet, so that the currents may be considered as flowing parallel to the surface and in opposite directions on the two sides and in that way forming a circuit. This is the case in most commercial machines as transformer cores, armature cores, etc. Let Fig. 137 be an edgewise (greatly enlarged) section of a thin plate or sheet of iron placed

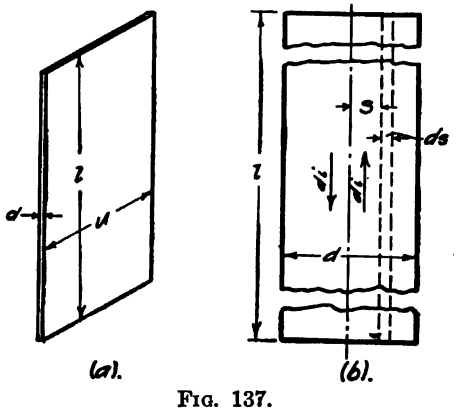



FIG. 137.

in a vertical position. Let $d$ be the thickness of the plate, $l$ the length (up and down) and $u$ the width of the thin plate or sheet of iron. Let $ds$ be an elemental zone, parallel to the surface of the plate as shown in cross-section in Fig. 137*b*.

For currents flowing up and down in the plate the conductance, $g$, for the zone $ds$ is:

$$g = \frac{\lambda u(ds)}{l} \tag{275}$$

The number of magnetic lines of force passing across $ds$ in a quarter cycle is $\mathfrak{B}sl$. Hence the voltage induced in this zone is:

$$_{e}E = \sqrt{2}\pi f l \mathfrak{B} s \text{ in c.g.s. units} \tag{276}$$

This voltage produces a current:

$$_{e}I = g\ _{e}E = \sqrt{2}\pi f \mathfrak{B} \lambda u s(ds) \tag{277}$$

Hence the power consumed in this zone:

$$d_e p = {}_e E_e I = 2\pi^2 f^2 \mathfrak{B}^2 l \lambda u s^2 (ds) \tag{278}$$

Therefore the total power consumed by the eddy currents in the sheet of thickness $d$, is:

$${}_e p' = \int_{-\frac{d}{2}}^{+\frac{d}{2}} ({}_e p) = 2\pi^2 f^2 \mathfrak{B}^2 \lambda\, ul \int_{-\frac{d}{2}}^{+\frac{d}{2}} s^2 (ds) = \frac{\pi^2 f^2 \mathfrak{B}^2 l \lambda u d^3}{6} \text{ ergs per sec.} \tag{279}$$

Power per cubic centimeter:

$${}_e p = \frac{{}_e p'}{lud} = \frac{\pi^2 f^2 \mathfrak{B}^2 \lambda d^2}{6} \text{ ergs per sec.} \tag{280}$$

Energy per cycle:

$${}_e w = \frac{{}_e p}{f} = \frac{\pi^2 f \mathfrak{B}^2 \lambda d^2}{6} \text{ ergs} \tag{281}$$

From equations (273) and (281):

$$\epsilon = \frac{\pi^2 d^2}{6} = 1.645 d^2, \text{ if } \lambda \text{ is given in c.g.s. units.}$$

As $\lambda$ is usually given in practical units the coefficient, $\epsilon$, includes the conversion factor $10^{-9}$ or:

$$\epsilon = 1.645 d^2 10^{-9} \text{ for } \lambda \text{ in practical units.}$$

In practical units $\lambda$ varies within the limits of $10^5$ and $10^4$. Equation (273) can therefore be restated:

$$\begin{aligned} {}_e w = \epsilon \lambda f \mathfrak{B}^2 &= 1.645 d^2 \lambda f \mathfrak{B}^2 10^{-9} \text{ ergs} \\ &= 1.645 d^2 \lambda f \mathfrak{B}^2 10^{-16} \text{ joules} \end{aligned} \tag{282}$$

Also equation (280) becomes:

$${}_e p = f\, {}_e w = 1.645 \lambda d^2 f^2 \mathfrak{B}^2 10^{-16} \text{ watts} \tag{283}$$

and the total loss by eddy currents for volume $V$:

$${}_e P = {}_e p V = 1.645 \lambda V d^2 f^2 \mathfrak{B}^2\, 10^{-16} \text{ watts} \tag{284}$$

2. *Iron Wire.*—The coefficient for eddy currents in an iron wire may be found in a similar manner. Let Fig. 138 represent the cross-section of a wire of diameter $d$. The magnetic lines of force lie parallel to the length of the wire and move at right angles to their direction, either toward or away from the center

of the wire. Hence the lines of force pass through the elemental sheet ($ds$) and generate voltages that tend to send currents in the sheet ($ds$) around the axis of the conductor. The resistance of an elemental circular zone ($ds$) in Fig. 138 at a distance $s$ from the center and $l$ cm. in length is $\frac{2\pi s}{l\lambda(ds)}$ and the flux crossing the zone in each quarter cycle is $\mathfrak{B}\pi s^2$.

Hence the voltage generated in the zone:

$$_eE = \sqrt{2}\pi^2 f\mathfrak{B}s^2 \text{ in c.g.s. units} \tag{285}$$

The current produced by this voltage:

$$_eI = g_eE = \frac{\pi f l\lambda\mathfrak{B}s(ds)}{\sqrt{2}} \tag{286}$$

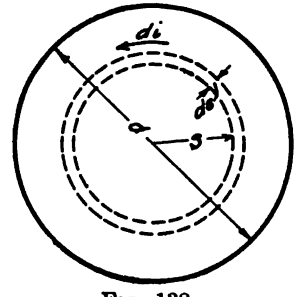



FIG. 138.

The power loss in this zone:

$$(d_ep) = {}_eE{}_eI = \pi^3\lambda l f^2\mathfrak{B}^2 s^3(ds) \tag{287}$$

The power consumed in $l$ cm. of wire:

$$_ep' = \pi^3\lambda f^2\mathfrak{B}^2 l\int_0^{\frac{d}{2}} s^3(ds) = \frac{\pi^3\lambda f^2\mathfrak{B}^2 l d^4}{64} \tag{288}$$

The power consumed in 1 c.c. of iron:

$$_ep = \frac{{}_ep'}{v} = \frac{\pi^2 f^2\lambda\mathfrak{B}^2 d^2}{16} \text{ ergs per second (c.g.s. units)} \tag{289}$$

Energy consumed by eddy currents per cycle, per cubic centimeter of iron:

$$_ew = \frac{p}{f} = \frac{\pi^2 f\lambda\mathfrak{B}^2 d^2}{16} \text{ ergs} \tag{290}$$

The coefficient of eddy currents is, therefore, from equations (273) and (290):

$$\epsilon = \frac{\pi^2 d^2}{16} = 0.617 d^2, \text{ if } \lambda \text{ is expressed in c.g.s. units} \quad (291)$$

For $\lambda$ in practical units the conversion factor $10^{-9}$ must be introduced and then:

$$\epsilon = 0.617 d^2 10^{-9} \quad (292)$$

With this value for $\epsilon$ equations (289) and (290) may be written as in (294) and (293).

Energy consumed by eddy currents in iron wire per cubic centimeter, per cycle:

$$_{e}w = \frac{\pi^2 f \lambda \mathfrak{B}^2 d^2 10^{-9}}{16} = 0.617 f \lambda \mathfrak{B}^2 d^2 10^{-9} \text{ ergs} \quad (293)$$

Power consumed at frequency, $f$:

$$_{e}p = f_{e}w = 0.617 f^2 \lambda \mathfrak{B}^2 d^2 10^{-16} \text{ watts} \quad (294)$$

Total power consumed by volume $V$:

$$_{e}P = {}_{e}pV = 0.617 V f^2 \lambda \mathfrak{B}^2 d^2 10^{-16} \text{ watts} \quad (295)$$

3. *Comparison of Power Consumed by Eddy Currents in Wires and Laminated Iron.*—With the same quality of iron, the same flux density, frequency and volume, the power loss is directly proportional to the eddy current coefficient, $\epsilon$.

For laminated iron:

$$\epsilon' = 1.645\ d_1^2\ 10^{-9}$$

For iron wire:

$$\epsilon'' = 0.617\ d_2^2\ 10^{-9}$$

For the same loss of power $\epsilon' = \epsilon''$ and hence:

$$\frac{\text{diameter of wire}}{\text{thickness of sheet}} = \frac{d_2}{d_1} = \sqrt{\frac{1.645}{0.617}} = 1.63 \quad (296)$$

For equal power losses due to eddy currents the diameter of the iron wire is therefore 1.63 times the thickness of the sheet, provided the quality of the iron is the same.

(*c*) **Mutual Induction.**—The eddy currents, discussed in the previous paragraphs, are produced by the alternating field passing into the laminated iron or the iron core. If the lines of force cut another conductor, an e.m.f. will also be induced therein. If the conductor forms a circuit, a current will flow and energy will be consumed. This energy must necessarily come from the circuit that produced the alternating magnetic flux. Thus in Fig. 139, if an alternating current flows in circuit

$A$ and a portion of the circuit lies parallel to a conductor forming part of circuit $B$, an e.m.f. will be induced causing a current to flow in $B$. The power consumed by the current flowing in circuit $B$ is supplied by the generator in circuit $A$. The voltage induced in circuit $B$ and the resulting current are in the opposite direction to the voltage and current in $A$. If circuit $B$ has both resistance and inductance, the current will lag behind the voltage, and either the current may be considered as consisting of a power component in phase with the voltage and a reactive component in quadrature with the voltage, or the voltage may be divided into components in phase and in quadrature with the current. Where the amount of energy transferred is considerable it is usually best to take the two circuits separately, and to determine the

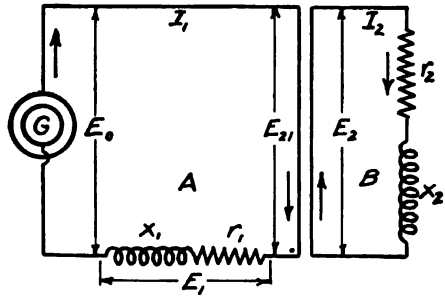



FIG. 139.

effect of mutual induction in terms of resistance and reactance constants for both the primary and the secondary circuits. In Fig. 139 let $r_1$, $x_1$, $E_1$ and $I_1$ for circuit $A$, and $r_2$, $x_2$, $E_2$ and $I_2$ for circuit $B$, represent the resistance, reactance, voltage and current respectively. The reactance $x_1$ refers to circuit $A$ alone and similarly $x_2$ to circuit $B$ without taking into consideration the existence of the other circuit.

If $L_1$ represents the induction due to all the interlinkages of the magnetic flux from unit current in $A$ with the number of turns in circuit $A$, and likewise $L_2$ the induction due to all the interlinkages of the magnetic flux from unit current in $B$ with the number of turns in circuit $B$, then $x_1 = 2\pi f L_1$, and $x_2 = 2\pi f L_2$. However, part of the lines of force produced by $I_1$ in circuit $A$ will also interlink with circuit $B$, and similarly the same proportion of the lines produced by current $I_2$, in circuit $B$ will interlink with circuit $A$, thus forming an additional inductance between

the two circuits. The mutual inductance $L_m$ is the total number of interlinkages of the magnetic flux produced by unit current in circuit $A$ with circuit $B$, or the total number of interlinkages of the magnetic flux produced by unit current in circuit $B$ with circuit $A$. Since in either case part of the magnetic flux will lie between the two circuits, $L_m^2$ must be less than $L_1L_2$. If $L_m$ be the mutual inductance, $x_m = 2\pi f L_m$ = the mutual reactance. The voltage induced in circuit $B$, is:

$$\dot{E}_2 = -jx_m I_1 \tag{297}$$

The current in $B$ is therefore:

$$\dot{I}_2 = \frac{\dot{E}_2}{z_2} = \frac{-jx_m I_1}{r_2 + jx_2} \tag{298}$$

The current $I_2$ in circuit $B$ similarly induces a voltage $E_{21}$, in the circuit $A$.

$$\dot{E}_{21} = -jx_m\dot{I}_2 = \frac{j^2x_m^2 I_1}{r_2 + jx_2} = \frac{-x_m^2 I_1}{r_2 + jx_2}$$
$$= I_1\left(\frac{-r_2x_m^2}{r_2^2 + x_2^2} + \frac{jx_m^2x_2}{r_2^2 + x_2^2}\right) \tag{299}$$

The impressed voltage necessary to overcome this induced counter e.m.f. is therefore:

$$\dot{E}_{12} = I_1\left(\frac{r_2x_m^2}{r_2^2 + x_2^2} - j\frac{x_m^2x_2}{r_2^2 + x_2^2}\right), \tag{300}$$

This shows that the effect of the mutual inductance is equivalent to a circuit having a resistance and condensive reactance, or that the mutual inductance increases the resistance and decreases the inductive reactance.

Let $r_3 = \frac{x_m^2r_2}{r_2^2 + x_2^2}$, the equivalent resistance of mutual induction and

$x_3 = \frac{x_m^2x_2}{r_2^2 + x_2^2}$, the equivalent reactance of mutual induction, then:

$$\dot{E}_{12} = \dot{I}_1(r_3 - jx_3) \tag{301}$$

Since circuit $A$ has resistance, $r_1$ and reactance, $x_1$, the voltage impressed by the generator, causing a current $I_1$ to flow through $A$, must be the vector sum of the voltages consumed by $r_1I_1$, $x_1I_1$ and $E_{12}$.

$$\dot{E}_0 = \dot{I}_1(r_1 + jx_1) + \dot{E}_{12} \tag{302}$$
$$= \dot{I}_1[r_1 + r_3 + j(x_1 - x_3)] \tag{303}$$

The equivalent resistance is therefore increased and the equivalent reactance decreased in circuit $A$ by the mutual inductance between circuits $A$ and $B$. The phase relations of the current and the components of the generated voltage consumed by the resistance and reactance in circuit $A$ and by the equivalent resistance and reactance due to the mutual inductance between the two circuits are illustrated by the vector diagram in Fig. 140.

(*d*) **Effective Resistance, Reactance, Conductance and Susceptance.**—From the preceding it is evident that the phenomena occurring in the space surrounding an alternating circuit, and due to the rapid changes in the magnetic field, consume power in

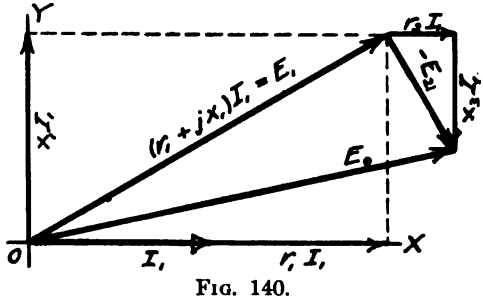



FIG. 140.

addition to the ordinary $ri^2$ losses in the circuit. Hence the transmission losses in a circuit are greater for alternating than for direct currents of equal magnitude. A similar loss is caused by dielectric hysteresis, corona, etc., as is shown in Chap. XX.

Taking the power loss by means of a wattmeter for direct and alternating currents of equal magnitude and in the same circuit, we have two values for the power.

For direct currents:

$$_dP = {}_dr I^2 \tag{304}$$

For alternating currents:

$$_aP = EI \cos\theta = zI^2 \frac{{}_ar}{z} = {}_ar I^2 \tag{305}$$

$$_aP > {}_dP, \text{ hence } {}_ar > {}_dr \tag{306}$$

The value as measured by direct currents, $_dr$, is called the *true* or *ohmic resistance;* and the quantity as measured by alternating currents, $_ar$, the *effective resistance.* The effective resistance is always larger than the ohmic resistance as it is composed of the ohmic resistance plus a quantity representing the power

losses due to dielectric and magnetic hysteresis, eddy currents, mutual magnetic and dielectric induction, skin effect, corona, etc. Accordingly the reactance of a circuit is influenced by its proximity to other circuits, and the actual value determining the phase relation of the currents and voltages is the algebraic sum of the reactances due to the inductance and condensance of the circuit itself and the effects produced by the neighboring conductors. This mutual induction decreases the inductive reactance while dielectric induction increases the condensive reactance of the circuit. The resultant value is called the *effective reactance* of the circuit. As several of these outside-the-conductor-phenomena, affecting the value of the effective resistance and effective reactance, are directly proportional to the impressed voltage, these are in a sense parallel circuits and therefore more conveniently expressed in terms of conductance and susceptance. As the relation between resistance, reactance, conductance and susceptance is fixed by the equations given in Chap. V, the terms *effective conductance* and *effective susceptance* correspond to *effective resistance* and *effective reactance.*

In Chap. II the term *effective* as applied to current and voltage was used to indicate $\frac{1}{\sqrt{2}}$ times the maximum value of a sine wave. In a broader sense, with waves of different shapes, it means the value actually measured by a voltmeter or ammeter indicating the square root of the average of the squared instantaneous values (see page 424). Likewise the term *effective* when applied to resistance, reactance, conductance and susceptance refers to the quantities measured in the alternating circuit whether the quantities are in the conductor of the circuit itself or in the surrounding magnetic and dielectric fields.

It is important to note that while the phenomena both in the conductor and in the space surrounding an alternating-current circuit are of great variety the mutual relations of the current, voltage and power can be expressed in a very simple manner by using two constants, either effective resistance and effective reactance, or, effective conductance and effective susceptance.

$$\text{Power (real or true)} = EI \cos\theta = rI^2 = {}_rEI \qquad (307)$$

${}_rE$ = power component of the voltage = effective resistance by current.

$$\text{Power (real or true)} = EI \cos\theta = gE^2 = {}_gIE \qquad (308)$$

$_gI$ = power component of current = effective conductance by voltage.

Power (reactive or wattless) $= EI \sin \theta = xI^2 = {}_gEI$ (309)

$_gE$ = reactive component of voltage = effective reactance by current.

Power (reactive or wattless) $= EI \sin \theta = bE^2 = {}_bIE$ (310)

$_bI$ = reactive component of current = effective susceptance by voltage.

The effective values can usually be measured in an existing circuit and may also, with a fair degree of accuracy, be predetermined by calculation. It must be kept in mind, however, that in general, effective values of $r$, $x$, $g$ and $b$ are not constants but depend upon the frequency, voltage, current, flux density, etc. The notation rule for *effective* values of current and voltage applies to the resistance, reactance, conductance and susceptance; namely, that unless specifically stated otherwise the *effective* values are understood when dealing with alternating-current circuits.

## PROBLEMS

**1.** Given sheet steel of thickness $d = 0.07$ cm.; $\lambda = 3 \times 10^4$; $\eta = 0.0015$; $f = 60$ cycles.

(*a*) Find $_hP$ and $_eP$ for 1 c.c., 1 cu. ft., 1 kg., and 1 lb. for $\mathfrak{B} = 8{,}000$.

(*b*) Plot curves for $_hP$ and $_eP$ as ordinates and $\mathfrak{B}$ as abscissæ, for 1 kg. and for values of $\mathfrak{B}$ from 4,000 to 18,000.

(*c*) Find $_hW$ and $_eW$ for 24 hr. in 100 lb. of the sheet steel with $\mathfrak{B} = 10{,}500$.

**2.** The iron losses in a 50-kv.a. transformer for $\mathfrak{B} = 8{,}500$ are found to be $_eP = 56$ watts and $_hP = 182$ watts. $f = 60$ cycles. Find $_eP$ and $_hP$ for $\mathfrak{B} = 6{,}000$, 7,500, 9,500, 11,000 and 12,000.

**3.** Two transformers have iron cores of the same kind of sheet steel of equal volumes. The windings differ so as to give values to $\mathfrak{B}$ equal to 6,500 and 10,500 respectively.

Find the ratios of the hysteresis losses and the eddy-current losses.

**4.** The total volume of iron in a certain transformer = 9,850 c.c. $\eta = 0.001$; $\lambda = 5 \times 10^4$; $d = 0.035$ cm.

Plot curves for hysteresis and eddy-current losses for values of $\mathfrak{B}$ from 4,000 to 16,000 for $f = 50$ and 60 cycles.

**5.** A laboratory test shows that a certain transformer has 181 watts iron loss at 30 cycles and 446 watts at 60 cycles per second for the same maximum flux density. (*a*) What is the hysteresis loss at each frequency? (*b*) What would be the total iron loss at double the flux density at 50 cycles per second?

For Experiments see page 535.

# CHAPTER XII

## TRANSFORMERS

### A. THE "CONSTANT-POTENTIAL" TRANSFORMER

The chief advantage of alternating over direct currents comes from the simple manner in which the alternating voltage may be raised or lowered. Transmission-line losses are, as stated by Joule's law, fundamentally proportional to the product of the resistance and the current squared. Since the power transmitted equals $EI \cos \theta$, it is evident that with constant power factor, an increase in voltage means that more power can be transmitted for the same line loss. For example, if the voltage be increased

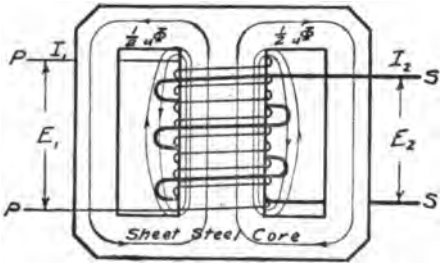



FIG. 141.—Shell-type transformer.

four-fold the power that can be transmitted over the same line at a given line loss will be sixteen times as large as at the lower voltage. At the receiver end of the line the safety of the customer and the economical design of lamps, motors and other appliances receiving the electric power require low voltages. Hence the fundamental requirement for economical transmission and distribution of electric energy is comparatively high potential on the transmission line combined with low voltage at both ends. This requirement is admirably met in alternating-current circuits by the use of the potential transformer.

The single-phase transformer is the simplest and most efficient of alternating-current apparatus. It consists of a magnetic circuit interlinked with two electric circuits, a primary and a secondary. Two fundamental designs for the magnetic circuit

are in commercial use, the *core* and *shell* types, as illustrated in Figs. 141 and 142. In some designs features of both the shell and core types are included. The electric circuits are insulated from each other and the iron core so that all the energy passing from the primary to the secondary circuit is transmitted through the transformer by the magnetic flux. In both the core and shell types the reluctance of the iron or steel cores is very small, so that almost all of the magnetic flux is inside of the steel core with only a weak stray field in the surrounding space.

(*a*) **The "Ideal" Transformer.**—At full load both the leakage flux and the power loss in the transformer are small as compared to the total flux and the power transmitted. Therefore, in a

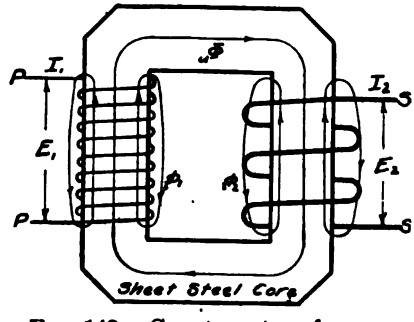



FIG. 142.—Core-type transformer.

preliminary discussion of the relations between the flux, impressed and induced voltages and load current it may be assumed:

1st. That all the magnetic flux passes through the steel core, thus neglecting the leakage flux.

2d. That all the power supplied to the primary is delivered by the secondary, thus neglecting the losses in the transformer.

Let $n_1$ = number of turns in the primary.
$n_2$ = number of turns in the secondary.
$_{u}^{\prime\prime}\Phi$ = maximum useful magnetic flux in the steel core, expressed in lines of force.
$_{\phi}E_1$ = voltage induced in the primary by the flux $_{u}^{\prime\prime}\Phi$.
$_{\phi}E_2$ = voltage induced in the secondary by the flux $_{u}^{\prime\prime}\Phi$.
$f$ = frequency in cycles per second.

From the first assumption it follows that the total flux cuts all the turns in both the primary and secondary.

$$_{\phi}E_1 = \sqrt{2}\pi n_1 f\, {}_{u}^{\prime\prime}\Phi 10^{-8} \text{ volts} \qquad (321)$$

$$_{\phi}E_2 = \sqrt{2}\pi n_2 f\, {}_{u}^{\prime\prime}\Phi 10^{-8} \text{ volts} \qquad (322)$$

Hence,

$$\frac{\phi E_1}{\phi E_2} = \frac{n_1}{n_2} \tag{323}$$

From the second assumption,

$$\text{power} = \phi E_1 I_1 \cos\theta = \phi E_2 I_2 \cos\theta, \text{ or } n_1 I_1 = n_2 I_2 \tag{324}$$

Hence,

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} \tag{325}$$

Therefore in an "ideal" transformer the voltage ratio is directly and the current ratio inversely proportional to the number of turns. The ratio between the number of turns may be any value and in commercial designs it varies within wide limits.

It is difficult to represent clearly on the same vector diagram quantities differing much in magnitude, as, for instance, the primary and secondary voltage on a 20 : 1 ratio transformer. Hence

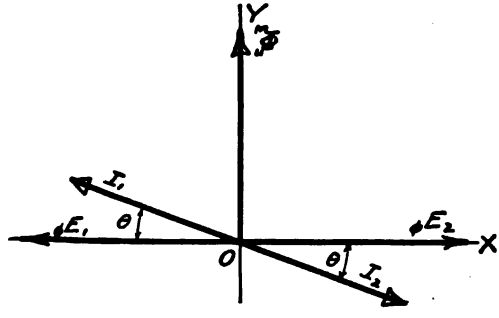



FIG. 143.

it is customary either to consider a transformer having a ratio of 1 : 1, for general illustrative purposes, or to let the vectors represent ampere-turns and voltage per turn instead of current and total voltage. This gives vectors of equal magnitude for the primary and secondary because $n_1 I_1 = n_2 I_2$, by equation (325) and $\frac{\phi E_1}{n_1} = \frac{\phi E_2}{n_2}$ by equation (323).

For an "ideal" transformer the vector diagram in Fig. 143 represents phase relations of the flux, currents and voltages. Let time be counted from the instant when the flux wave is passing through zero in the positive direction; then the magnetic flux vector is drawn upward along the $Y$-axis, and since the induced voltage in the secondary coil lags behind the flux by 90° it is repre-

sented by the vector $O_\phi E_2$ along the $X$-axis. The impressed voltage in the primary is equal and opposite to the voltage induced by the flux in the primary and is represented by the vector $O_\phi E_1$. Let the current in the secondary lag behind the voltage by $\theta^\circ$ and be represented by the vector $OI_2$. Then the corresponding current flowing in the primary is the vector $OI_1$, lagging behind the impressed voltage by $\theta^\circ$.

(*b*) **Magnetizing Current.**—While the basic relations of the magnetic flux, the voltages and currents are given in the description of the "ideal" transformer, even an elementary discussion of the actual transformer must take into account the magnetizing current and the effect of the resistance and reactance in both the primary and the secondary windings. With the secondary open and hence no current flowing in the secondary winding, a small current, sometimes incorrectly called the *leakage current*, flows in the primary providing the magnetomotive force, $n_{1m}I$, necessary to produce the flux, ${}_{u}^{n}\Phi$, in the transformer. This is the *magnetizing current.* As explained in Chap. XI, it is not a simple sine function of the time, but can be represented, approximately, by an equivalent sine wave, of equal effective value and equal power, leading the magnetic flux by an angle $\alpha$, the hysteretic angle of advance. The magnetizing current lags behind the impressed voltage, ${}_\phi E_1$, by $90^\circ - \alpha$ and can be divided into components in time phase and time quadrature with the voltage.

${}_mI \cos (90^\circ - \alpha)$ = the power component, and represents the losses from hysteresis and eddy currents, ${}_hP + {}_eP$.

${}_mI \sin (90^\circ - \alpha)$ = the magnetizing or wattless component, and represents the reactive power in the magnetic field.

In the symbolic notation:

$$ {}_mI = {}_mg_\phi E_1 - j_m b_\phi E_1 \tag{326} $$

$$ {}_mg_\phi {E_1}^2 = {}_hP + {}_eP, \text{ or } {}_mg = \frac{{}_hP + {}_eP}{{}_\phi {E_1}^2} = \frac{V\eta f \mathfrak{B}^{1.6} + V\epsilon\lambda f^2 \mathfrak{B}^2}{{}_\phi {E_1}^2} \tag{327} $$

The susceptance, ${}_mb$, can be found from the dimensions of the magnetic circuit, the quality of the iron and maximum flux density.

$A$ = cross-section of iron core in square centimeters.

${}_{u}^{M}\Phi$ = maximum useful lines of force

$\mathfrak{B} = \frac{{}_{u}^{M}\Phi}{A}$ = maximum flux density.

$\mathfrak{R}$ = the reluctance of the magnetic circuit for the density $\mathfrak{B}$.

The maximum magnetomotive force required to magnetize the iron core is equal to the product of the reluctance of the magnetic circuit and the maximum core flux.

$$\frac{4}{10}\pi n_1\sqrt{2}{}_{m}I\sin(90^\circ-\alpha)=\frac{4}{10}\pi n_1\sqrt{2}{}_{m}b_{\phi}E_1={}_{u}^{M}\Phi\,\mathfrak{R} \qquad (328)$$

$${}_{m}b=\frac{10\,{}_{u}^{M}\Phi\mathfrak{R}}{4\pi\sqrt{2}n_{1\phi}E_1}=0.56\,\frac{{}_{u}^{M}\Phi\mathfrak{R}}{n_{1\phi}E_1} \qquad (329)$$

It should be noted that ${}_{\phi}E_1$, in equations (326) to (329) is that part of the total voltage $E_1$ impressed on the primary terminals which is balanced by the inducing action of the magnetic flux, ${}_{u}^{M}\Phi$, cutting the $n_1$ turns of the primary winding. However, as will be shown later, the loss of voltage in the primary winding is small, and therefore in most cases the terminal voltage $E_1$ may be substituted for ${}_{\phi}E_1$ in equations dealing with the magnetizing current, without introducing appreciable errors.

(*c*) **Magnetic Leakage.**—In the preceding paragraph the magnetizing current has been discussed under no load or open secondary circuit conditions. As the maximum flux interlinking both the primary and the secondary circuits, that is the useful flux in the iron core, is approximately the same for all load conditions, the magnetizing current remains practically the same under load as when the secondary is open. With the transformer loaded and hence a current flowing in the secondary circuit, magnetomotive forces equal to the product of the currents into the respective number of turns appear in both the secondary and the primary. In the primary circuit an m.m.f. equal to $n_1I_1$ produces a flux which surrounds the primary winding. This flux may be separated into two components, one part in the iron interlinking all the turns of both the primary and secondary circuits; and a second part, the leakage flux ${}_{l}\phi_1$, in Fig. 142, very much smaller, lies in the space outside the iron core, occupied by the insulation and interlinks part or all of the primary turns, but does not interlink with the secondary. Similarly the secondary m.m.f. produces a magnetic flux in the opposite direc-

tion, one part in the iron and a second part, ${}_{l}\phi_2$, in Fig. 142, passing through the space occupied by the insulation and interlinking part or all of the secondary turns but not interlinking with the primary winding. In the shell type, Fig. 141, only that part of the flux in the air lying between the primary and secondary windings is the leakage flux. All lines of force cutting both the primary and secondary windings are useful and hence grouped as ${}_{u}^{n}\Phi$. Since the m.m.f. of the primary is opposed by the m.m.f. of the secondary, the resultant or difference is at all loads, the m.m.f. of the magnetizing current which produces the useful flux, ${}_{u}^{n}\Phi$, interlinking both the primary and secondary windings.

$$n_2\dot{I}_2 = n_1\dot{I}_{11} \tag{330}$$

$$n_1\dot{I}_1 = n_{1m}\dot{I} + n_1\dot{I}_{11} \tag{331}$$

Hence the resultant m.m.f. for both primary and secondary, in the iron core:

$$n_1\dot{I}_1 - n_2\dot{I}_2 = n_1\dot{I}_{11} + n_{1m}\dot{I} - n_2\dot{I}_2 = n_{1m}\dot{I} \tag{332}$$

The lines of force outside of the iron core and interlinking all or part of the turns in either the primary or the secondary circuit, but not both constitute what is called the leakage flux.

${}_{l}\phi_1$ = primary leakage flux, proportional to $I_1$.
${}_{l}\phi_2$ = secondary leakage flux, proportional to $I_2$.

The leakage flux linkages are therefore a simple inductance in each circuit and can be represented by a primary and a secondary reactance.

In general transformers are designed for minimum leakage reactance, but under certain conditions the service required necessitates the insertion of a considerable amount of reactance in the circuit, as for example in the operation of rotary converters on a rapidly varying load. For this service the transformer is designed with a magnetic bridge between the primary and secondary windings which gives a large leakage reactance.

(*d*) **Circuit Diagram.**—In Fig. 144 is shown a circuit diagram for a potential transformer receiving power over a line from a generator and supplying a load.

$E_0$ = generator voltage.
$E_1$ = voltage impressed on the transformer primary.
$\phi E_2$ = voltage induced in the secondary by the flux ${}_{s}^{n}\Phi$.

$_{\phi}E_1 = \frac{n_1 {}_{\phi}E_2}{n_2}$, voltage in primary balancing the voltage induced by the flux, $_{m}\Phi$, in the primary.

$E_2$ = terminal voltage of secondary.

$r_0$ = line resistance.

$x_0$ = line reactance.

$r_1$ = primary resistance.

$x_1$ = primary reactance.

$r_2$ = secondary resistance.

$x_2$ = secondary reactance.

$r_3$ = load resistance.

$x_3$ = load reactance.

$_{m}g$ = equivalent conductance in magnetizing circuit.

$_{m}b$ = equivalent susceptance in magnetizing circuit.

$I_1$ = primary current.

$I_2$ = secondary current.

$_{m}I$ = magnetizing current.

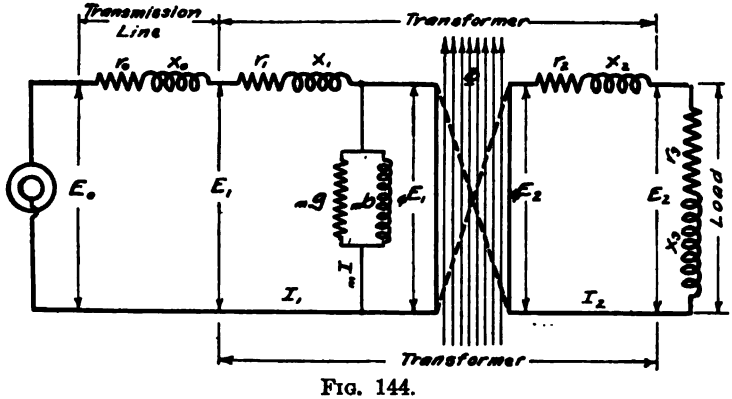



FIG. 144.

The broken lines across the magnetic flux lines are intended to indicate that while the power is transmitted from the primary to the secondary winding by the flux, the directions of the current and voltage in the two windings are reversed. The magnetizing current flows in an equivalent parallel circuit having a conductance, $_{m}g$, for the hysteresis and eddy-current losses, and a susceptance, $_{m}b$, for the reactive power in the magnetic field. Starting at the load end, it is evident that the voltage $_{\phi}E_2$ induced in the secondary is equal to the vector sum of the terminal voltage, $E_2$, and the impedance drop in the secondary caused by $r_2$ and $x_2$.

$$_{\phi}\dot{E}_2 = \dot{I}_2(r_2 + jx_2) + \dot{E}_2 \qquad (333)$$

The voltage in the primary, $_{\phi}E_1$, balanced by the induced voltage in the primary and equivalent to $-\frac{n_1}{n_2} {}_{\phi}E_2$, combined vec-

torially with the impedance drop in the primary is equal to the voltage $E_1$ impressed on the primary.

$$\dot{E}_1 = \dot{I}_1(r_1 + jx_1) + {}_{\phi}\dot{E}_1 = \dot{I}_1(r_1 + jx_1) - \frac{n_1}{n_2}{}_{\phi}E_2 \quad (334)$$

The voltage impressed on the transformer, $E_1$, combined vectorially with the impedance drop of the line equals the generator voltage, $E_0$.

$$\dot{E}_0 = \dot{I}_1(r_0 + jx_0) + \dot{E}_1 \quad (335)$$

The current in the primary, $I_1$, consists of the equivalent primary load current, $I_{11} = -\frac{n_2}{n_1}I_2$, combined vectorially with the magnetizing current, ${}_{m}I$.

$$\dot{I}_1 = {}_{m}\dot{I} + \dot{I}_{11} = {}_{m}\dot{I} - \frac{n_2}{n_1}\dot{I}_2 \quad (356)$$

(*e*) **Vector Diagrams.**—The time is selected so that the magnetic flux vector lies in the positive direction on the *Y*-axis,

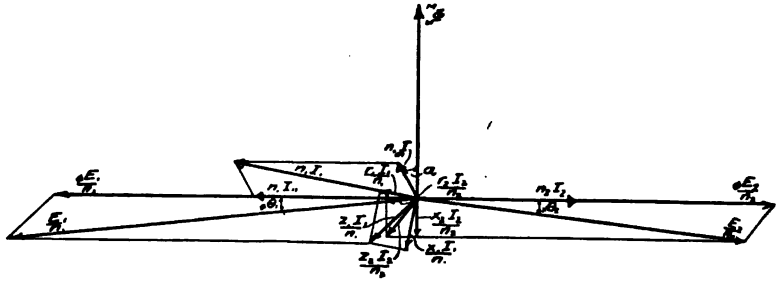


FIG. 145.

Fig. 145. The voltage, $\frac{\phi E_2}{n_2}$, induced per turn in the secondary, by the flux ${}_{u}^{n}\Phi$, lags behind the flux by 90° and therefore is drawn in the positive direction along the *X*-axis. The impressed voltage in the primary, $\frac{\phi E_1}{n_1}$, required to balance the voltage induced per turn in the primary by the flux ${}_{u}^{n}\Phi$ is equal to $\frac{\phi E_2}{n_2}$ and opposite in direction. Hence $\frac{\phi E_1}{n_1}$ is drawn along the *X*-axis in the negative direction. Let the current in the secondary be in phase with the induced voltage, ${}_{\phi}E_2$, then the ampere-turns are represented by the vector $n_2I_2$. The impedance drop per

turn in the secondary is given by the vectors $\frac{r_2}{n_2}I_2$ and $\frac{x_2}{n_2}I_2$, in time phase and in time quadrature, respectively, with the current $I_2$. The secondary terminal or load voltage, $\frac{E_2}{n_2}$, is the vector sum of $\frac{{}_\phi E_2}{n_2}$, $\frac{r_2 I_2}{n_2}$ and $\frac{x_2 I_2}{n_2}$. Hence $\frac{E_2}{n_2}$ closes the voltage triangle in the secondary.

The current in the primary $I_1$ is the vector sum of the magnetizing current ${}_m I$ and $I_{11}$, the current equivalent to the secondary and differing by 180° in phase, as given by equation (336). The impedance drop per turn in the primary is given by $\frac{r_1}{n_1}I_1$ and $\frac{x_1}{n_1}I_1$ in time phase and in time quadrature, respectively, with the primary current. The impressed voltage $\frac{E_1}{n_1}$ per turn in the primary is therefore the vector sum of $\frac{{}_\phi E_1}{n_1}$, $\frac{r_1}{n_1}I_1$ and $\frac{x_1}{n_1}I_1$.

In illustrative vector diagrams for transformers the phase relations are of much more importance than the relative magnitudes of the currents and voltages. Therefore, in Figs. 146 to 155 inclusive, the resistance and reactance drops, both in the primary and secondary and the magnetizing currents are, relatively to the impressed voltages and load currents, represented as from ten to twenty times as large as in ordinary commercial transformers. Also the current and voltage notation represents ampere-turns and volts per turn, or the ratio of the number of turns has been taken as unity.

In the series, Figs. 146 to 155, the power factor of the load is varied so as to illustrate the effect of the transformer impedance on the terminal voltages for lagging and leading currents. The angle ${}_\phi\theta_2$ refers to the phase difference of the secondary current and the induced secondary voltage. It is therefore the time lag in the secondary circuit due to the total reactance, that is to the sum of the internal reactance in the secondary circuit and the external or load reactance.

The following values are constant:

${}_u^n\Phi$, the maximum flux.
$f$, the frequency.
$n_1$ and $n_2$, the number of turns in primary and secondary.

$r_1$ and $r_2$, the resistance in the primary and secondary.
$x_1$ and $x_2$, the reactance in the primary and secondary.
${}_mI$, the magnetizing current (ampere-turns).
$\alpha$, the hysteretic and eddy-current angle of advance assumed to be 30°

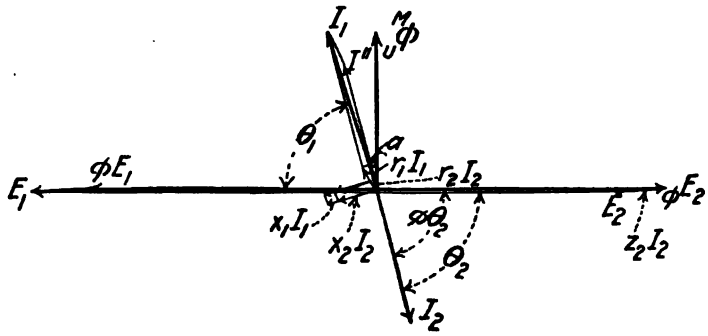



FIG. 146.—$_\phi\theta_2 = 75°$, lagging current.

${}_mg$ and ${}_mb$, the conductance and susceptance of magnetizing circuit.
$I_2$, the secondary current (ampere-turns).
$I_{11}$, component of primary current opposite and equal to $I_2$ (ampere-turns).

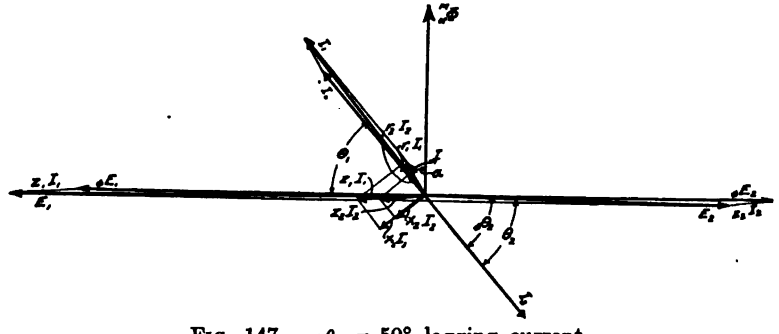


FIG. 147.—$_\phi\theta_2 = 50°$, lagging current.

$_\phi\theta_2$, the angle between the load current and the induced voltage in the secondary is assumed to have the following values:
Fig. 146, $_\phi\theta_2 = 75°$ lagging current.
Fig. 147, $_\phi\theta_2 = 50°$ lagging current.
Fig. 148, $_\phi\theta_2 = 25°$ lagging current.

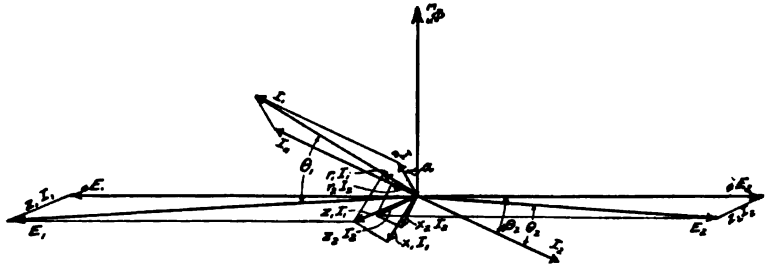


FIG. 148.— $\phi\theta_2 = 25°$, lagging current.

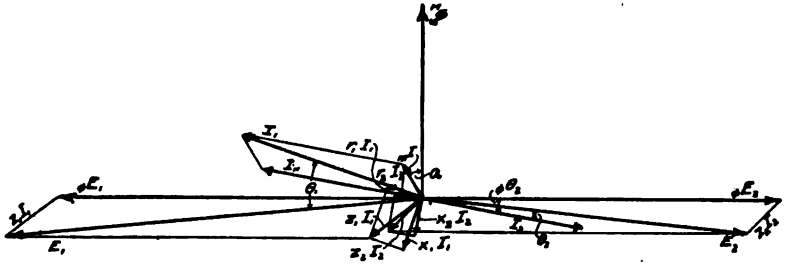


FIG. 149.— $\phi\theta_2 = 10°$, lagging current.

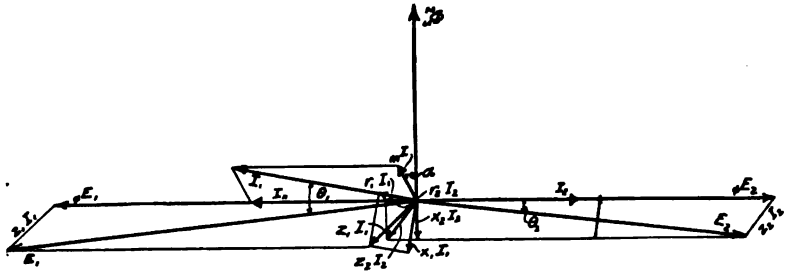


FIG. 150.— $\phi\theta_2 = 0°$.

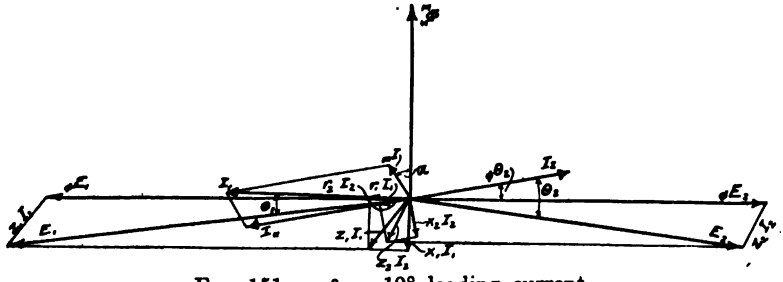


FIG. 151.— $\phi\theta_2 = 10°$, leading current.

Fig. 149, $_{\phi}\theta_2 = 10°$ lagging current.
Fig. 150, $_{\phi}\theta_2 = 0°$ unity power factor.

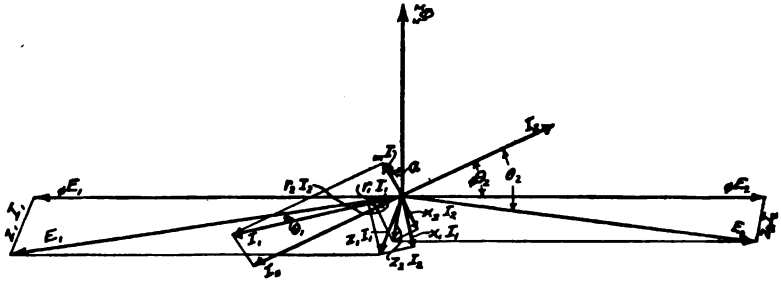


FIG. 152.—$_{\phi}\theta_2 = 25°$, leading current.

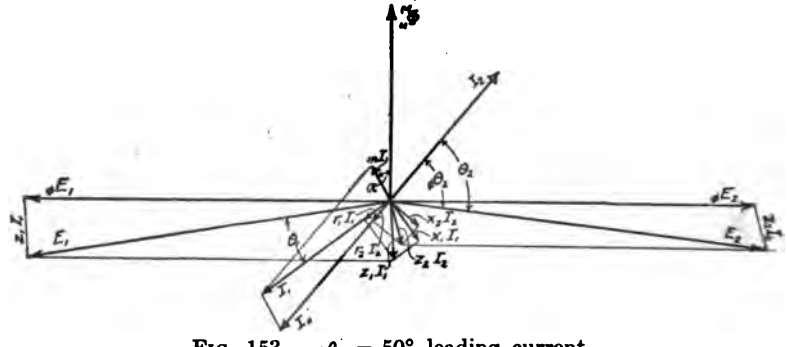


FIG. 153.—$_{\phi}\theta_2 = 50°$, leading current.

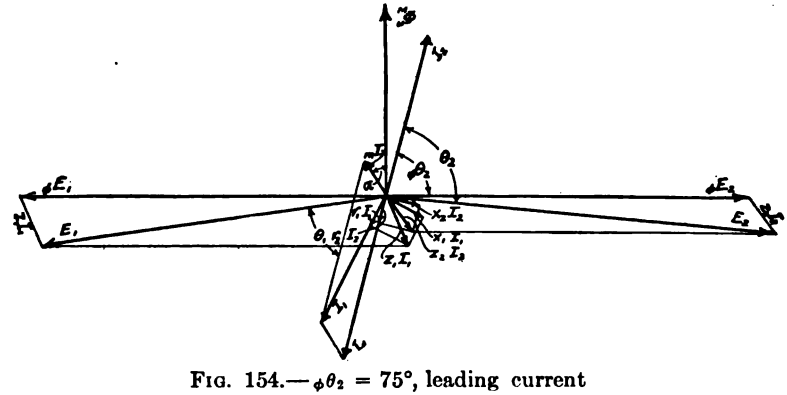


FIG. 154.—$_{\phi}\theta_2 = 75°$, leading current

Fig. 151, $_{\phi}\theta_2 = 10°$ leading current.
Fig. 152, $_{\phi}\theta_2 = 25°$ leading current.

Fig. 153, $_{\phi}\theta_2 = 50°$ leading current.
Fig. 154, $_{\phi}\theta_2 = 75°$ leading current.
Fig. 155, $_{\phi}\theta_2$ varies from 90° lagging to 90° leading.

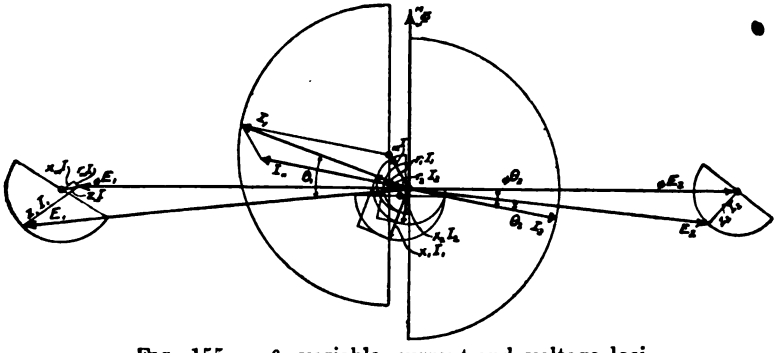


FIG. 155.—$_{\phi}\theta_2$ variable, current and voltage loci.

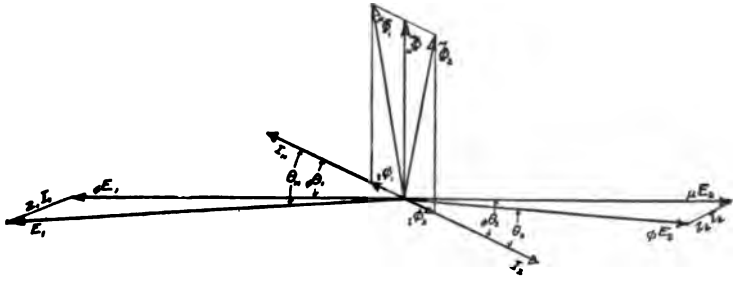


FIG. 156.—$_{\phi}\theta_2 = 25°$, lagging current.

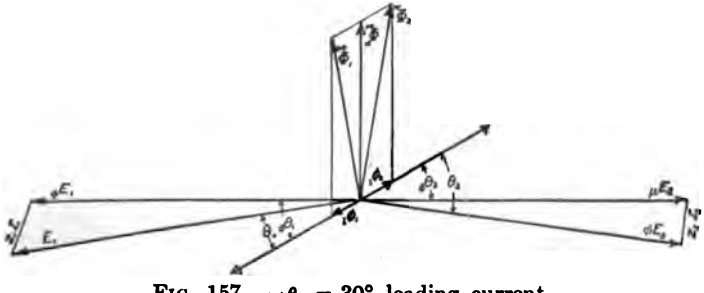


FIG. 157.—$_{\phi}\theta_2 = 30°$, leading current.

In the preceding diagrams $^{\mu}_{\mu}\Phi$ represents the maximum useful magnetic flux, or the lines of force passing through both the primary and secondary coils. The total flux in each coil is the vector sum of $^{\mu}_{\mu}\Phi$ and the leakage flux, $_{1}\phi_1$. In Figs. 141,

142, 156 and 157 are shown useful flux, ${}_{u}{}^{M}\Phi$, the primary leakage flux ${}_{L}\phi_1$, the secondary leakage flux, ${}_{L}\phi_2$, the total primary flux ${}^{M}\Phi_1$ and the total secondary flux, ${}^{M}\Phi_2$. Since the leakage flux, representing the reactive power in the transformer, is necessarily in phase with the current, the total primary and secondary fluxes vary both in magnitude and phase position for changes in the angle of lead or lag of the load current.

For the assumed values of ${}_{\phi}\theta_2$ the following quantities are found graphically:

$I_1$ = the primary current (ampere turns).
$E_1$ = the primary impressed voltage (volts per turn).
$\theta_1$ = the phase angle of $E_1$ and $I_1$.
$E_2$ = the secondary terminal voltage (volts per turn).
$\theta_2$ = the phase angle of $E_2$ and $I_2$.

(*f*) **Equivalent Circuits.**—The transfer of power from the generator to the load through the transformer is much the same as if the primary and secondary formed a series circuit. With a change in the secondary constants the transformer circuit in Fig. 144 may be represented by the equivalent continuous circuit in Fig. 158. In the secondary circuit electric energy is changed into heat by the $r_2I_2^2$ losses. In the equivalent circuit in Fig. 158 the same loss must be represented by the product of a resistance $r'$ into the square of the equivalent primary current.

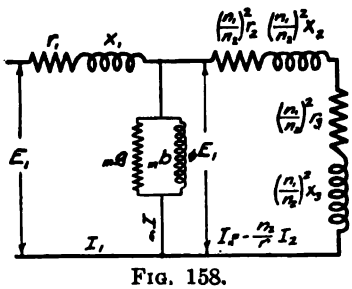


FIG. 158.

$$r'I_{11}^2 = r_2I_2^2 \tag{337}$$

$$I_2 = \frac{n_1}{n_2}I_{11} \tag{338}$$

Hence the equivalent resistance $= \left(\frac{n_1}{n_2}\right)^2 r_2$ (339)

Similarly the equivalent secondary reactance $= \left(\frac{n_1}{n_2}\right)^2 x_2$ (340)

Likewise the equivalent load resistance $= \left(\frac{n_1}{n_2}\right)^2 r_3$ (341)

And the equivalent load reactance $= \left(\frac{n_1}{n_2}\right)^2 x_3$ (342)

Since the resistance and reactance of the primary are small,

the difference between the impressed voltage $E_1$ and the voltage inside the transformer $_\phi E_1$ is small compared to $E_1$. Hence the circuit may be simplified by transferring the magnetizing current circuit to the primary terminals, as shown in Fig. 159*a*. A further simplification in notation may be made by letting $r_t$

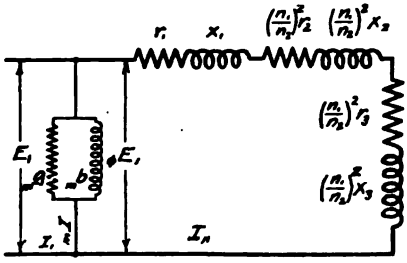



FIG. 159*a*.

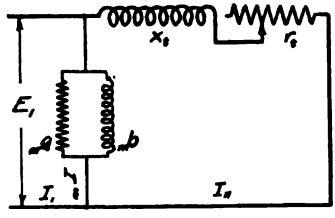



FIG. 159*b*.

equal the equivalent total resistance and $x_t$ the equivalent total reactance of the primary and secondary windings and of the load.

$$r_t = r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2 + \left(\frac{n_1}{n_2}\right)^2 r_3 \qquad (343)$$

$$x_t = x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2 + \left(\frac{n_1}{n_2}\right)^2 x_3 \qquad (344)$$

The equivalent simple circuit diagram which closely approximates the actual transformer is shown in Fig. 159*b*.

(*g*) **The Circle Diagram.**—The conductance, $_m g$, and susceptance, $_m b$, are constant, and hence for constant impressed voltage, $E_1$, the magnetizing current, $_m I$, is constant. The equivalent total resistance, $r_t$, and reactance, $x_t$, vary with the load. For non-inductive load the total reactance, $x_t$, is constant. The locus of the extremity of the vector representing the current flowing through a constant reactance, $x_t$, and a variable resistance, $r_t$, for a constant impressed voltage, $E_1$, is a semicircle, as explained in Chap. VI. The corresponding vector diagram, called the *transformer circle diagram*, is shown in Fig. 160. Counting time

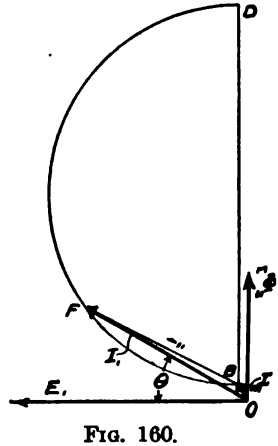



FIG. 160.

as in the previous diagrams the flux, ${}_{u}^{n}\Phi$, is drawn upward along the $Y$-axis. The magnetizing current, ${}_{m}I$, leads the flux by $\alpha$, the hysteretic angle of advance. The impressed voltage leads the flux by 90°. From the end of the magnetizing current vector draw the line $BD$ parallel to the magnetic flux and of length equal to $\frac{E_1}{x_t}$. Upon $BD$ as diameter describe the semicircle $BFD$. This is the locus of the equivalent load current, $I_{11}$. The total primary current, $I_1$, is the vector sum of ${}_{m}I$ and $I_{11}$.



FIG. 161.—Assembling the coils of a shell type air-blast transformer. (*Westinghouse Elec. & Mfg. Co.*)

(*h*) **Regulation.**—For constant-potential transformers, regulation is defined as the per cent. increase of secondary voltage from full load to no load, the primary impressed voltage being constant. In the vector diagrams the induced voltage has been assumed constant, thus indicating a change in both the primary and secondary terminal voltage. Assuming that the drop of voltage in the primary due to the magnetizing current is negligible, then at no load the terminal voltages are directly proportional to the number of turns. Hence, at no load the secondary terminal voltage is equal to $\frac{n_2}{n_1}E_1$.

Let $E_2$ be the secondary terminal voltage at full load, the primary voltage, $E_1$, remaining constant.

$$\text{Regulation} = 100\frac{\frac{n_2}{n_1}E_1 - E_2}{E_2}. \tag{345}$$

To determine the regulation from transformer constants either $E_1$ or $E_2$ must be found in terms of the primary and secondary resistances and reactances and the power factor of the load. The vector diagram in Fig. 162 represents the full-load voltage,

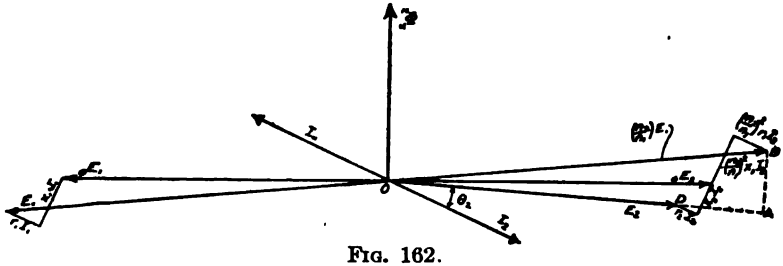


FIG. 162.

$E_2$, and corresponding no-load voltage, $\frac{n_2}{n_1}E_1$, with the load current, $I_2$, lagging $\theta_2°$.

$r_2I_2$ = voltage consumed in secondary by $r_2$.

$x_2I_2$ = voltage consumed in secondary by $x_2$.

$\left(\frac{n_2}{n_1}\right)^2 r_1I_2$ = equivalent voltage consumed in secondary by $r_1$.

$\left(\frac{n_2}{n_1}\right)^2 x_1I_2$ = equivalent voltage consumed in secondary by $x_1$.

Let the line $BA$ be drawn perpendicular to $OD$ extended.

$$DA = \left(x_2I_2 + \left(\frac{n_2}{n_1}\right)^2 x_1I_2\right)\sin\theta_2 + \left(r_2I_2 + \left(\frac{n_2}{n_1}\right)^2 r_1I_2\right)\cos\theta_2 \tag{346}$$

$$BA = \left(x_2I_2 + \left(\frac{n_2}{n_1}\right)^2 x_1I_2\right)\cos\theta_2 - \left(r_2I_2 + \left(\frac{n_2}{n_1}\right)^2 r_1I_2\right)\sin\theta_2 \tag{347}$$

$$\frac{n_2}{n_1}E_1 = \sqrt{(E_2 + DA)^2 + (BA)^2} \tag{348}$$

The substitution of (346) and (347) gives an unwieldy equation and it is simpler to solve for the numerical values of $DA$ and $BA$ before substitution in (348). Approximate formulæ are generally used in commercial calculations.

(*i*) **Losses.**—The transformer consists of a laminated iron core, copper conductors and insulation material or the dielectric.

The iron losses were discussed in Chap. XI and found to be

$$_hP = \eta V f \mathfrak{B}^{1.6} 10^{-7} \text{ watts due to hysteresis, and} \quad (349)$$

$$_eP = \epsilon \lambda V f^2 \mathfrak{B}^2 10^{-7} \text{ watts due to eddy currents.} \quad (350)$$

For certain grades of silicon steel $_hP = \eta V f \mathfrak{B}^{1.7} 10^{-7}$ watts.

The dielectric losses are similar to the iron losses but relatively very small as explained in Chap. XIX. In commercial tests they are usually included with the hysteresis and eddy-current losses of the iron. In the copper conductors the losses are given by Joule's law:

$$P_c = r_1 I_1^2 + r_2 I_2^2 \quad (351)$$

These losses are slightly larger than for direct currents on

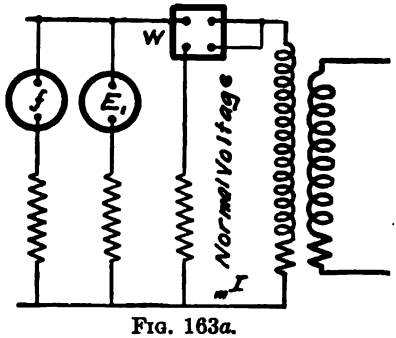



FIG. 163*a*.

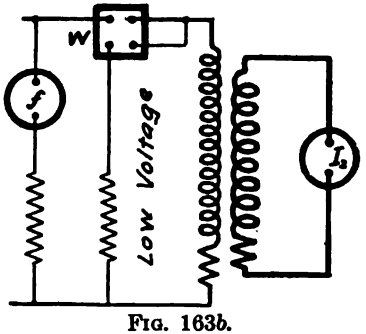



FIG. 163*b*.

account of eddy currents in the copper conductors. The effective values of $r_1$ and $r_2$ in equation (351) are therefore proportionately larger than the ohmic resistance of the primary and secondary windings.

$$\text{Total loss} = {}_hP + {}_eP + P_c \quad (352)$$

The power lost in the transformer is changed into heat.

In commercial tests the losses are determined by wattmeter readings for the specified primary voltage, full-load current, and frequency. The circuit diagram for finding the iron losses is shown in Fig. 163*a*. The secondary is open. The copper loss in

the primary due to the magnetizing current is negligible, and hence the wattmeter reading gives the hysteresis and eddy-current losses in the iron and the dielectric.

The circuit diagram for finding the copper losses is shown in Fig. 163*b*. The secondary is short-circuited through an ammeter.



FIG. 164.—Building up of iron of a sheet type air-blast transformer.

On the primary is impressed sufficient voltage, at the specified frequency, to send approximately full-load current through the ammeter in the secondary. The magnetization and voltage are so low that the iron and dielectric losses are negligible. The wattmeter reading therefore gives the copper losses. The sum of the two readings is the total power lost in the transformer; the losses in the ammeter itself are considered negligible.

(*j*) **Efficiency.**—The efficiency is the ratio between the output and the input.

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{E_2 I_2 \cos \theta_2}{E_2 I_2 \cos \theta_2 + R I_2^2 + {}_hP + {}_eP} \quad (353)$$

$R$ = the equivalent resistance of the secondary and primary. The efficiency is a maximum when $\frac{d\eta}{dI_2} = O$.

$$(E_2I_2 \cos \theta_2 + RI_2^2 + {}_hP + {}_eP)E_2 \cos \theta_2 - E_2I_2 \cos \theta_2(E_2 \cos \theta_2 + 2RI_2) = O$$

or

$${}_hP + {}_eP = RI_2^2 \qquad (354)$$

That is, the iron loss is equal to the copper loss.

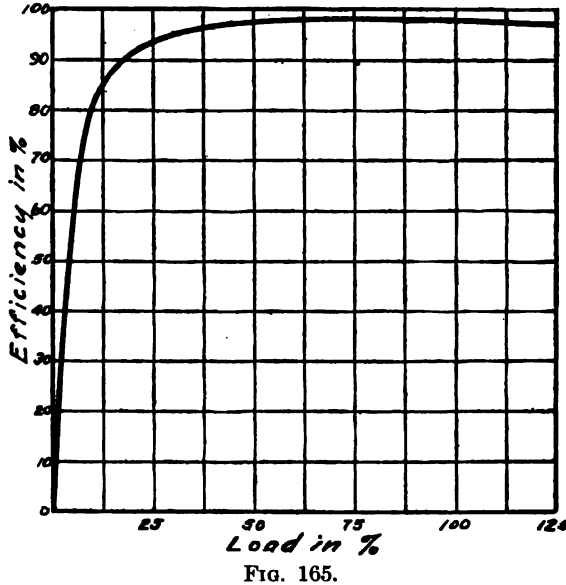



FIG. 165.

Transformers are usually so designed as to have the maximum efficiency between three quarters and full load. While the efficiency at no load is zero the losses at all loads are small. The curve rises rapidly, and lies nearly horizontal from quarter to full load, as is shown in the typical efficiency curve in Fig. 165. Potential transformers have high efficiencies; large commercial units give 99 per cent. at full load. In commercial constant-potential systems the primary voltage is impressed on the transformer continuously, while power is taken from the secondary only part of the time. Thus in residence lighting the lamps are lighted only a few hours out of the twenty-four. The average efficiency, or all-day efficiency as it is called, will there-

fore be lower since the iron losses are constant throughout the 24 hr. If the transformer carries a steady unity power factor load, for $k$ hours each day, the all-day efficiency will be expressed by equation (355).

All-day energy efficiency:

$$\frac{E_2 I_2 k}{E_2 I_2 k + R I_2^2 k + 24({}_iP + {}_cP)} \tag{355}$$

Distributing transformers are designed with a smaller iron loss than full-load copper loss, thereby increasing the average efficiency.

(*k*) **Rating.**—Transformers are designed for service at specified terminal voltage, frequency and load. For a properly designed transformer the temperature becomes the limiting factor in determining the load which the transformer can carry. The copper and iron losses are converted into heat inside the transformer. The temperature will necessarily rise until the rate of removal and generation of heat are equal. In small transformers the heat is dissipated by radiation and air conduction. The windings and core are immersed in oil to assist in the conduction of the heat to the surface of the casing. To increase the radiating surface the casing is often corrugated. Large transformers are usually *water-cooled*; that is, the heat is removed by means of cold water flowing through a system of pipes inside the transformer. The essential point is the fact that the temperature limit is the most important factor in determining the kilowatt or kv.a. rating of transformers. By the A.I.E.E. rules the temperature of the transformer should not rise more than 50°C. above the surrounding air temperature.

(*l*) **Voltage Ratios.**—In order to manufacture and operate transformers, lamps, motors and other electrical appliances in an economical manner, uniformity in voltage is necessary; and certain standard voltages have been adopted for commercial systems. Thus for residence lighting and for small motors 110 and 220 volts are considered standard. The distributive network between the substation and the transformer near the customer is generally operated at 2,200 volts. Hence the pole transformer transmitting energy to the lighting and small motor load in any small division of the distribution system has a 10 and 20:1 voltage ratio. Circuits connecting the several substations in large systems are operated at voltages from 11,000

to 22,000. Hence the substation transformer ratio is from 5 : 1 to 10 : 1.

Long-distance transmission lines operate at higher voltages as 60,000, 110,000 or some similar voltage best adapted to the particular system, and this again requires transformers giving the required voltage ratio, stepping up at the generating station and stepping down at the substation end of the high-tension line. While the transformer can be built for any voltage ratio within wide limits, most standard types are designed for a 5, 10 or 20 : 1 ratio.

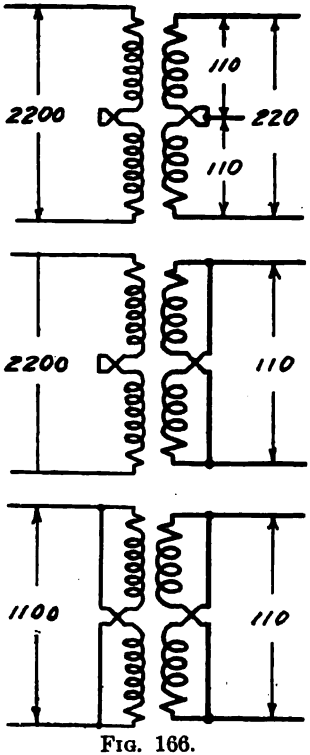


FIG. 166.

By dividing both the primary and secondary coils into two parts, different ratios may be obtained by connecting the coils in series or in parallel as shown in Fig. 166.

(*m*) **Connections.**—Although innumerable ways for connecting transformers may be devised, only a few forms are in general commercial use. Simplicity in wiring, efficiency in the use of apparatus, minimum insulation stresses, standardization of voltages and distributing apparatus have made uniformity in transformer connections necessary.

1. *Single-phase, Three-wire System.*—The secondary is divided into two coils, or a tap is brought out at the middle point as shown in Fig. 167. The middle or neutral wire is grounded in order to protect the customer as well as the apparatus. In general the 2,200-volt side has only two wires and no neutral connection. In the vector diagram *OD* represents the voltage from either line to neutral and *OH* the voltage between the mains *A* and *B*. The system is widely used for general distribution, residence lighting, small motors, etc.

2. *Two-phase, Four-wire System.*—This system consists essentially of two separate single-phase circuits both on the primary and secondary side, Fig. 168. The voltages in the two phases

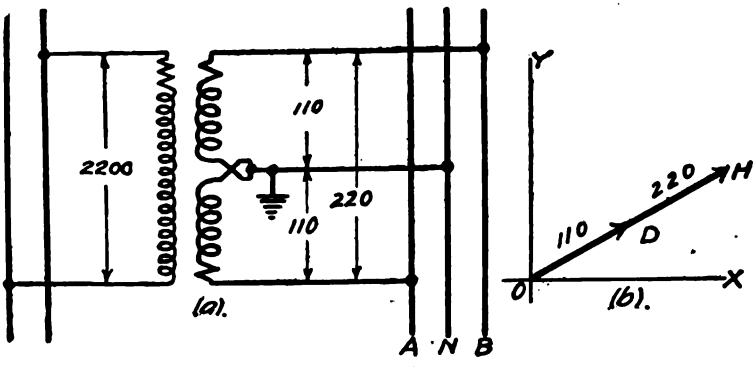
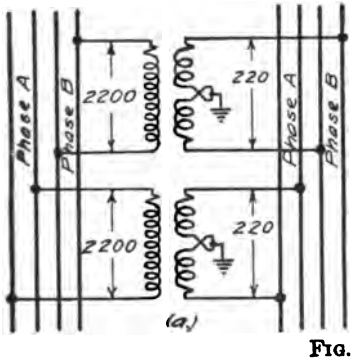



FIG. 167.



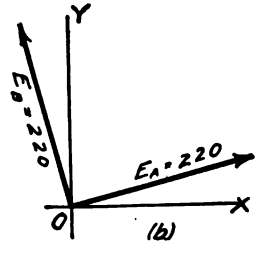




FIG. 168.

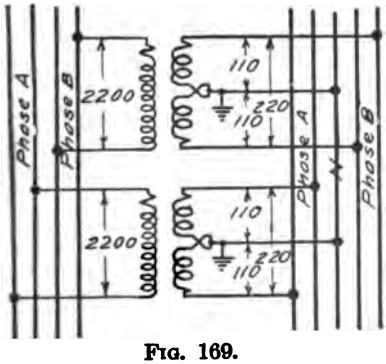



FIG. 169.

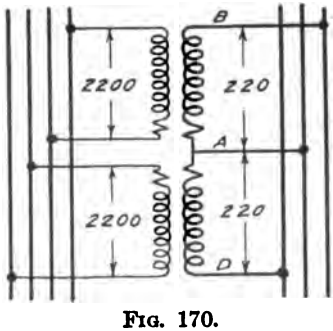



FIG. 170.

differ by 90 electrical degrees. The secondary circuits often are three-wire systems for each phase similar to Fig. 167. To save copper the neutrals for both phases are combined, Fig. 169. The five wires on the secondary side give 110 and 220 volts in both phases.

3. *Two-phase, Three-wire System.*—By combining one wire from each phase in a two-phase system less copper is required,

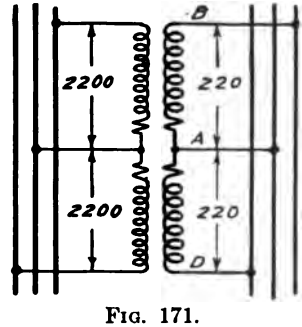



FIG. 171.

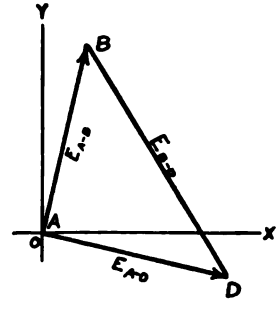
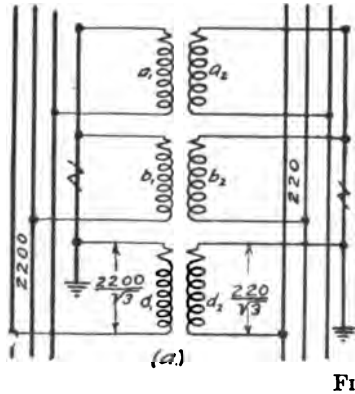



FIG. 172.

since the current in the common wire is the vector sum of the currents in the two phases. In a balanced system the current in the common wire is $\sqrt{2}$ times the current in either of the



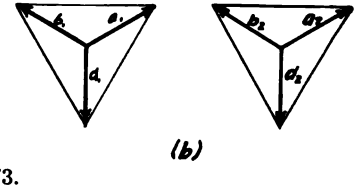




FIG. 173.

other wires. Hence for the same voltage drop the copper required for the third wire is in the ratio of 1.41:2 as compared to the corresponding two wires in the four-wire system. Wiring and vector diagrams are shown in Figs. 170, 171 and 172.

4. *Three-phase, Y Connection, Three-wire System.*—The con-

nections and voltage relation are shown in Fig. 173. This system is well adapted for general power transmission but if one transformer is disabled the whole system is interrupted. The *Y* arrangement of the transformers provides a desirable neutral or ground connection.

5. *Three-phase, Y Connection, Four-wire System.*—This system is sometimes used for a combination of power and lighting loads,



FIG. 174.—Single phase, 60 cycle, 120,000/15,000. To carry 5,000 kv.a. as water-cooled and to carry 2,500 kv.a. as self-cooled (no water).

especially in connection with rotary converters in combination with the Edison three-wire system. The connections are shown in Fig. 175, secondary side.

The chief advantages of this system are (*a*) increased flexibility and (*b*) the neutral wire carries all unbalanced currents.

6. *Three-phase, Δ Connection, Three-wire System.*—The connections are shown in Fig. 176. One advantage of the Δ connection is the possibility of supplying power on all three phases

by the use of only two transformers as explained in the next paragraph.

7. *Three-phase, Open Delta or V Connection.*—In three-phase circuits, single-phase transformers are used in each of the three phases. In the delta connection continuous operation may be

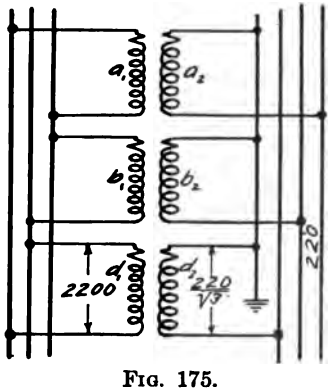



FIG. 175.

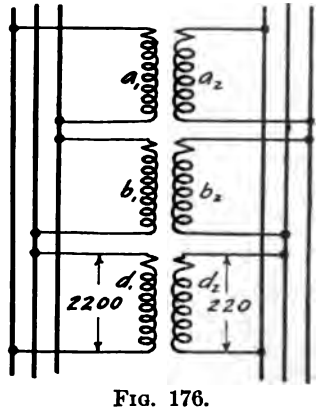



FIG. 176.

maintained by means of any two of the three transformers. This is sometimes desirable in a new district with light load. The third transformer may be added as soon as the load increases beyond the capacity of the two transformers. Also in case one transformer is damaged the *V* or open delta connection becomes

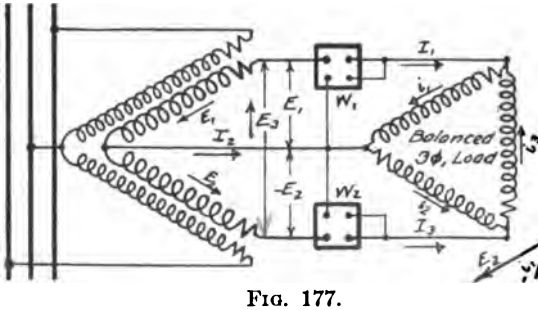



FIG. 177.

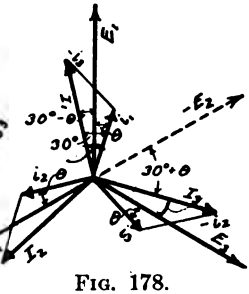



FIG. 178.

a convenient arrangement for continuing the service until the damaged unit is repaired or replaced. However, the *V* connection introduces certain undesirable features that may cause dangerous disturbances in the system. It should also be noted that the two transformers in open delta will heat faster for the

same load than when in the closed delta connection. As proved below, the capacity is reduced to 86.7 per cent. of the full-load rating; so that two transformers in open delta will carry, with the same rise in temperature, only $86.7 \times \frac{2}{3} = 57.7$ per cent. of the load carried by the three transformers in closed delta. The circuit diagram is shown in Fig. 177 and the corresponding vector diagram in Fig. 178.

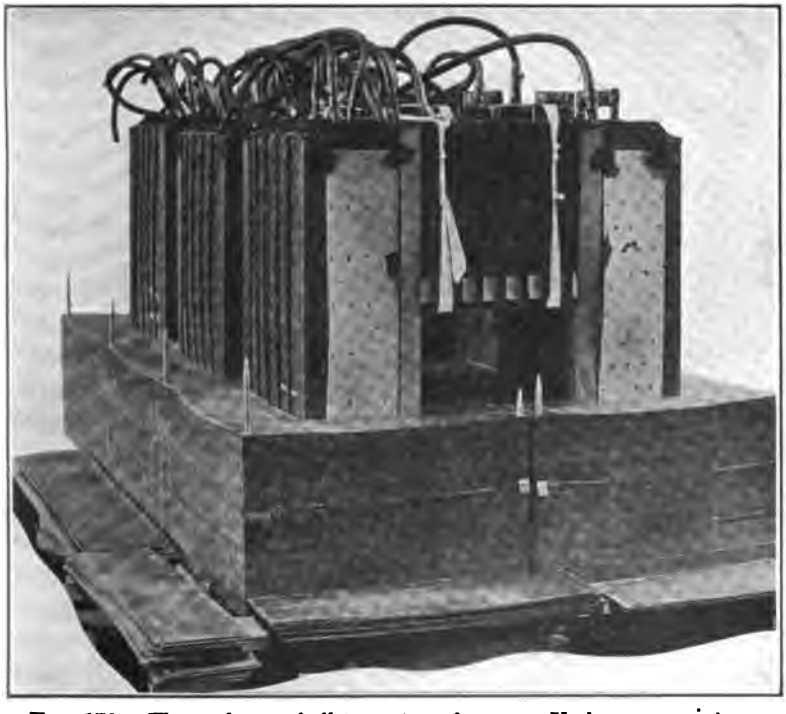


FIG. 179.—Three-phase, shell-type transformer. Under construction.

The load is balanced and hence,

$$i_1 = i_2 = i_3,\ I_1 = I_2 = I_3 \text{ and } \theta_1 = \theta_2 = \theta_3.$$

The power passing through the transformers is measured by the two wattmeters connected as shown in Fig. 177. The same current flows through the transformer as in the current coil of the wattmeter, and hence the same as in the mains instead of in the circuits as would be the case with three transformers connected in delta.

If the connection of the voltage coil for wattmeter $W_1$ be

taken as indicated by the arrows in the figure, then the connection for wattmeter $W_2$ is in the reverse direction.

$$W_1 = E_1 I_1 \cos (30° - \theta) \text{ and}$$
$$W_2 = E_2 I_3 \cos (30° + \theta) \qquad (356)$$

Total power (two transformers)

$$= EI[\cos (30° - \theta) + \cos (30° + \theta)]$$
$$= 2EI \cos \theta \cos 30° = 2EI \cos \theta \times 0.867 \qquad (357)$$

For three transformers the total power for full load would be:

$$W = 3EI \cos \theta$$



FIG. 180.—Three-phase, core-type transformer. Cover removed.

Therefore, the full-load capacity of two transformers in open delta is only 57.8 per cent. of the three transformers in closed delta.

8. *Distributed Y for Three-wire Edison System.*—(See Fig. 313, Chap. XVI.)

9. *Parallel Operation and the Δ and Y Three-wire Connection.*—If two transformer banks operate in parallel on both the primary

and secondary sides, ten combinations of the Δ and *Y* connections are possible. In six of these combinations the time-phase relations are the same for the two banks while the remaining four will not work in parallel. This is shown in condensed form in Table III.

TABLE III

| Combination | Primary | | Secondary | |
|---|---|---|---|---|
| | *A* | *B* | *A* | *B* |
| 1 | Δ | Δ | Δ | Δ |
| 2 | *Y* | *Y* | *Y* | *Y* |
| 3 | *Y* | Δ | *Y* | Δ |
| 4 | Δ | *Y* | Δ | *Y* |
| 5 | *Y* | *Y* | Δ | Δ |
| 6 | *Y* | Δ | Δ | *Y* |
| 7 | Δ | Δ | Δ | *Y* |
| 8 | Δ | Δ | *Y* | Δ |
| 9 | *Y* | *Y* | Δ | *Y* |
| 10 | *Y* | *Y* | *Y* | Δ |

Numbers 7 to 10 will not work, since the secondary voltages will be 30° out of time phase, as is evident by inspection.

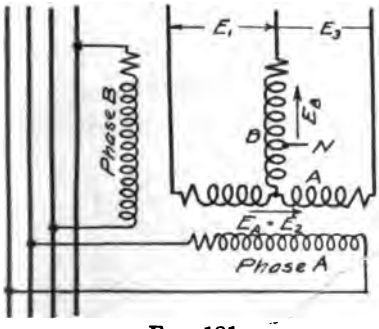



FIG. 181.

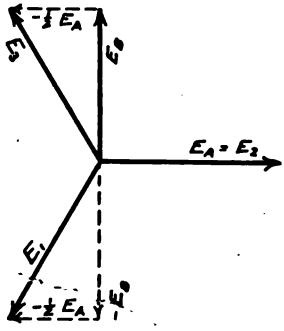



FIG. 182.

10. *Phase Transformation. Two-phase, Three-phase. The "T" or Scott Connection.*—Let the primaries of two single-phase transformers be connected to a two-phase four-wire system as indicated in Fig. 181. Let the ratio of transformation in the two transformers be such that the secondary voltages are in the ratio of $1:\frac{\sqrt{3}}{2}$; hence, if $E_A = 100$ volts, $E_B = 86.7$ volts. Connect one of the terminals of phase *B* to the middle point of phase *A* as indicated in Fig. 181. The voltage between the three

remaining terminals will be equal in magnitude and 120° different in phase, forming a three-phase circuit. This is readily seen from the vector diagram in Fig. 182 in connection with the circuit diagram in Fig. 181. The direction of the arrows and the notation in the two diagrams make it evident that $E_A = E_2$ both in phase and magnitude; that $E_3$ is the vector difference of $E_B$ and $\frac{E_A}{2}$; and that $\dot{E}_1 = -\dot{E}_B - \frac{\dot{E}_A}{2}$. Since $E_B = \frac{\sqrt{3}}{2} E_A$ the vectors $E_1$, $E_2$ and $E_3$ are equal in magnitude and differ by 120° in phase. By this simple method a two-phase circuit is

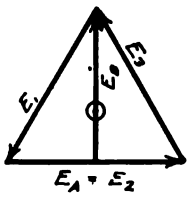


FIG. 183.

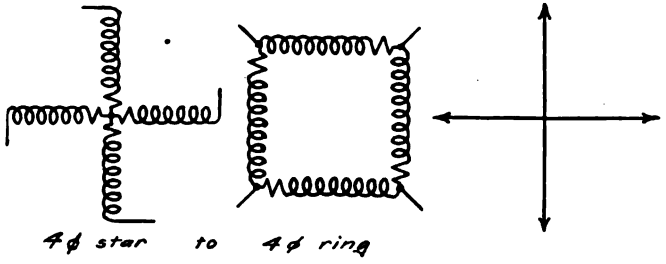


FIG. 184.

changed to three-phase. It is apparent that by the same arrangement a three-phase system can be changed into a two-phase.

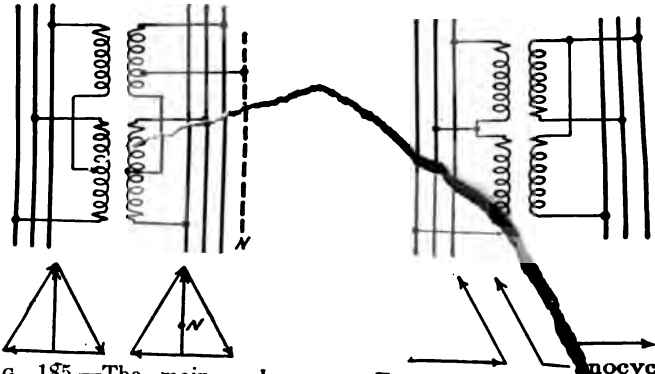


FIG. 185.—The main and teaser connection.

FIG. 186.—The mo nocyclic connection.

A neutral connection can be made by tapping transform er $B$ at a point $N$, one-third the number of turns from the junct on with

the middle point of $A$. This excellent method for phase transformation was invented by Charles F. Scott, and is often called the *Scott connection.*

The diagram in Fig. 183, is a convenient method for showing the relation of the voltages or currents in two-phase or three-phase systems. Instead of drawing the vectors through the origin, a triangle is formed by connecting the free end of phase $B$ to the terminals of phase $A$. Thus the base and perpendicular represent the two-phase voltages and the sides of the triangle the three-phase voltages. The directions of the arrows indicate phase relations.

11. Other transformer connections in commercial use are the *ring four-phase*, Fig. 184; the *main and teaser*, Fig. 185; and the *monocyclic*, Fig. 186. Several phase transformation connections from two-phase to six-phase and from three-phase to six-phase are discussed in Chap. XVI. See circuit and vector diagrams in Figs. 307 to 313 inclusive.

($n$) **Autotransformer.**—If a single winding be placed around an iron core, and taps brought out as shown in Fig. 187, power can

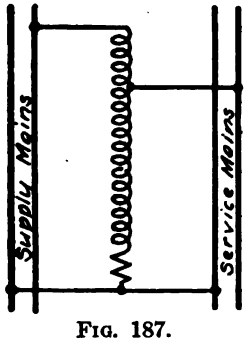



FIG. 187.

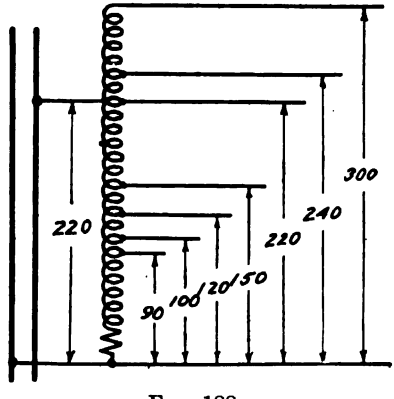



FIG. 188.

be transmitted with a change in voltage in much the same way as in a transformer having two windings. The voltage on the generator and load leads will be in proportion to the number of turns. Thus in Fig. 187, if between the supply mains there are $n_1$ turns and between the service mains $n_2$ turns, the voltage and current relations are the same as if the transformer had two windings, a primary of $n_1$ turns and a secondary of $n_2$ turns.

However, in the winding itself, and in the part carrying the current for both the supply and service circuits, the actual current flowing will be the difference between the currents in the outside mains. On this account the copper losses are less in an autotransformer for the same load than in the ordinary transformer having two windings. However, while the voltage between the wires in the service circuit may be low, the difference of potential to ground is the same as on the supply circuit. For this reason autotransformers can be used only on circuits where the voltages on both sides are low or where the necessary protection is provided in other ways. In places where the danger factor is absent it is economical to use autotransformers, as both first cost and operating losses are less than for a regular transformer of equal rating. By means of several taps any number of voltages may be obtained from one autotransformer, as in Fig. 188. This

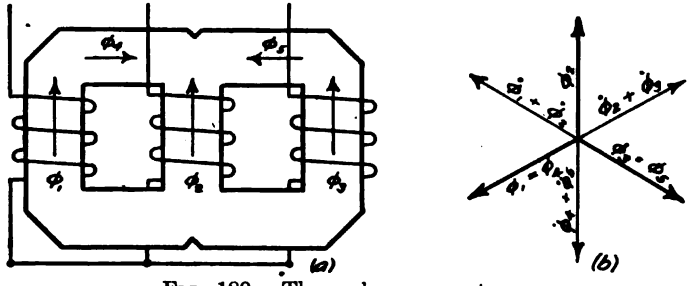


FIG. 189.—Three-phase, core type.

autotransformer receives power at 220 volts and delivers at 300, 240, 220, 150, 120, 100 and 90 volts. Power may be taken from any or all of the secondary circuits at the same time.

(*o*) **Three-phase Transformers.**—Since the magnetic fluxes in three single-phase transformers, connected to a three-phase system, bear the same time-phase relation as the voltages, a saving in iron may be secured by combining the three transformer cores forming one unit, called a three-phase transformer. In Fig. 189*a* are shown the electric and magnetic circuits, diagrammatically, of a *core type* three-phase transformer. The corresponding magnetic flux vector diagram is shown in Fig. 189*b*. It is apparent that the vector sum of the fluxes in the cores is the same in magnitude as the resultant fluxes in the yokes. Hence for the same flux density the cross-sectional area of the iron must be the same in all parts of the magnetic circuit.

A similar diagram for the electric and magnetic circuits of a *shell-type* transformer is shown in Fig. 190*b* and the corresponding magnetic flux vector diagram in Fig. 190*c*. Note that the winding in the middle section is reversed in direction, compared to the two end windings. From the vector diagram it is evident that the fluxes in the cross-bar portions of the yoke, $\phi_7$, $\phi_8$, $\phi_9$, are equal in magnitude to $\phi_4$, $\phi_5$ and $\phi_6$. The cross-section of the iron core, for the same flux density, is in proportion to the total magnetic flux in each of the magnetic circuits. Hence the cross-sections of the central cores for $\phi_1$, $\phi_2$ and $\phi_3$, are twice the cross-sections of the cross-bars for $\phi_7$, $\phi_8$ and $\phi_9$, or for the yokes $\phi_4$, $\phi_5$ and $\phi_6$.

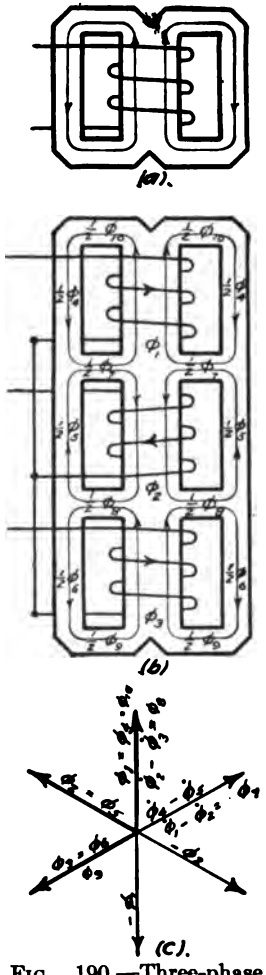



FIG. 190.—Three-phase, shell type.

A modified form of the *shell type*, called the *hexagonal type*, is shown in Fig. 191*a*, and the corresponding magnetic flux vector diagram in Fig. 191*b*. From the diagram it is seen that the flux magnitude in each core, $\phi_1$, $\phi_2$, or $\phi_3$, is $\sqrt{3}$ times the flux in any part of the yoke, $\phi_4$, $\phi_5$ or $\phi_6$; and therefore the corresponding cross-sectional areas should be in the proportion of $\sqrt{3}:1$.

The more important advantages gained by using one three-phase transformer in place of three single-phase transformers of equal rating and magnetic flux density are:

(*a*) The weight of the iron core is less by about 16 per cent.

(*b*) The efficiency is increased by 0.15 to 0.4 per cent.

(*c*) The required floor space is reduced by about 50 per cent.

(*d*) The first cost is less by about 25 per cent.

The chief disadvantage is in the probability of having more frequent interruptions in the service. Damage on either of the three phases in the three-phase transformer interrupts the service

until the repairs have been completed. When three single-phase transformers are used any one may be removed for repairs without interrupting the service; the two remaining transformers (open delta connection, Fig. 177) supplying the three-phase load.

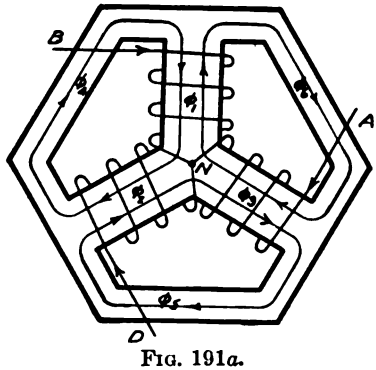



FIG. 191a.

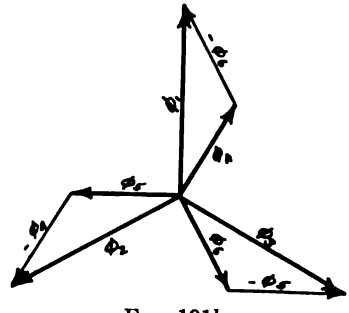


FIG. 191b.

### B. THE "CONSTANT-CURRENT" TRANSFORMER

In constant-potential systems variations in the load are accompanied by corresponding changes in the current. Similarly in constant-current systems the voltage varies with the load. In America practically all electric systems operate on the constant-potential basis; and the current varies with the load. The important exceptions are the series arc and incandescent lamp circuits for street lighting. In most cases the street lighting is only a small part of the total load, and hence the electric energy is generated and distributed by constant-potential circuits, except the particular feeders supplying the street-lighting circuits. The connecting link between the constant-potential busbars and the constant-current series lamp circuit is the constant-current transformer. In design this transformer differs from the constant-potential types chiefly by having the secondary winding movable with respect to the primary, as shown in Figs. 192 and 193. Since the currents in the primary and secondary

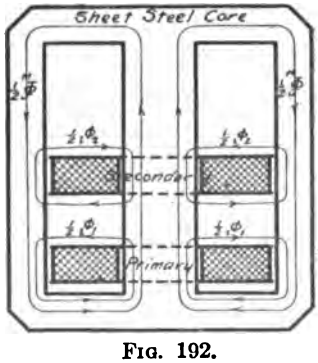



FIG. 192.

coils flow in opposite directions the magnetic fluxes between the two coils pass in the same direction. The two magnetic fields therefore produce a force tending to push the coils apart. By properly counterbalancing most of the weight of the secondary coil the magnetic repulsion may be balanced against gravity so

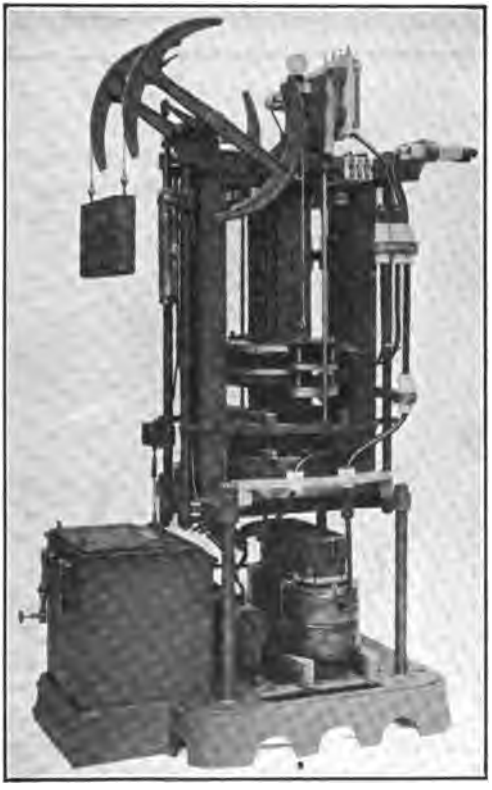


FIG. 193.—Constant-current transformer combined with mercury-arc rectifier. (*General Electric Co.*)

as to control the position of the coil by small changes in the current.

If the resistance in the series lamp circuit is decreased, the current will increase; this causes an increase in the magnetic repulsion and the secondary coil is pushed further away from the primary. As a consequence the leakage flux is increased and the induced voltage in the secondary is decreased. Thus the voltage in the secondary may be made very nearly proportional

to the resistance in the series lamp circuit and the current kept approximately constant. In the primary coil the increased leakage flux increases the reactance and hence supplies the required increase in the primary impedance. For an increase in the resistance in the series lamp circuit the current tends to decrease, the secondary coil is drawn nearer to the primary by the force of gravity and the secondary induced voltage increases so as to keep the current practically constant.



FIG. 194*a*.—Type "A" current transformer, 600 volts maximum.



FIG. 194*b*.—Type "KA" current transformer, 3000 volts maximum.

By this simple device the energy from the constant-potential busbars is transformed and delivered to the series lamp circuit under constant-current conditions. The transformer is automatically self-regulating but its efficiency is low as compared to the constant-potential transformer.

### C. INSTRUMENT TRANSFORMERS

By means of current and voltage transformers the range of alternating-current ammeters, voltmeters and wattmeters can

be extended and these meters used upon circuits having larger currents and higher voltages than could be applied to the terminals of the instruments. The voltage that may be applied to most types of switchboard voltmeters is from 0 to 150 volts, and the range of the current in the ammeters is from 0 to 5 amp. In principle the instrument transformers are similar to the larger power units, but the dimensions must be proportioned so as to comply with the required service. The transformer is a source of error in the instrument readings and must be carefully calibrated. The voltage drop in the transformer introduces a time-phase lag which may affect the current and voltage-phase displacement in the wattmeter. In properly designed transformers the impedance drop is very small, generally less than one-tenth of 1 per cent. of the voltmeter terminal voltage.

The current transformer consists of one or more turns on the primary and a secondary of as many turns as the desired ratio requires. The resistance and leakage reactance are very small. The secondary is short-circuited through the ammeter or through a short-circuiting switch before the ammeter is removed from the circuit.

## PROBLEMS

**1.** In a 50-kv.a., 2,200/220 transformer the hysteresis loss is 198 watts and the eddy-current loss 34 watts. At full load the copper loss is 508 watts.

(*a*) Find the efficiency for 25 per cent., 50 per cent., 75 per cent., 100 per cent., 125 per cent. and 150 per cent. of full load. Load having unity power factor.

(*b*) Same as (*a*) but for a load having 85 per cent. power factor, lagging current.

(*c*) Plot the efficiency curves for (*a*) and (*b*).

**2.** At full load the equivalent reactance drop of the transformer in problem 1 is 3.1 per cent. and the resistance drop 1.1 per cent. of the rated terminal voltage. Find the regulation.

(*a*) For $\cos \theta_2 = 1.00$.

(*b*) For $\cos \theta_2 = 0.85$.

**3.** The average flux density of the transformer in problem 1 is 64 kilolines per sq. in.

Find the core loss for $\mathfrak{B} = 30$, 40, 50, 60, and 70 kilo-lines.

**4.** Given a 5-kv.a., 2,200–220/110-volt transformer.

$_hP = 31$ watts; $_eP = 14$ watts; $RI^2$ (full load) = 85 watts.

Average $\mathfrak{B} = 8500$ per cm.$^2$; equiv. resistance drop 1.7 per cent. and reactance drop 2.8 per cent. of the rated terminal voltage. Core weight 92 lb.

(*a*) Plot efficiency curve from 0 to 150 per cent. load; $\cos \theta_2 = 95$ per cent.

(*b*) Find the regulation for loads having power factors = 1.00; 95 per cent.; 90 per cent., 85 per cent., and 80 per cent. Lagging current.

(*c*) Find core loss $\mathfrak{G}$ = 6,000, 7,000, 8,000, 9,000, 9,500, 10,000 and 10,500 per cm.$^2$

**5.** Let the following data apply to a transformer:

$_{\phi}E_1$ = 1,100 volts; ratio of $_{\phi}E_1 : {}_{\phi}E_2 = 5:1$.
$r_1$ = 3.3 ohms; $r_2$ = 0.13 ohm; $x_1$ = 6.5 ohms; $x_2$ = 0.26 ohm.
$_{m}I$ = 0.3 amp.; $I_2$ (full load) = 125 amp. (lagging); $_{\phi}\theta_2 = 30°$;
$_{h}P + {}_{e}P$ = 148 watts.

(*a*) Find $E_1$, $E_2$, $\theta_1$, $\theta_2$ and $I_1$ for full load.

(*b*) Draw the vector diagram to scale.

**6.** With the same data as in problem 5 except the current leads the voltage, $_{\phi}\theta_2 = 30°$, obtain answers for (*a*) and (*b*).

**7.** Two transformers are used in changing 2,200 two-phase to 220-volt three-phase. (*a*) Neglecting losses, find the ratios of the number of turns in the two transformers.

(*b*) Draw in rectangular coördinates the sine waves for the voltages between mains in the two-phase and three-phase circuits.

(*c*) Draw vector diagram of the voltages and currents in the three-phase and two-phase circuits with the load current lagging 40°.

**8.** Draw a vector diagram for a constant-current transformer taking power from constant-potential mains. (*a*) When under full load (all the series lamps in circuit); (*b*) for one-fourth load (one-fourth of the lamps in circuit).

**9.** Three 2,200/220-volt transformers, connected in delta, supply a lighting load of 150 kw. One of the transformers is damaged and removed for repairs.

(*a*) What current was flowing in each transformer when the three transformers were in service?

(*b*) What current flows in each of the two transformers after the third has been removed?

(*c*) What is the rating of each transformer if the two transformers on open delta carry full load?

**10.** In a 500 kv.a. 60$\sim$. transformer, $_{e}P$ = 820 watts; $_{h}P$ = 1,780 watts; $RI^2$ (full load) = 3,250 watts; equivalent resistance drop = 0.65 per cent. and equivalent reactance drop 5 per cent. of the terminal voltage (full load). $\mathfrak{G}$ = 10,800 per cm.$^2$ $E_2$ = 220 volts.

(*a*) Plot the efficiency curve from 0 to 150 per cent. full load. Unity power factor.

(*b*) Find the regulation for power factors = 1.00, 0.90, 0.85 and 0.75. Leading and lagging currents.

(*c*) Find core loss for $\mathfrak{G}$ = 5,000, 7,000, 9,000, 10,000, 11,000 and 12,000 per cm.$^2$

**11.** Two transformers having constants as given in problem 5 are connected in "V." Balanced three-phase voltage (1100 volts) is supplied to the primary and three equal load resistances of 1 ohm each are connected in delta on the secondary. Neglect the exciting current and calculate the equivalent resistance and reactance of each transformer. Calculate the secondary voltages and currents and plot a complete vector diagram.

**12.** The following data are for single-phase transformers, oil insulated, self cooled, 60 cycle, 2,300 volts primary, 230 and 115 volts secondary, used on distribution systems for lighting load.

| Size of transformer, kv.a. | | 5 | 10 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|
| Flux density in gausses | Inside | 10,350 | 10,350 | 10,900 | 11,380 | 12,330 |
| | Outside | 5,560 | 5,500 | 6,250 | 5,975 | 9,570 |
| Apparent iron losses in volt-amperes | | 95 | 140 | 320.0 | 590.0 | 1570.0 |
| True iron losses in watts | | 44 | 70 | 120.0 | 240.0 | 480.0 |
| Weight of iron in lbs. | | 85 | 135 | 195.0 | 395.0 | 435.0 |
| Weight of copper in lbs. | | 45 | 69 | 105.0 | 186.0 | 315.0 |
| Copper loss per lb. at full load in watts | | 2 | 3 | 2.8 | 2.9 | 3.2 |

(*a*) Plot efficiency curves 0 to 125 per cent. full load.
(*b*) Find the regulation from no load to full load.
(*c*) Draw full load vector diagrams.

**13.** The following data are for single-phase transformers, oil insulated, 60 cycle power load distribution systems.

| Size of transformer in kv.a. | Self cooled | | | | Water cooled |
|---|---|---|---|---|---|
| | 25 | 75 | 250 | 1000.0 | 1000.0 |
| Primary voltage | 11,000.0 | 23,000.00 | 4,400.0 | 13,200.0 | 33,000.0 |
| Secondary voltage | 440.0 | 2,200.00 | 440.0 | 2,300.0 | 2,300.0 |
| | 11,650.0 | | | | |
| Flux density in gausses | 8,350.0 | 12,600.00 | 12,300.0 | 11,000.0 | 12,500.0 |
| Apparent iron losses in kv.a. | 0.7 | 8.72 | 12.6 | 30.5 | 69.6 |
| True iron losses in kw. | 0.2 | 0.83 | 1.7 | 5.1 | 7.0 |
| Weight of iron in lbs. | 190.0 | 434.00 | 954.0 | 3,735.0 | 3,660.0 |
| Weight of copper in lbs. | 100.0 | 190.00 | 435.0 | 1,186.0 | 619.0 |
| Copper losses at full load watts per lb. | 4.0 | 6.00 | 5.2 | 4.2 | 13.6 |

(*a*) Plot efficiency curves from 0 to 125 per cent. full load.
(*b*) Find regulation from no load to full load.
(*c*) Draw full load vector diagrams.

For Experiments see page 537.

## CHAPTER XIII

## INDUCTION MOTORS

As already noted the magnetic field passing between the primary and secondary windings of a transformer produces mechanical forces tending to push the coils apart. In the potential transformer the two windings are rigidly fixed in position and hence, although the forces exist, no motion is produced and no energy changed into the mechanical form. On short-circuits with excessive currents flowing, these forces may tear the transformers to pieces. In the constant-current transformer the secondary winding is, to a certain extent, movable, and the reaction between the two fields furnishes the means for automatic regulation. In the induction motor the windings of the successive phases are so arranged in space around a rotating spider that a continuous torque is produced in one direction. The induction motor was invented by Ferrari and Tesla in 1887 and is essentially an apparatus for transforming *electric energy into mechanical energy*. Some forms of the induction motor are extremely simple in design, requiring very little attention and still give excellent service. Like the transformer, the induction motor has a primary and a secondary. Either may be placed on the rotating spider, but as the outside offers more space it is generally desirable to wind the primary with the highest voltage on the stationary drum or *stator*, and place the secondary, low-voltage winding, on the rotating spider, or *rotor*. The basic principles of the induction motor are more nearly like those of the transformer than those of the direct-current motor and hence it is customary to refer to the *primary* and *secondary*, or the *stator* and *rotor* instead of the *armature* and *field* windings.

(*a*) **The Revolving Field in Polyphase Motors.**—Let the coils be placed with their sides parallel in the position shown in Fig. 195. If they be connected in parallel from the same pair of busbars the magnetic fluxes are in the vertical direction and vary from a positive to a negative maximum, equal in magnitude to the sum of the maxima for the separate coils. If the two coils

be connected to a two-phase circuit the maxima of the separate coils come 90° apart in time and the total resultant maximum in between, equal in magnitude to the square root of the sum of the squares of the component maximum fluxes. If the maximum

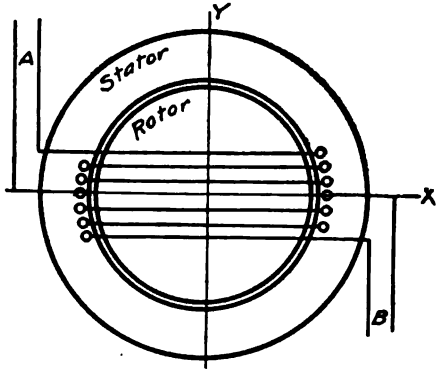



FIG. 195.—Coils in space-phase.

fluxes in the two coils are equal the direction of the flux in space is always either up or down. In Fig. 196 is shown the vector diagram for the two coils on the same phase, and in Fig. 197 the corresponding diagram with the two coils connected to a two-

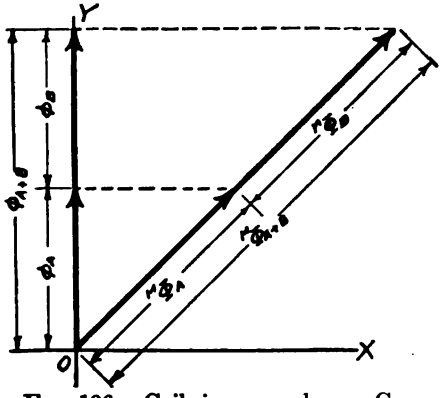



FIG. 196.—Coils in space-phase. Currents in time-phase. Flux: stationary, pulsating.

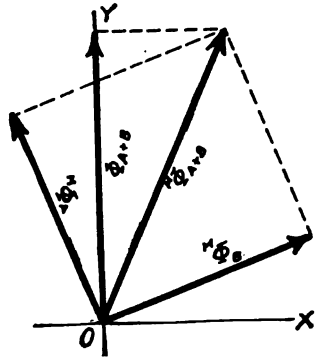



FIG. 197.—Coils in space-phase. Currents in time-quadrature. Flux: stationary, pulsating.

phase circuit. The instantaneous flux in both cases is the projection of the resultant vector on the *Y*-axis. If the two coils be placed mechanically at right angles, as in a two-pole, two-phase induction motor, shown diagrammatically in Fig. 198, the flux in

coil $A$ will be along the $Y$-axis as before, but for coil $B$ the direction of the flux will be horizontal. If the two coils are connected in parallel on the same phase or to a single-phase circuit the instantaneous values for the flux in $A$ are the projections along the $Y$-axis and for coil $B$ along the $X$-axis as shown in Fig. 199. If the two coils are alike the resultant flux, $\Phi_r$ lies at 45 space degrees or midway between the two axes. Since the fluxes in both coils are in time phase and therefore reach their maximum and zero values simultaneously, the resultant flux also reaches its maximum and zero values coincident with the two components. The positions in space of the three fluxes are,

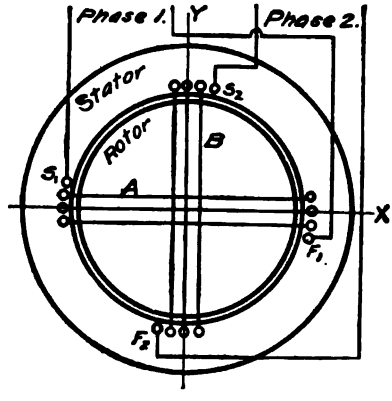



FIG. 198.—Coils in space-quadrature.

however, constant; for coil $A$ along the $Y$-axis, for coil $B$ along the $X$-axis and for the resultant at an angle of 45 space degrees.

Let the coils be connected to a two-phase system; then the two fluxes are in *time quadrature* since the currents in the two phases are a quarter cycle out of phase, and in *space quadrature* because the two windings are at right angles. The instantaneous values of the flux for coil $A$ are along the $Y$-axis and similarly for coil $B$ along the $X$-axis, on account of the *space quadrature* of the two coils. Since the fluxes are also in *time quadrature* the instantaneous values are as the sine and cosine of the same time angle because the maximum values of the two coils come a quarter cycle apart. The resultant flux therefore moves in space position from the $X$-axis when the flux in coil $B$ is a maximum to the $Y$-axis in a quarter cycle. Thus the resultant

flux is constant in magnitude but rotates in space 360 electrical degrees for each cycle of the two-phase current. This is shown in Fig. 200. The field for phase 1 lies vertically and for phase 2 in the horizontal position. The magnitude of the instantaneous values depends on the time elapsed.

$$\phi_2 = {}^{\kappa}\Phi_2 \sin \omega t \tag{365}$$

$$\phi_1 = {}^{\kappa}\Phi_1 \sin (\omega t + 90°) = {}^{\kappa}\Phi_1 \cos \omega t \tag{366}$$

Equations (365) and (366) give the instantaneous values and the positions of the coils determine the directions. In Fig. 200, $\phi_1$, and $\phi_2$ are drawn along the $Y$- and $X$-axes respectively.

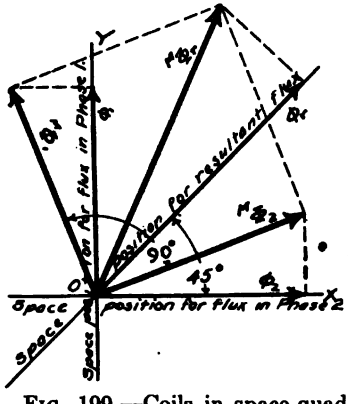



FIG. 199.—Coils in space-quadrature. Currents in time-phase. Flux: stationary, pulsating.

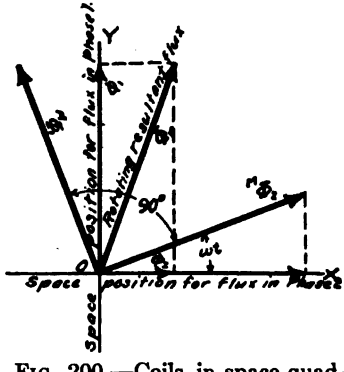



FIG. 200.—Coils in space-quadrature. Currents in time-quadrature. Flux: rotating, constant.

The resultant flux is constant in magnitude and continually changes its position with ($\omega t$).

In the two-phase induction motor the coils are arranged in space as shown in Fig. 198. A revolving field is produced by the combination of *space-phase* and *time-phase* factors as determined by the relative space positions on the motor windings and the conditions of the circuit from which power is derived. This is shown diagrammatically in Fig. 201 for a two-phase, two-pole induction motor. The vertical lines mark the time and are spaced one-eighth of a cycle apart. Neglecting losses, the magnetic flux is in phase with the primary current in each phase. The cross and dot indicate the direction of the currents, and the arrows show the corresponding directions of the magnetic fields.

The rotation of the resultant field is shown by the series of magnetic flux vectors.

The stator winding diagram for a three-phase, two-pole induc-

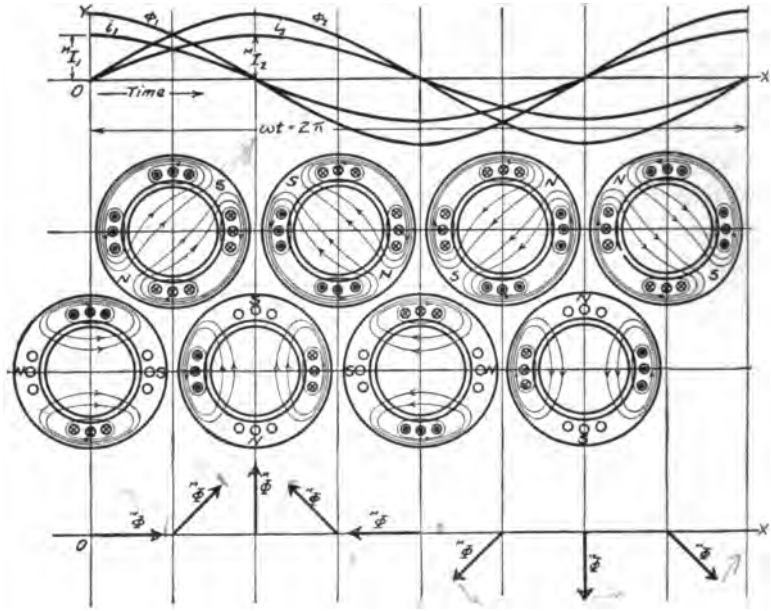



FIG. 201.—Diagrammatical and vector representation of revolving field in 2-$\phi$, two-pole, induction motor.

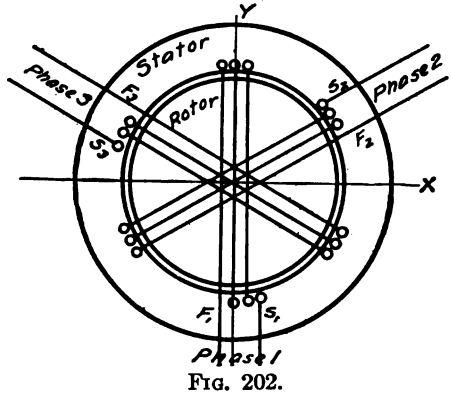



FIG. 202.

tion motor is shown in Fig. 202 and the corresponding revolving field in Fig. 203. The resultant flux is constant in magnitude and rotates in space position as in the two-phase motor. In a

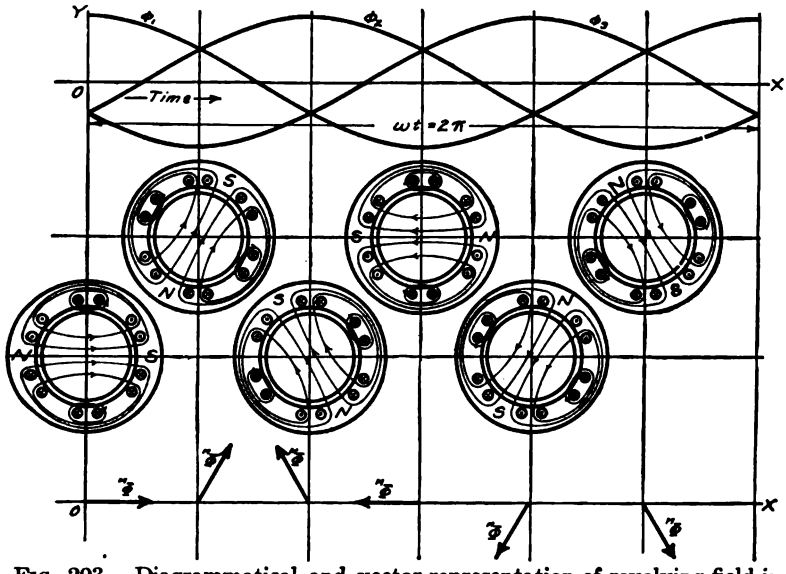



FIG. 203.—Diagrammatical and vector representation of revolving field in 3-$\phi$, two-pole induction motor.

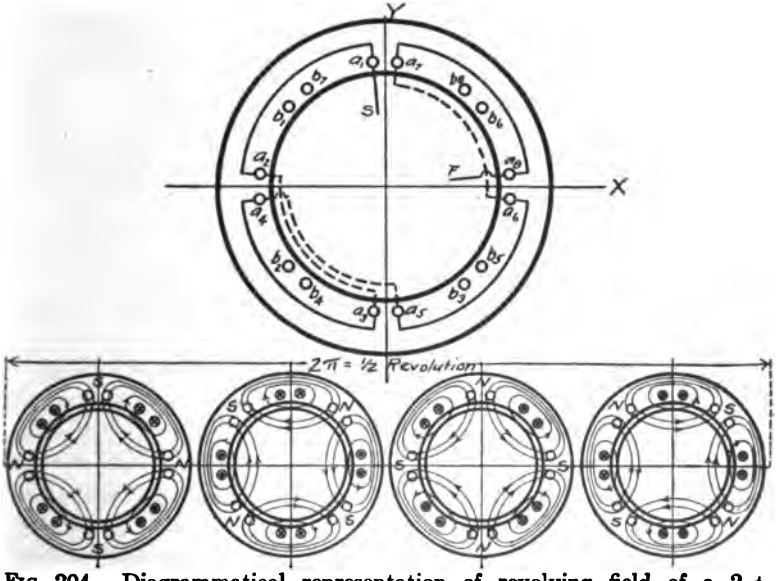



FIG. 204.—Diagrammatical representation of revolving field of a 2-$\phi$, four-pole, induction motor.

four-pole, two-phase motor two pairs of north and south poles are formed and a complete revolution in space requires two cycles of the magnetic flux as shown in Fig. 204. Precisely similar action takes place in the three-phase, four-pole motor as indicated in Fig. 205. The rotation of the magnetic field should therefore be measured in electrical degrees as each pair of poles represents a complete wave in the magnetic circuit. The primary windings on the stator must, therefore, also be spaced in electrical degrees; 90° for two-phase circuits, and 120° for three-phase systems. In order to reduce the magnetic reluctance the

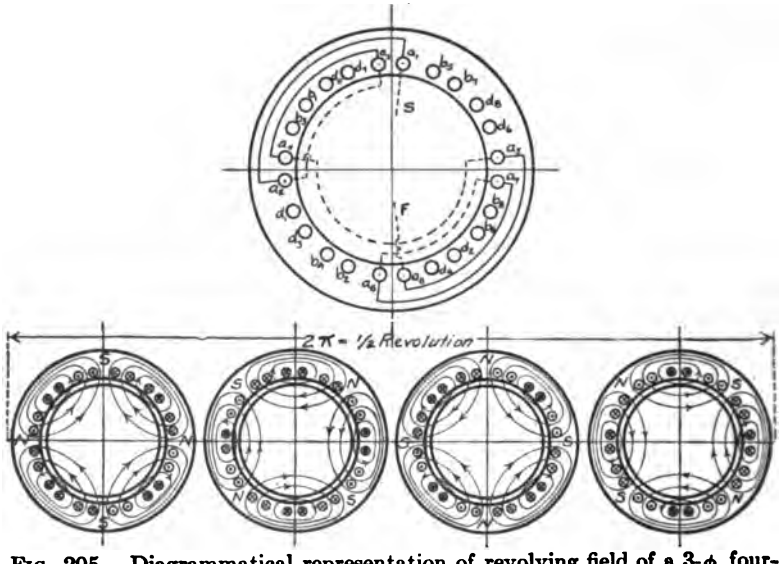



FIG. 205.—Diagrammatical representation of revolving field of a 3-$\phi$, four-pole, induction motor.

primary winding is placed in slots on the inside of a hollow cylindrical core made of sheet steel.

(*b*) **Two-phase Motor Obtained from Four-phase Design.**—It should be noted that the so-called two-phase motor is designed as a four-phase machine. By merely rearranging the connections between the conductors inside the machine all the advantages of four-phase design may be gained for the two-phase motor. The fundamental principles involved are more easily explained by first replacing the rotor by a two-pole rotating field, as in Fig. 206*a*, and considering the direction of the induced

voltages in the several stator conductors. In the figure, four conductors are shown, spaced 90° apart. The voltage induced when the bipolar field rotates may be represented by the vector diagram in Fig. 206*b*; four vectors, differing successively by 90 time degrees and hence four-phase. With four conductors in

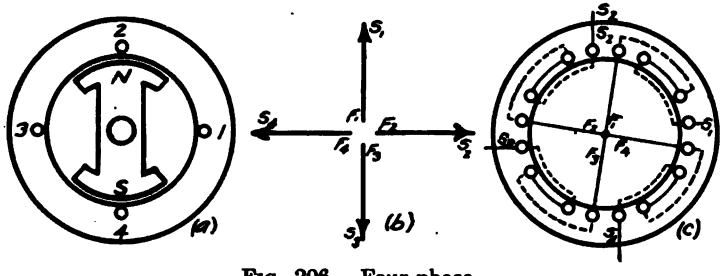



FIG. 206.—Four-phase.

each group connected into four phases as in Fig. 206*c*, the stationary part of the machine can be used either as the armature of a four-phase generator or as the stator of a four-phase induction motor. The voltages in the four phases, taken in order, are in time quadrature or 90 time degrees apart, and the corresponding vector diagram is similar to Fig. 206*b*. In multi-

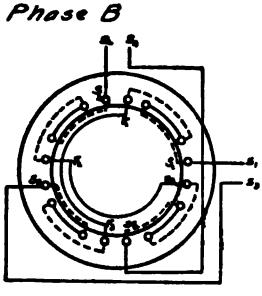



FIG. 207.—So-called two-phase.

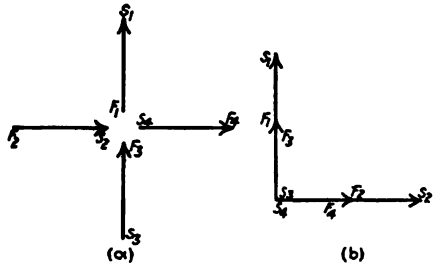



FIG. 208.

polar machines the windings are spaced 90 electrical degrees; that is, 360 electrical degrees divided by 4, the number of phases.

By reversing the connections of coils $S_3F_3$ and $S_4F_4$, as in Fig. 207, or using connections as in Fig. 198, the directions of the voltage vectors for the third and fourth phases are reversed. The corresponding vector diagram is shown in Fig. 208*a*. From the connections it is seen that the first and third, and the second

and fourth phases are in series between the corresponding pairs of leads. The vector diagram for the voltages between the leads is therefore two-phase as shown in Fig. 208*b*. Hence by the simple expedient of changing the winding connections inside the machine, the four-phase motor is converted into the so-called two-phase motor. All two-phase motors in commercial use are fundamentally four-phase machines with the conductors connected inside the machine so as to operate on two-phase systems.

(*c*) **Three-phase Motors, Six-phase Motor Design.**—The actual three-phase motor has the windings for the three phases

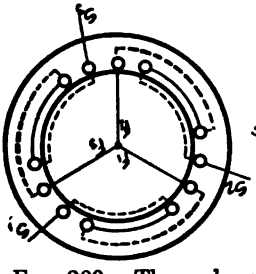


FIG. 209.—Three-phase.

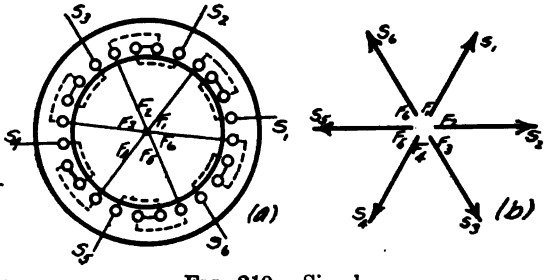


FIG. 210.—Six-phase.

spaced 120 electrical degrees, with the conductors connected in some such way as indicated in Fig. 209. Only a few real three-phase motors have ever been built. The so-called

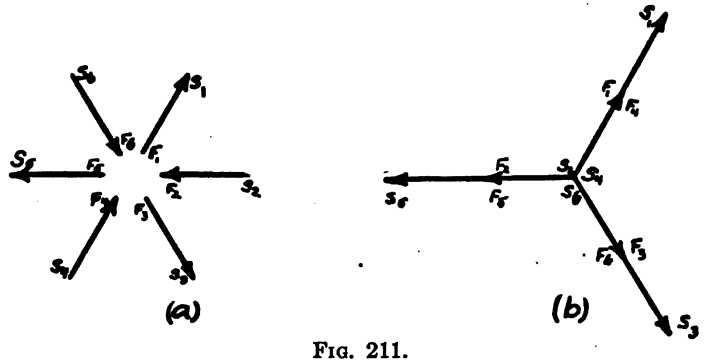


FIG. 211.

three-phase induction motor, so extensively used in industrial power applications, is essentially a six-phase motor with the conductors so connected inside the machine that with three leads

the machine operates on three-phase systems. In a six-phase motor design the conductors for the six phases are spaced 60 electrical degrees; that is, 360 electrical degrees divided by 6, the number of phases. The windings may be connected to six leads, as indicated in Fig. 210*a* with a common neutral. The corresponding vector diagram for the voltages between the leads is shown in Fig. 210*b*. From the vector diagram it is apparent that if the connections for the conductors in phases two, four and six be reversed the voltage vector diagram is represented by Fig. 211*a*. If the windings for phases one and four, three and six, and five and two be connected in series as indicated in Fig. 212 (vector diagram Fig. 211*b*), and the terminals $F_2$, $F_4$, $F_6$, joined at a common point, then the terminals $S_1$, $S_3$, $S_5$, form the three leads of the so-called three-phase motor. Phase one is produced by coils $S_1F_1$ and $S_4F_4$ reversed; phase two by coils $S_3F_3$ and $S_6F_6$ reversed; and phase three by coils $S_5F_5$ and $S_2F_2$ reversed. The three resultant phases with the terminals $S_1$, $S_3$, and $S_5$ connected to the leads are spaced 120 electrical degrees, but the individual coils are only 60 electrical degrees apart.

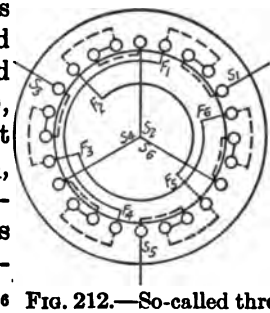



FIG. 212.—So-called three-phase.

(*d*) **The Rotor.**—The rotor, or secondary, consists of a grooved cylinder of laminated steel mounted on a spider. The conductors lie in grooves in the steel core and the nature of the rotor winding largely determines the type of motor. In order to keep the magnetic reluctance as low as possible the air gap between the stator and rotor must be a minimum, consistent with safe clearance. For this reason the shaft is extra stiff and the bearings comparatively large.

The secondary may be wound on the rotor in much the same way as the armature of a direct-current motor or may simply consist of parallel copper bars short-circuited by brass rings at both ends. The latter design, known as the *squirrel-cage* type, has a wide application and is in most general use. In the wound rotor the terminals are brought out to collector rings in order that outside variable resistance may be inserted in the secondary. The conditions requiring additional resistance in the secondary will be discussed in the paragraph on *Torque*.

Whether the rotor be of the *squirrel-cage* or *wound rotor* types, it consists of a resistance and an inductive reactance and its electrical properties may be stated quantitatively in the form of an equation.

(*e*) **Rotor Voltage and Current. Slip.**—With the rotor at standstill, the revolving field produced by the primary induces voltages in the secondary of the same frequency as in the primary. This induced voltage causes a secondary current to flow in the same way as in a transformer.

If the rotor runs in synchronism with the revolving field, called the synchronous speed, no lines of force cut the secondary conductors and hence no secondary voltage is generated.

The operating speeds of the rotor and of the revolving field are usually expressed in terms of the *slip* and are often given in per cent. *Slip* is defined as the ratio of the difference between the synchronous and rotor speeds to the synchronous speed. Thus if the synchronous speed is 1,800 and the rotor speed 1,764 r.p.m., the slip is:

$$s = \frac{1{,}800 - 1{,}764}{1{,}800} = 0.02 = 2 \text{ per cent.}$$

Since the voltage generated depends upon the rate of cutting lines of force, the secondary voltage depends on the rotor slip. Likewise the reactance in the rotor circuit is directly proportional to the frequency of the secondary current, and hence, also dependent upon the slip.

Let

$f$ = frequency of primary current.
$E_2$ = secondary voltage at standstill.
${}_{s}x_2$ = secondary reactance at standstill.
$r_2$ = secondary resistance.

Then

$$s{}_{s}x_2 = \text{secondary reactance at slip } s \tag{361}$$

$$\sqrt{r_2^2 + s{}_{s}x_2^2} = \text{secondary impedance at slip } s \tag{362}$$

$$sE_2 = \text{secondary voltage at slip } s \tag{363}$$

$$I_2 = \frac{sE_2}{\sqrt{r_2^2 + s{}_{s}x_2^2}} = \text{secondary current at slip } s \tag{364}$$

$$\cos \theta_2 = \frac{r_2}{\sqrt{r_2^2 + s{}_{s}x_2^2}} = \text{power factor in secondary at slip } s \tag{365}$$

The resistance of the secondary is small when compared to $x_2$, the reactance at standstill, but large enough to make the reactance, $sx_2$, negligible near synchronism. It is therefore evident that the secondary current depends very largely upon the slip. Near synchronism the secondary current varies directly with the speed; and near standstill, aside from any changes in the primary flux, the secondary current is practically independent of the speed since both the secondary impedance and the induced secondary voltage are affected in the same proportion by changes in the speed. The secondary current in *squirrel-cage motors* is about 5.5 times as large at standstill as at full load.

(*f*) **Torque in Polyphase Motors.**—Torque is produced by the reaction of the secondary current upon the primary field. The

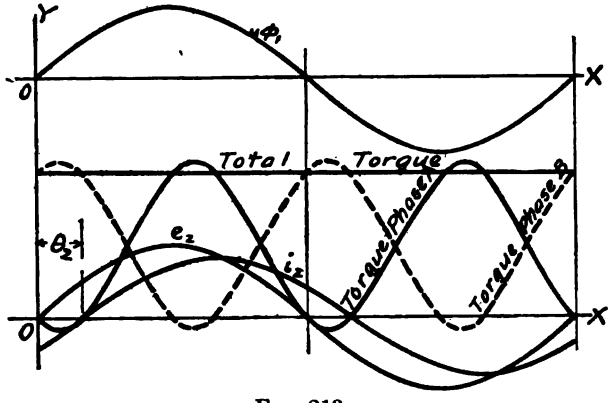



FIG. 213.

quantitative value of both the instantaneous and average values depends both upon the magnitude and phase relations of the current and field. From equation (365) the power factor of the secondary circuit depends upon the slip, being approximately unity near synchronism and decreasing with an increase of the slip. The secondary voltage is in time-phase with the flux and hence both the time- and space-phase angles between the primary flux and secondary current increase with the slip. In Fig. 213 is shown by the drawn curves the phase relations of the primary magnetic flux, ${}_u\Phi_1$, the secondary voltage, $e_2$, secondary current, $i_2$, and torque, $T$, for one phase of a two-pole induction motor under full-load conditions. The current and field are nearly in time-phase and the torque curve is positive except for a very

short time in each half cycle. The average torque produced in one phase winding is the quotient of the difference between a

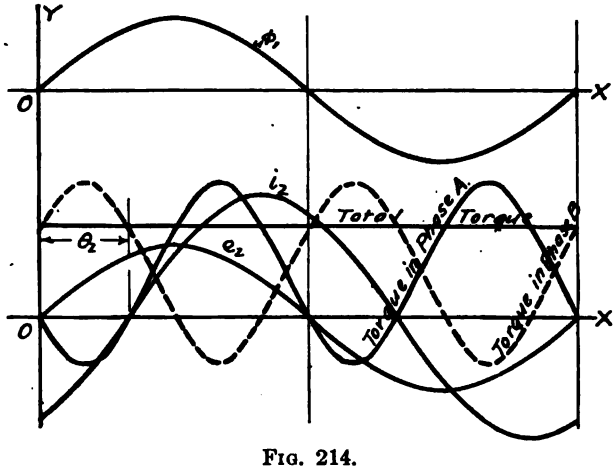



FIG. 214.

positive and a negative area divided by the time of one-half cycle. The other phases produce similar torque curves (broken

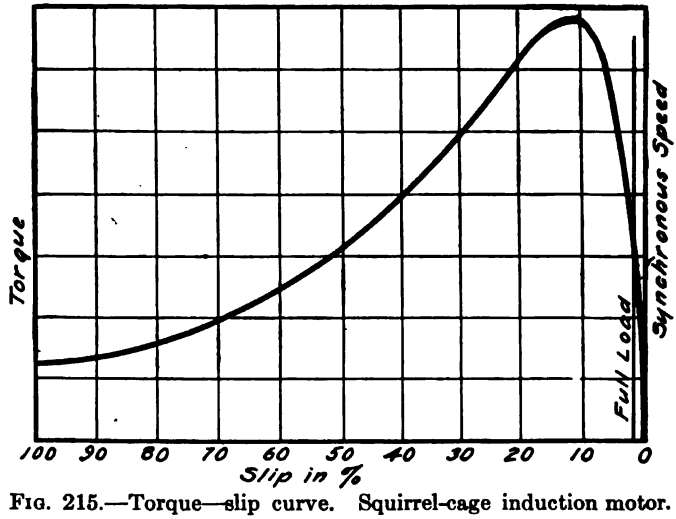



FIG. 215.—Torque—slip curve. Squirrel-cage induction motor.

curve in Fig. 213), displaced 90° for two-phase and 120° for three-phase motors. The instantaneous torque in both two-phase and three-phase motors is constant since the torque is a sine wave of

double frequency. The relations of ${}_{u}\Phi_1$, $e_2$, $i_2$ and torque for phase $A$ and phase $B$ with the motor near standstill are shown in Fig. 214. The secondary current and primary flux differ in phase by a large angle and therefore the torque has large negative as well as positive values. The difference, representing the resultant torque, is less than at full load although the standstill current is much larger. In two-phase and three-phase motors the torque curves are displaced 90° and 120° respectively, and hence produce a constant total torque. It is evident that while the secondary current increases from zero at synchronous speed to a maximum at standstill the torque has its maximum at a point between synchronous and standstill conditions. A typical torque curve for a squirrel-cage induction motor is shown in Fig. 215.

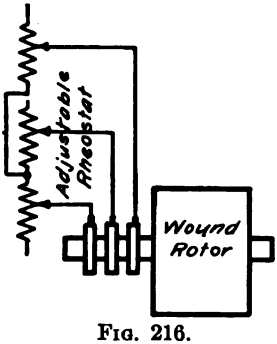


FIG. 216.

The torque, while zero at synchronous speed, rapidly rises to a maximum value with the increase of slip, and then falls to a comparatively low value at standstill. This decrease is due to the change in the power factor in the secondary circuit as explained above and to a decrease of the primary field with the slip. The action is the same as already discussed for the decrease in secondary voltage for inductive load in transformers. The variation is, however, small as compared to the change in secondary current and power factor, and will be neglected in the preliminary discussion. Assuming then, with a constant impressed voltage of constant frequency, on the primary, a constant primary flux $\Phi_1$, the equation for the torque, $T_1$, at slip, $s$, is:

$$T_1 = \Phi_1 I_2 \cos\theta_2 = \frac{\Phi_1 s E_2}{\sqrt{r_2^2 + s^2 x_2^2}} \times \frac{r_2}{\sqrt{r_2^2 + s^2 x_2^2}} = \frac{s E_2 r_2 \Phi_1}{r_2^2 + s^2 x_2^2} \quad (366)$$

For variable slip the torque is a maximum when the rotor resistance and reactance are equal.

$$\text{For maximum torque, } r_2 = s x_2 \quad (367)$$

In the wound rotor, shown diagrammatically in Fig. 216, the secondary resistance may be varied by means of the outside rheostat. By continually keeping the total resistance in the secondary equal to $sx_2$ the same maximum torque may be ob-

tained at all speeds. A series of torque curves for the wound rotor motor having variable secondary resistance is shown in Fig. 217. From equations (366) and (367) it is evident that all the maximum values are of the same magnitude; and hence the maximum torque is, within limits, independent of the secondary resistance. Substituting the value of the resistance for maximum torque in equation (366):

$$T = \frac{\Phi_1 s^2 E_2 x_2}{2 s^2 x_2^2} = \frac{\Phi_1 E_2}{2 x_2} \tag{368}$$

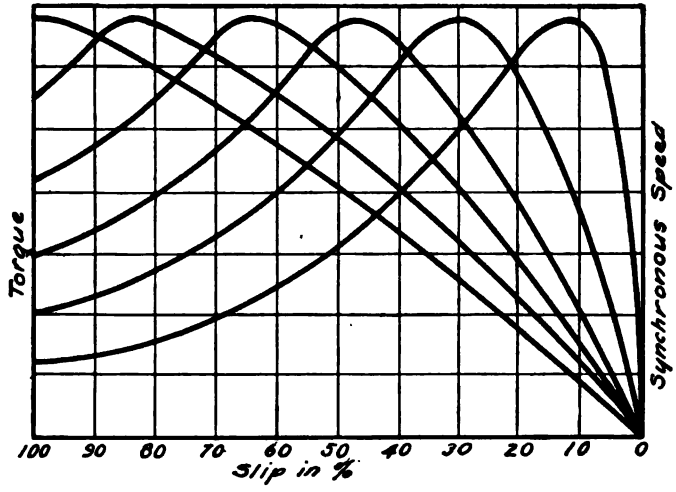



FIG. 217.—Torque—slip curves. Induction motor. Wound rotor. Secondary resistance variable.

From equations (364) and (367) the secondary current for maximum torque is constant and independent of the resistance.

$$I_2 = \frac{s E_2}{\sqrt{r_2^2 + s^2 x_2^2}} = \frac{s E_2}{\sqrt{2 s^2 x_2^2}} = \frac{E_2}{\sqrt{2} x_2} \tag{369}$$

The secondary power factor for maximum torque is a constant.

$$\cos \theta_2 = \frac{r_2}{\sqrt{r_2^2 + s^2 x_2^2}} = 0.707 \tag{370}$$

At standstill $s = 1$, and hence $r_2 = x_2$ for maximum torque.

$$\text{Maximum torque at standstill} = \frac{\Phi_1 E_2 r_2}{r_2^2 + x_2^2} = \frac{\Phi_1 E_2}{2 r_2} \tag{371}$$

If $r_2$ is less than $x_2$ the starting torque can be increased by increasing the resistance. The maximum possible torque is

reached with $r_2$ equal to $x_2$. Any further increase of the resistance decreases the torque. Load conditions determine the choice between the squirrel-cage and wound-rotor types of induction motors. As the cost of construction of the squirrel-cage type is less than for wound-rotor design the cheaper type is used for all cases where the load conditions fall within the torque-speed limitations. For loads having a large starting torque the more expensive wound-rotor type must be used. From Fig. 217 it is apparent that by inserting a variable resistance in the armature circuit the maximum starting torque can be made available from standstill up to full speed.

The running position curve lies between the synchronous speed and the maximum point on the torque curve. Ordinarily the full load requires less than one-third of the maximum torque. If the load be increased the speed decreases slightly with a rapid increase in the torque. If the load be increased past the maximum value of the torque the motor stops, or *breaks down*. While an induction motor will carry momentarily, overloads up to the *breakdown* point, it can not do so continuously without overheating. For overloads the efficiency is slightly lower and the losses are greatly increased, producing more heat than can be dissipated at safe operating temperatures. As a consequence the temperature rises and if the excessive overload be continued the insulation may be damaged and the motor *burned out*. For excessive overloads the rotor temperature may rise rapidly until the soldered joints give way. Motors having the copper bars welded or brazed to the short-circuiting rings on the rotor, Fig. 222, can operate at higher temperatures, for short periods, without serious damage.

(*g*) **Power and Efficiency.**—The power output of the motor is proportional to the product of the torque and the speed.

$$P_2 \propto (1 - s)T \propto \frac{(1 - s)sE_2r_2\Phi_1}{r_2^2 + x_2^2s^2} \qquad (372)$$

The power is a maximum at a slip less than that at maximum torque since it is the product of speed and torque. When the speed is near synchronism, as is the case under normal loads, the iron losses in the rotor are small due to the low frequency of the secondary currents, and may be neglected. The copper losses in the secondary are:

$$r_2I_2^2 = \frac{s^2E_2^2r_2}{r_2^2 + s^2x_2^2} = \frac{sE_2T}{\Phi_1} \qquad (373)$$

The copper loss is therefore proportional to the product of the slip and the torque; or for a given torque the slip is proportional to the rotor copper loss and independent of the secondary reactance. The copper losses in the secondary may also be expressed in terms of the voltage, current and power factor.

$$\text{Copper losses in secondary} = sE_2I_2 \cos \theta_2 \qquad (374)$$

The electrical input into the secondary in terms of the secondary current, secondary voltage and power factor:

$$\text{Input for rotor} = E_2I_2 \cos \theta_2 \qquad (375)$$

The mechanical output is equal to the electrical input minus the electrical losses in the secondary.

$$\begin{aligned} \text{Mechanical output} &= E_2I_2 \cos \theta_2 - sE_2I_2 \cos \theta_2 \\ &= E_2I_2(1 - s) \cos \theta_2 \qquad (376) \end{aligned}$$

Considering only the copper losses in the rotor and neglecting the primary losses and iron losses in the secondary, a partial efficiency may be obtained from equations (375) and (376).

$$\begin{aligned} \text{Approximate efficiency} &= \frac{\text{mechanical output}}{\text{rotor input}} \\ &= \frac{E_2I_2\,(1 - s) \cos \theta_2}{E_2I_2 \cos \theta_2} = 1 - s \qquad (377) \end{aligned}$$

Hence the approximate efficiency, under the stated restrictions, is equal to the speed in per cent. of synchronism. However, to obtain the actual efficiency of the motor all the losses must be taken into account. In addition to the copper losses in the secondary there are iron losses in both the primary and secondary, copper losses in the primary, rotor windage and friction losses. The expression in equation (377) is of some interest as it shows that the actual efficiency must always be less than the speed expressed in per cent. of synchronism.

(*h*) **Transformer Features of Polyphase Motors.**—After this preliminary discussion it is evident that by inserting in the secondary a resistance, like the load resistance in the secondary of the transformer, sufficient to consume the same power as the mechanical load, the circuits for the induction motor are the same as for the transformer. The circuit diagram for one phase is shown in Fig. 218.

$\dot{E}_1$ = impressed voltage on the primary.
$r_1$ = primary resistance.
$x_1$ = primary reactance, due to the leakage flux.
$r_{21}$ = equivalent of the secondary resistance in the primary circuit.
$x_{21}$ = equivalent of the secondary reactance in the primary circuit due to leakage flux.
$r_{31}$ = equivalent load resistance. This is assumed to be of such magnitude as to make $r_{31}I_{21}^2$ equal to the mechanical load.
$E_{21}$ = equivalent of secondary voltage.
$I_1$ = primary current.
$_mI$ = magnetizing current.
$I_{21}$ = equivalent of secondary current.
$_mg$ = conductance for $_mI$.
$_mb$ = susceptance for $_mI$.

The magnetizing current is much larger in the induction motor than in a transformer of like capacity, due to the larger leakage flux caused by the air gap in the magnetic circuit. With the rotor at synchronous speed the total current flowing in the primary is the magnetizing current.

$$_m\dot{I} = {}_mg\dot{E}_{21} - j{}_mb\dot{E}_{21} \qquad (378)$$

The component taken by the conductance, $_mg$, represents the part supplying the core losses and is in time phase with the voltage, while the exciting current is in time quadrature with the voltage and in the diagram is indicated by the susceptance, $_mb$. On account of the air gap between the stator and rotor the magnetic leakage is much larger in the induction motor than in the transformer. The secondary reactance is proportional to the frequency of the secondary current, hence to the slip, and it therefore varies with the load. As the slip, under operating conditions of squirrel-cage induction motors, is only a small per cent. of the speed, the reactance of the secondary is necessarily small as compared to the reactance of the primary. It may therefore

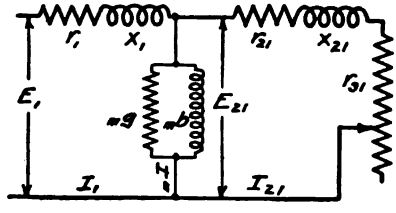


FIG. 218.

be assumed that the equivalent total reactance for both stator and rotor is a constant and equal to $x_1 + x_{21}$. By moving the magnetizing current circuit from the position between the primary and secondary to the position shown in Fig. 219 the solution of the problem is greatly simplified. The errors introduced in the calculations by this change are negligible.

In the equivalent circuit diagram, Fig. 218, $E_1$, $r_1$, $x_1$, $r_{21}$, $x_{21}$, ${}_mg$, ${}_mb$ and ${}_mI$ are constant. The load resistance, $r_{31}$, varies and consequently, likewise $I_1$ and $I_{21}$. The simplified equivalent circuit diagram in Fig. 219 gives a very close approximation of the actual conditions.

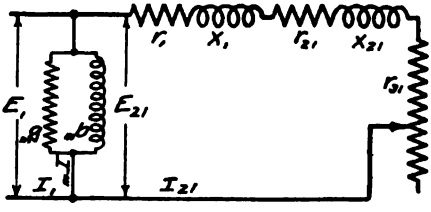



FIG. 219.

The symbolic equations for the circuit in Fig. 219 are:

$$ {}_m\dot{I} = {}_mg\dot{E}_1 - j{}_mb\dot{E}_1 \qquad (379) $$

$$ \dot{I}_1 = {}_m\dot{I} + \dot{I}_{21} \qquad (380) $$

$$ \dot{E}_1 = (r_1 + r_{21} + r_{31})\dot{I}_{21} + j(x_1 + x_{21})\dot{I}_{21} \qquad (381) $$

(*i*) **The Circle Diagram for Polyphase Motors.**—An induction motor with a variable load is equivalent to a circuit having constant reactance and variable resistance as in Figs. 76 and 77; hence for variable load the locus for $I_{21}$ is a circle and the complete performance of the polyphase induction motor may be derived from the circle diagram. This can best be illustrated by describing a common commercial test on induction motors. Two sets of readings are required.

(1) At no load, corresponding to open secondary on a transformer.

(2) Rotor locked, corresponding to short-circuit conditions for a transformer.

(1) *At No Load.*—The rated voltage is applied to the terminals and readings taken of current and power input. The ammeter gives the magnetizing current, ${}_mI$, and the wattmeter the core losses, windage and friction. The losses due to windage and friction are small; so the power indicated by the wattmeter may be considered as due to conductance in the magnetizing circuit as shown in Fig. 219. The magnetizing current has therefore components in time phase and time quadrature with the

voltage and their magnitudes are determined by the amperes, volts and watts taken at no load.

(2) *At Standstill.*—For the second set of readings the rotor is clamped and sufficient voltage applied to give about twice the full-load current in the primary. Readings are taken to obtain the volts, amperes and watts. At standstill the rotor current is very nearly directly proportional to the impressed voltage and hence the value of the equivalent current in the primary at the rated voltage of the motor is found by multiplying the observed value by the voltage ratio. Let ${}_{s}I_{21}$ be the equivalent secondary current in the primary at standstill and at the rated voltage of the motor.

$$ {}_{s}I_{21} = \frac{E_1}{x_1 + x_{21}} \text{ (approximately)} \qquad (382) $$

The wattmeter reading represents the copper loss $(r_1 + r_{21}){}_{s}I_{21}^2$ and hence is proportional to the square of the current.

From the two sets of readings, quantitative values are obtained for the power and reactive components of the currents at no load and at standstill, and from these data the constant and variable losses may be calculated.

At no load:

$ {}_{m}\dot{I} = {}_{m}g\dot{E}_1 - j\,{}_{m}b\dot{E}_1. $

${}_{m}I$ = magnetizing current.

${}_{m}gE_1$ = power component, due to core loss, windage and friction; the constant losses in the motor.

${}_{m}bE_1$ = reactive component providing the revolving field.

At standstill:

$$ {}_{s}\dot{I}_1 = ({}_{s}g + {}_{m}g)\dot{E}_1 - j({}_{s}b + {}_{m}b)\dot{E}_1 \qquad (384) $$

${}_{s}I_1$ = current at standstill in the primary.

${}_{s}gE_1$ = power component, due to copper losses in stator and rotor; the variable losses in the motor.

${}_{s}bE_1$ = reactive components, due to leakage flux, or the stator and rotor reactance.

From these data the circle diagram may be constructed as in Fig. 220.

$E_1$ = constant impressed voltage, drawn from $M$ along the $Y$-axis.

$MN$ = ${}_{m}gE_1$, in time phase with the voltage.

$NO = j_m bE_1$, in time quadrature with the voltage.
Hence
$MO = {}_mI$, the magnetizing current.
$OS = j_{s_s} bE_1$, in time quadrature with the voltage.
$SQ = {}_{s_s}gE_1$, in time phase with the voltage.
Hence
$MQ = {}_{s_s}I_1$, the primary current at standstill.
$OQ = {}_{s_s}I_{21}$, the equivalent secondary current at standstill.

With the center on the line $NO$ extended, draw the circle $OPQV$ passing through the points $O$ and $Q$.

Divide the line $QS$ at $G$ so that $QG$ and $GS$ are in the same ratio as the secondary and primary copper losses. Connect $O$ and $G$.

The complete performance of the motor may be determined from the diagram, Fig. 220. Since the circle is the locus of the

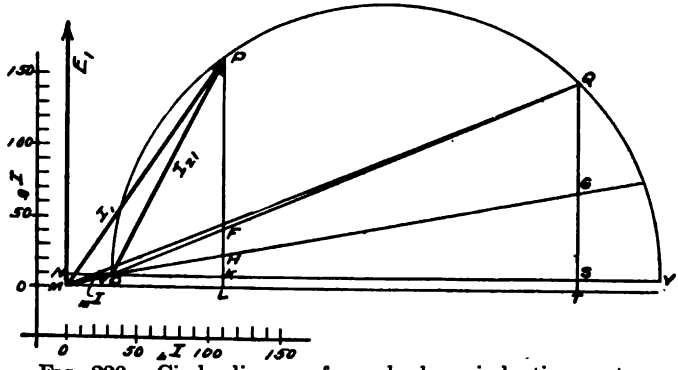



FIG. 220.—Circle diagram for polyphase induction motor.

secondary current, for any point $P$ the following values are given directly from the diagram:

$PM = I_1$, the primary current.
$PO = I_{21}$, the secondary current.
$MO = {}_mI$, the magnetizing current.

If the quantities in the circle diagram represent values per phase, the corresponding quantities for the motor should be multiplied by $n$, the number of phases.

$PL = g_1E_1$; $nE_1(PL)$ = input in watts.
$MN = {}_mgE_1$; $nE_1(MN)$ = core loss, windage and friction.
$HK = {}_{s_s}g_1E_1$; $nE_1(KH)$ = primary copper losses.

$FH = {}_{s}g_2E_1$; $nE_1(HF)$ = secondary copper loss.
$FP = g_2E_1$; $nE_1(FP)$ = mechanical load, or output in watts.

$$\text{Power factor} = \cos(EMP) = \frac{PL}{PM} \tag{385}$$

$$\text{Efficiency} = \frac{PF}{PL} \tag{386}$$

$$\text{Slip} = \frac{\text{rotor loss}}{\text{rotor input}} = \frac{FH}{PH} \tag{387}$$

$$\text{Torque (in kg. at 1 meter radius)} = \frac{nE_1(PF)}{2\pi(9.806)\ (\text{r.p.m.})} \tag{388}$$

$$\text{Torque (in lb. at 1 ft. radius)} = \frac{nE_1\ (PF)\ 33{,}000}{2\pi(746)\ (\text{r.p.m.})} \tag{389}$$

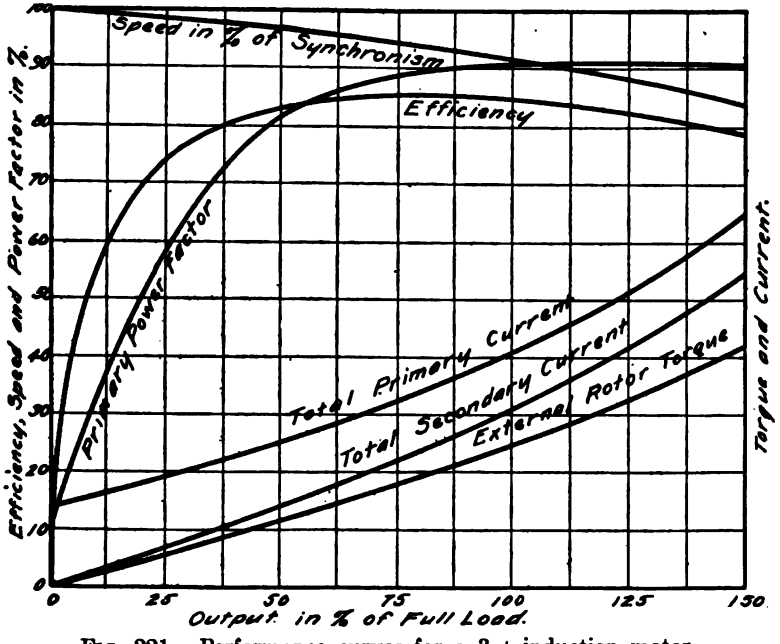



FIG. 221.—Performance curves for a 3-$\phi$ induction motor.

A convenient way of expressing torque is in terms of *synchronous watts* or *synchronous horsepower*, which is the power that is developed by the torque at synchronous speed.

$$\text{Torque (in synchronous watts)} = nE_1(PH) \tag{390}$$

$$\text{Torque (in synchronous horsepower)} = \frac{nE_1(PH)}{746} \tag{391}$$

$$\text{Torque (in ft. lbs.)} = nE_1(PH)\ \frac{7.04}{\text{syn. r.p.m.}}$$

The maximum power output of the motor is found from the circle diagram by taking $P$ at the point of tangency of a straight line tangent to the circle and parallel to $OQ$. The value of the losses, efficiency, power factor, torque, etc., at maximum output may be found for the given position of $P$ in the same manner as above. Complete performance curves for the motor may be developed by plotting the values for a succession of positions of the point $P$ on the circle diagram as in Fig. 221.

(*j*) **The Revolving Field of a Single-phase Induction Motor.**—In order to produce a revolving field two alternating-current



FIG. 222.—Squirrel-cage rotor, showing welded end rings. (*General Electric Co.*)

circuits differing both in time and space phase are required. In the single-phase motor only one of the component fields can be supplied directly from the mains. The single-phase current coming from the mains passes through the stator winding and produces a magnetic field at right angles to the plane of the coil. This flux, known as the *transformer field* varies with the current from a maximum in the positive to an equal maximum in the negative direction. This is shown diagrammatically in Fig. 223. Throughout the whole cycle the direction of the flux is always in the up or down direction in the figure. With the rotor at standstill this is the only flux in the motor. In Fig. 225 is shown the

vector diagram with the instantaneous values as the projections on the *Y*-axis. The pulsations of the transformer flux, $\overset{\sim}{\Phi}$, generate a voltage and this in turn produces currents in the rotor conductors in the region marked $AA'$ in Figs. 223 and 224. The

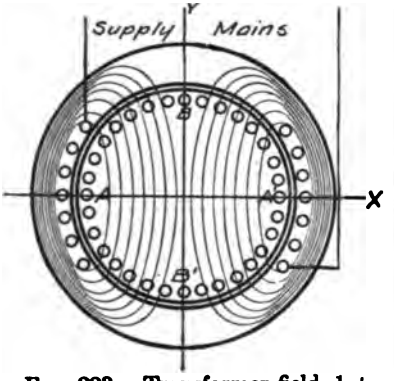



FIG. 223.—Transformer field, 1-$\phi$ induction motor.

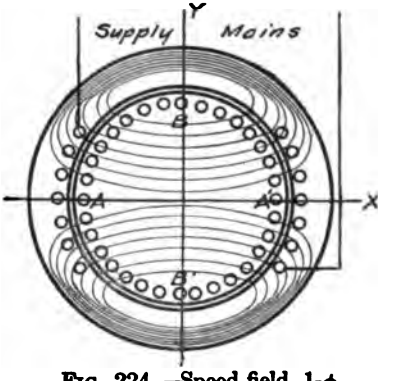



FIG. 224.—Speed field, 1-$\phi$ induction motor.

maximum voltage is induced by transformer action in the conductors at $AA'$ and no voltage at $BB'$.

With the rotor in motion the conductors at $BB'$ cut the transformer flux and thereby generate a voltage in the conductors in the region $BB'$. The resulting current in $BB'$ creates a field at right angles to $BB'$ as shown in Fig. 224. This field, due to the rotation of the rotor, is known as the *speed field*. From Figs. 223 and 224 it is seen that the *transformer field* and *speed field* are in *space quadrature*. The voltgae induced at $AA'$ due to transformer action is in time quadrature with the flux in the transformer field. The maximum voltage generated in $BB'$ due to rotation is necessarily coincident with the maximum strength of the transformer flux. As the maximum rate of cutting lines of force in the speed field must be

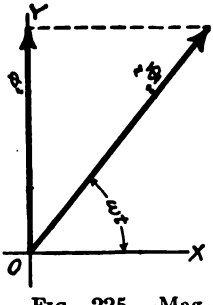



FIG. 225.—Magnetic flux, 1-$\phi$ induction motor at standstill.

coincident with the maximum voltage, the speed field is in *time quadrature* with the transformer field. Hence the two fields are in *space* and *time quadrature* and combined produce a revolving

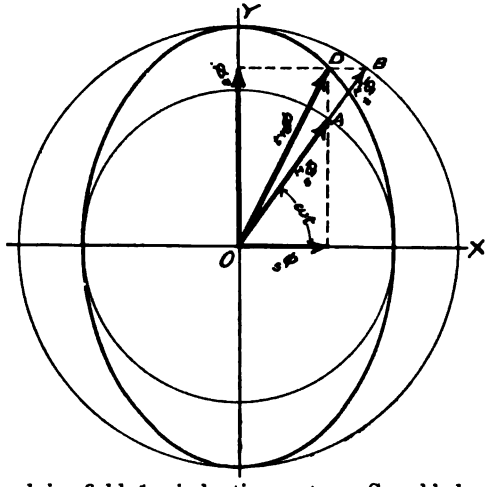



FIG. 226.—Revolving field, 1-$\phi$ induction motor. Speed below synchronism.

field. This is shown graphically in Figs. 226, 227 and 228. By drawing the rotating vectors $\frac{m}{i}\Phi$ and $\frac{m}{s}\Phi$, representing the maxi-

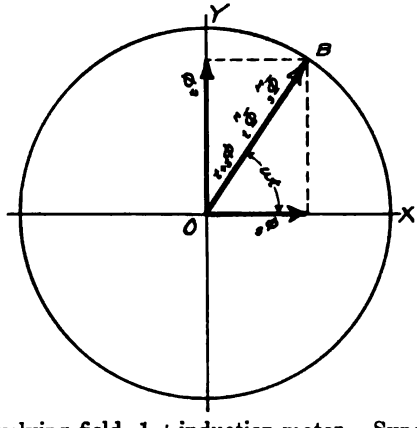



FIG. 227.—Revolving field, 1-$\phi$ induction motor. Synchronous speed.

mum values of the transformer and speed fields, along the same line the diagram correctly expresses both the time phase and space phase relations if the projections of $\frac{m}{i}\Phi$ along the $Y$-axis,

and of ${}^{\kappa}_{s}\Phi$ along the $X$-axis, represent the instantaneous values of the two fields. If the speed is less than synchronous the speed flux, ${}^{\kappa}_{s}\Phi$, is less than the transformer flux, ${}^{\kappa}_{t}\Phi$, and the resulting locus of the revolving field, $_{t+s}\phi$, is an ellipse with the major axis as shown in Fig. 226. At synchronous speed the two fields are of equal magnitude and each equal to the resultant revolving field.

$$ {}^{\kappa}_{t}\Phi = {}^{\kappa}\Phi = {}_{t+s}\phi \text{ (at synchronous speed)} \quad (392) $$

The revolving field is constant and its locus a circle as in Fig. 227. By increasing the speed of the rotor above synchronism

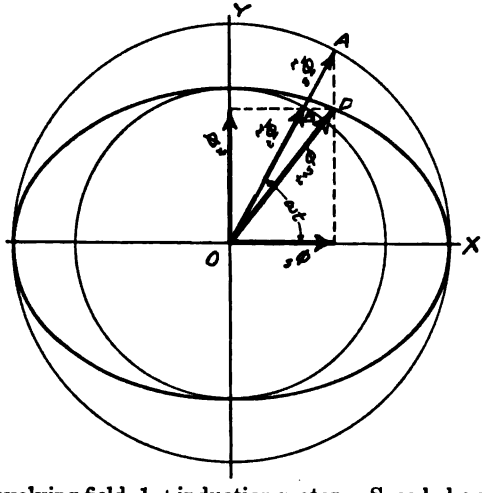



FIG. 228.—Revolving field, 1-$\phi$ induction motor. Speed above synchronism.

the speed field becomes stronger than the transformer field and the locus of the revolving field again becomes an ellipse with the major axis along the speed field as shown in Fig. 228.

Again referring to Figs. 223 and 224 it is evident that voltage is produced in the rotor conductors by two methods:

1. By transformer action; that is, the lines of force produced by the primary current cut the rotor conductors.

2. By the conductors cutting lines of force due to the rotation of the rotor.

In the conductors at $AA'$ voltage is generated: first, by the pulsation of the transformer field; and second, by the cutting of the speed flux by the rotation of the rotor. Likewise in the conductors at $BB'$ voltage is generated: first, by the cutting of

the transformer flux; and second, by the pulsation of the speed field.

$_{pt}E_A$ = voltage in $AA'$ due to pulsation of transformer field.
$_{rs}E_A$ = voltage in $AA'$ due to rotor cutting speed field.
$_{rt}E_B$ = voltage in $BB'$ due to rotor cutting transformer field.
$_{ps}E_B$ = voltage in $BB'$ due to pulsation of speed field.

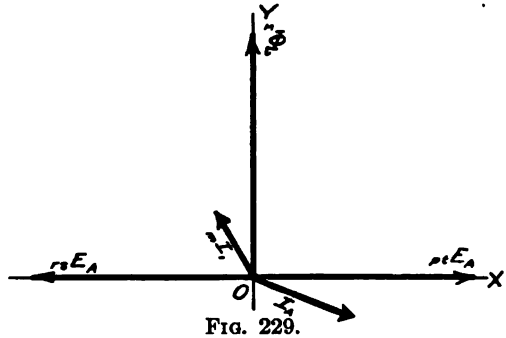



FIG. 229.

By noting the direction of the motion of the rotor conductors and also of the magnetic fluxes in the fields at any instant it is

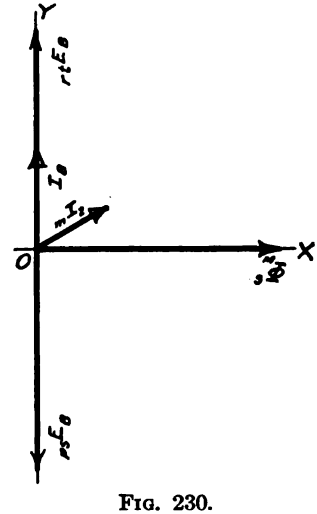



FIG. 230.

seen that the voltage induced at $AA'$, and likewise at $BB'$, by pulsation of one field is opposed by the voltage induced by rotation of the rotor. At synchronous speed the pairs of voltages are equal and a balanced condition exists. At any speed below synchronism the difference in the opposing voltages causes a

current to flow in the conductors $AA'$ and $BB'$. The phase relations are shown in Fig. 229 for $AA'$ and in Fig. 230 for $BB'$.

The magnetizing currents are shown as separate vectors to indicate the similarity of the action to that in the transformer and polyphase induction motor. Also the leakage or reactance, in

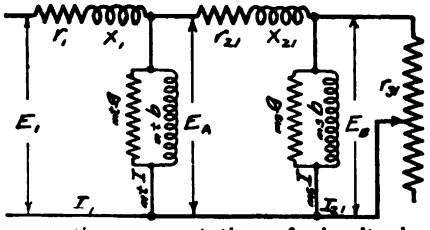


FIG. 231.—Diagrammatic representation of circuits in a 1-$\phi$ induction motor.

each circuit is omitted. By comparing Figs. 223, 224, 229 and 230 it is seen that $I_A$ and $\frac{1}{2}\Phi$, and likewise $I_B$ and $\frac{1}{2}\Phi$ are both in time-phase and space-phase. Hence there is a reaction or torque produced both at $AA'$ and $BB'$ when the rotor is in motion. At standstill $\frac{1}{2}\Phi$ equals zero and hence $I_B$ is also zero, and no torque is produced. Hence the single-phase induction motor has no starting torque and must be brought up to speed by means of special devices.

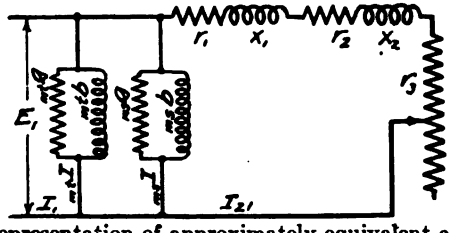


FIG. 232.—Representation of approximately equivalent circuits of a 1-$\phi$ induction motor.

Circuit diagrams similar to the polyphase motor and transformer may be drawn for the single-phase induction motor. In Fig. 231 is shown the circuit diagrams representing the equivalent conditions of the single-phase motor. The transformer and speed fields require separate magnetizing currents as shown in the diagram. By modifying the diagram into the approximately equivalent arrangement of Fig. 232 the solution of the problem is greatly simplified while the errors introduced are relatively small.

$_{M}I$ is the magnetizing current for the transformer field and

has components ${}_{mt}gE_1$ in time-phase and ${}_{mt}bE_1$ in time-quadrature with the voltage.

Similarly ${}_{ms}I$ is the magnetizing current for the speed field and has components ${}_{ms}gE_1$ and ${}_{ms}bE_1$ in time-phase and in time-quadrature respectively with the voltage. With the same notation as used in the polyphase equivalent circuit diagrams the symbolic equations for the equivalent circuits in Fig. 232 are:

$$ {}_{mt}\dot{I} = {}_{mt}g\dot{E}_1 - j\,{}_{mt}b\dot{E}_1 \tag{393} $$

$$ {}_{ms}\dot{I} = {}_{ms}g\dot{E}_1 - j\,{}_{ms}b\dot{E}_1 \tag{394} $$

$$ \dot{I}_1 = {}_{mt}\dot{I} + {}_{ms}\dot{I} + \dot{I}_{21} \tag{395} $$

$$ \dot{E}_1 = (r_1 + r_2 + r_3)\dot{I}_{21} + j(x_1 + x_2)\dot{I}_{21} \tag{396} $$

(*k*) **The Circle Diagram for Single-phase Induction Motors.**—In the circuit in Fig. 232 the only variable is the load resistance,

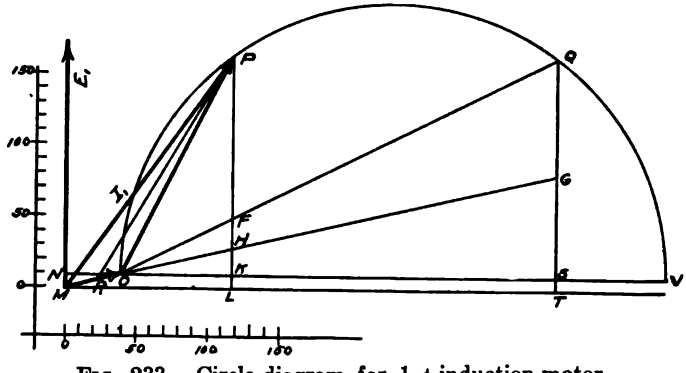



FIG. 233.—Circle-diagram for 1-$\phi$ induction motor.

$r_3$. Hence the locus of the load current, $I_{21}$, is a circle and the performance curves of the single-phase motor may be derived from circle diagram in Fig. 233.

With the motor running at synchronous speed the current in the primary is the vector sum of the magnetizing currents for the transformer and speed fields. In the diagram the power component at synchronous speed is represented by $MN$, and the reactive component by $NO$. The total magnetizing current is therefore $MO$ which is the vector resultant of the magnetizing currents for the transformer and speed fields. For convenience the two component currents are shown as in phase and hence $MO$ as the arithmetic sum. For actual machines this need not

be the case. Let $MO$ be divided at $R$ so that $MR$ represents the magnetizing current, ${}_{m}I$, for the transformer field, and $RO$ the similar current, ${}_{m}I$, for the speed field. With the rotor locked, the power component is represented by $QT$ and the reactive component by $MT$. The circle diagram is then constructed by drawing a circle through $O$ and $Q$ with its center on $NO$ produced, as in Fig. 233. It is often inconvenient to obtain the low voltage required for the locked rotor readings. As the center of the circle diagram must be in the line $OV$, the locus may be determined by finding the point $P$ for any convenient load. A perpendicular from the middle point of the line joining $O$ and $P$ intersects the line $OV$ at the center of the required circle $OPV$.

The complex equations are, at no load and synchronous speed:

$$ {}_{m}\dot{I} = {}_{mt}\dot{I} + {}_{ms}\dot{I} $$

$$ {}_{m}\dot{I} = ({}_{mt}g + {}_{ms}g)\dot{E}_1 - j({}_{mt}b + {}_{ms}b)\dot{E}_1 \qquad (397) $$

${}_{m}I$ = total primary current, at no load.

$({}_{mt}g + {}_{ms}g)E_1$ = power component due to core loss in both the transformer and speed fields.

$({}_{ms}b + {}_{ms}b)E_1$ = reactive component, or exciting component, for both the transformer and speed fields.

Similarly at standstill:

$$ {}_{ss}\dot{I}_1 = ({}_{mt}g + {}_{s}g_1 + {}_{s}g_2)\dot{E}_1 - j({}_{mt}b + {}_{s}b_1 + {}_{s}b_2)\dot{E}_1 \qquad (398) $$

${}_{s}g_1E_1$ = power component due to copper losses in stator.

${}_{s}g_2E_1$ = power component due to copper losses in rotor.

$({}_{s}b_1 + {}_{s}b_2)\dot{E}_1$ = reactive component due to leakage or stator and rotor reactance.

It should be noted that the current at standstill does not contain the magnetizing current for the speed field. In constructing the circle diagram this error is neglected and the construction made as if the power component of the current at standstill were due to the total losses in the motor while running.

After the circular locus of the current is found, complete performance curves may be determined directly from the circle diagram.

$E_1$ = impressed voltage.
$MN = ({}_{m}g + {}_{m}g)\dot{E}_1$
$NO = j({}_{m}b + {}_{m}b)\dot{E}_1$
$MO = {}_{m}\dot{I} + {}_{m}\dot{I} = {}_{m}\dot{I}$
$OS = j_{s}bE_1$
$QS = {}_{s}gE_1$, power component due to total copper losses at standstill.
$QG$ = component due to rotor copper losses at standstill.
$SG$ = component due to stator copper losses at standstill.

From any point $P$ on the circle draw $PL$, $PM$, $PR$ and $PO$, and connect $OQ$ and $OG$. Then from the diagram:

$PL = g_1E_1$; hence $E_1(PL)$ = input in watts.
$MN = ({}_{m}g + {}_{m}g)E_1$; hence $E_1(MN)$ = core loss windage and friction.
$HK = {}_{s}g_1E_1$; hence $E_1(HK)$ = stator copper loss.
$FH = {}_{s}g_2E_1$; hence $E_1(FH)$ = rotor copper loss.
$E_1 \times PH$ = total input to the rotor, not including speed field excitation loss.
$FP = g_2E_1$; hence $E_1(FP)$ = mechanical load or output in watts.
$E_1 \times FL = ({}_{m}g + {}_{m}g + {}_{s}g_1 + {}_{s}g_2)E_1^2$ = total losses in machine.

$$\text{Power factor} = \frac{PL}{PM} \tag{399}$$

$$\text{Efficiency} = \frac{PF}{PL} \tag{400}$$

In the polyphase motor the torque was taken directly from the diagram and when expressed in terms of synchronous watts was found to be equal to the rotor input. It must be noted that while in the polyphase motor the fields produced by both phases are always equal at any rotor speed, in the single-phase motor the speed field varies directly with the speed as indicated by the elliptical locus of the revolving field, shown in Figs. 226 and 228. As the strength of the speed field is one of the torque factors, then torque must vary with the speed.

$$\text{Torque in synchronous watts} = \frac{\text{rotor speed}}{\text{synchronous speed}}(E_1)(PH) \tag{401}$$

Torque is also equal to the output divided by the speed.

Hence torque in synchronous watts =

$$\frac{E_1(PF)\,\text{synchronous speed}}{\text{rotor speed}} \tag{402}$$

From (401) and (402):

$$\frac{\text{rotor speed}}{\text{synchronous speed}} = \left(\frac{PF}{PH}\right)^{\frac{1}{2}} \tag{403}$$

Substituting this value in equation (401):

$$\text{Torque} = E_1\sqrt{(PF)\ (PH)}, \text{ in synchronous watts} \tag{404}$$

(*l*) **Equivalent Single-phase Values.**—For convenience in solving numerical problems in polyphase systems, *equivalent single-phase* values for the current and the resistance are often used. By the *equivalent single-phase current* is understood that current which transmits the same power in a single-phase system *with the voltage between mains the same as in the corresponding polyphase systems.* Similarly, the *equivalent single-phase resistance* gives the same heat losses in the single-phase circuit with the *equivalent single-phase current* as is dissipated in the corresponding polyphase system.

Let $I_2$ = current in one of the mains of a balanced two-phase four-wire system.

$r_2$ = resistance in either phase of the two-phase, four-wire system.

$I_3$ = current in one of the mains of a balanced three-phase three-wire system.

$r_3$ = resistance of one phase (between terminals or mains) of the three-phase, three-wire system.

$I_1$ = equivalent single-phase current.

$r_1$ = equivalent single-phase resistance.

For two-phase, four-wire systems:

$$I_1 = 2I_2 \tag{405}$$

$$r_1 = \tfrac{1}{2}\, r_2. \tag{406}$$

For three-phase, three-wire systems, either $\Delta$ or $Y$:

$$I_1 = \sqrt{3}I_3 \tag{407}$$

$$r_1 = \tfrac{1}{2}\, r_3 \tag{408}$$

(*m*) **Performance Curves by One Wattmeter[1] and Voltmeter.**—The performance curves of an induction motor may be calculated from data giving the primary and secondary resistances and the voltmeter and wattmeter input readings from no load to full load. The primary resistance is usually determined by direct-current methods and the secondary resistance from standstill data. These are obtained by clamping the rotor and impressing a sufficient voltage on the motor terminals to send approximately full-load current through the stator circuit. Under these conditions the input may be considered as equal to the primary and secondary copper losses, and the equivalent

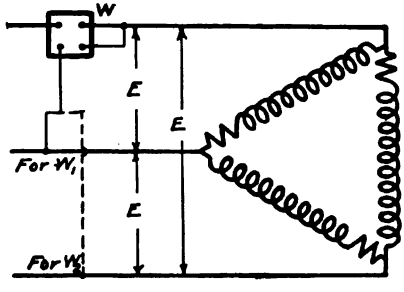



FIG. 234.

secondary resistance may be calculated. By taking voltmeter and wattmeter input readings for various loads, all the required points for determining the performance curves may be calculated. In three-phase circuits the readings of $W_1$ and $W_2$ may be obtained by means of one wattmeter by changing the voltage connection as shown in Fig. 234.

The method is best shown by an example with the calculations arranged in tabular form as in Table IV.

The equivalent resistance $r_1 = 0.23$ ohm. and $r_2 = 0.30$ ohm. The observed input data are given in columns 1, 2 and 3. The method for finding the calculated quantities is indicated at the top of each column. The corresponding performance curves are shown in Fig. 235.

The same method is applied to a single-phase motor in Table V. The *observed data* are given in columns 1, 2 and 3 and the calculated values are derived as indicated at the top of each column. The performance curves are plotted in rectangular

[1] MCALLISTER, "Alternating-current Motors," p. 25.

TABLE IV.—PERFORMANCE TEST. THREE-PHASE INDUCTION MOTOR

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Primary e.m.f., $E$ | Primary watts, $W_1$ | Primary watts, $W_2$ | (2)+(3) Tot. pri. input $\sqrt{3}EI\cos\theta$ | (2)−(3) $EI\sin\theta$ | $\frac{(5)}{(4)}\sqrt{3}$ $\tan\theta$ | $\tan^{-1}(6)$ Primary angle of lag $\theta$ | cos (7) Pri. P.F. $\cos\theta$ | $\frac{(5)}{(1)}$ $\sin\theta$ | $\frac{(5)}{(1)}$ $I\sin\theta$ | $\frac{(10)}{(9)}$ Pri. amp. per lead | (11)$\sqrt{3}$ Tot. pri. amp., $\sqrt{3}I$ | $\frac{(4)}{(1)}$ Power comp. pri. current | (10)$\sqrt{3}$ Quadrature comp. pri. current, $\sqrt{3}I\sin\theta$ | (0.231) (12)$^2$ Pri. cu. loss, $I^2R$ | (4)−(15) No-load data. Iron and friction fixed loss |
| 103.5 | 425 | −220 | 205 | 645 | 5.440 | 79° 35′ | 0.181 | 0.984 | 6.23 | 6.34 | 10.96 | 1.98 | 10.77 | 27.9 | 177 |
| 104.0 | 650 | 000 | 650 | 650 | 1.732 | 59° 58′ | 0.501 | 0.866 | 6.25 | 7.20 | 12.48 | 6.25 | 10.80 | 36.0 | 177 |
| 104.0 | 850 | +160 | 1,010 | 690 | 1.182 | 49° 45′ | 0.646 | 0.763 | 6.63 | 8.20 | 15.02 | 9.70 | 11.48 | 46.7 | 177 |
| 103.7 | 940 | 240 | 1,180 | 700 | 1.028 | 45° 48′ | 0.697 | 0.717 | 6.75 | 9.40 | 16.28 | 11.40 | 11.69 | 61.2 | 177 |
| 104.0 | 1,050 | 330 | 1,380 | 720 | 0.903 | 42° 5′ | 0.742 | 0.670 | 6.92 | 10.30 | 17.85 | 13.25 | 11.98 | 73.5 | 177 |
| 103.0 | 1,140 | 440 | 1,580 | 700 | 0.766 | 37° 28′ | 0.794 | 0.608 | 6.80 | 11.20 | 19.35 | 15.32 | 11.78 | 85.7 | 177 |
| 103.5 | 1,275 | 540 | 1,815 | 735 | 0.700 | 35° 6′ | 0.820 | 0.573 | 7.10 | 12.40 | 21.50 | 17.60 | 12.29 | 107.0 | 177 |
| 103.8 | 1,375 | 630 | 2,005 | 745 | 0.645 | 32° 50′ | 0.840 | 0.542 | 7.18 | 13.20 | 22.90 | 19.30 | 12.42 | 120.8 | 177 |
| 102.7 | 1,525 | 790 | 2,315 | 735 | 0.552 | 28° 54′ | 0.876 | 0.483 | 7.16 | 14.80 | 25.60 | 22.55 | 12.40 | 152.0 | 177 |
| 102.0 | 1,730 | 960 | 2,690 | 770 | 0.494 | 26° 18′ | 0.897 | 0.443 | 7.55 | 17.10 | 29.40 | 25.40 | 13.08 | 202.5 | 177 |

Equivalent single-phase resistance: $R_1 = 0.23$ ohm. $R_2 = 0.30$ ohm. Synchronous speed 1,800 r.p.m.

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (15) + (16) Total pri. losses with fric. | (4) − (17) Sec. input syn. watts, $W_s$ | $\frac{(18)}{255.6}$ Rotor external torque, in lb.-ft. | (13) − 1.98 Power comp. sec. amp. | (14) − 10.77 Quadrature comp. sec. amp., $I_2 \sin \theta_2$ | $\sqrt{(20)^2+(21)^2}$ Total sec. amp., $I_2$ | (0.3) $(22)^2$ Sec. cu. loss, $I_2^2R_2$ | $\frac{(23)}{(18)}$ Rotor slip, $S$ | (18) − (23) Motor output, watts | $\frac{(25)}{746}$ Motor output, hp. | $\frac{(25)}{(4)}$ 100 Motor efficiency, per cent. | 100[1 − ](24) Rotor speed, per cent. syn. |
| 205 | 000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0 | 0.0000 | 000 | 0.00 | 0.0 | 100.0 |
| 213 | 437 | 1.71 | 4.27 | 0.03 | 4.28 | 5.5 | 0.0126 | 432 | 0.58 | 66.3 | 98.7 |
| 224 | 786 | 3.08 | 7.72 | 0.71 | 7.75 | 18.0 | 0.0230 | 768 | 1.03 | 76.0 | 97.7 |
| 238 | 942 | 3.69 | 9.40 | 0.92 | 9.45 | 26.8 | 0.0285 | 915 | 1.23 | 77.5 | 97.1 |
| 250 | 1,130 | 4.42 | 11.27 | 1.21 | 11.35 | 38.5 | 0.0340 | 1,092 | 1.46 | 79.1 | 96.6 |
| 263 | 1,317 | 5.15 | 13.32 | 1.01 | 13.40 | 53.7 | 0.0408 | 1,263 | 1.69 | 79.9 | 95.9 |
| 283 | 1,532 | 6.00 | 15.60 | 1.52 | 15.67 | 73.7 | 0.0480 | 1,458 | 1.96 | 80.4 | 95.2 |
| 298 | 1,707 | 6.68 | 17.30 | 1.65 | 17.40 | 90.5 | 0.0530 | 1,617 | 2.16 | 80.7 | 94.7 |
| 329 | 1,986 | 7.77 | 20.60 | 1.63 | 20.70 | 128.1 | 0.0650 | 1,858 | 2.49 | 80.3 | 93.5 |
| 380 | 2,310 | 9.05 | 23.40 | 2.31 | 23.50 | 165.7 | 0.0720 | 2,144 | 2.87 | 79.6 | 92.8 |

coördinates, in Fig. 236. The simplicity of the experimental part of this method is apparent.

(*n*) **Starting Single-phase Induction Motors.**—The single-phase induction motor has no starting torque, and hence some other means must be provided to bring the machine up to speed. It is evident that for commercial purposes it is desirable to make the machine self-starting. Hence the starting device is part of

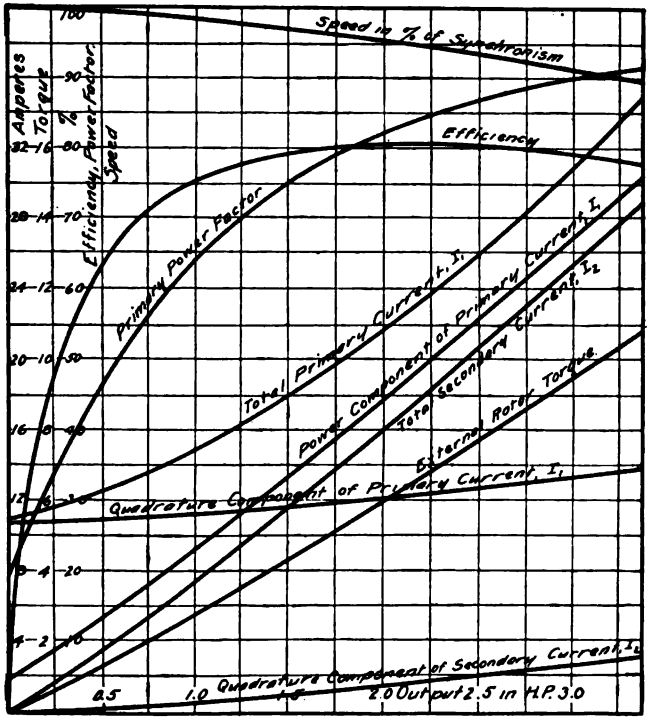



FIG. 235.—Performance curves of a 110-volt, 3-$\phi$, 60-cycle, 3-h.p., induction motor.

the machine and its action should be automatic. Some small motors are started by hand and have no self-starting arrangement. These are, however, unimportant exceptions. Of the automatic devices in commercial use, the repulsion motor, split-phase and shading coils are in general use.

*Repulsion Motor Starting.*—The repulsion motor is described in Chap. XVII. In order to make available the high starting torque of the repulsion motor for bringing the induction motor up to speed, a wound armature with a commutator like that of a

TABLE V.—PERFORMANCE TEST. SINGLE-PHASE INDUCTION MOTOR

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Primary e.m.f., $E$ | Primary amp., $I$ | Primary watts, $W$ | $\frac{(3)}{(1)\times(2)\cos\theta}$ | $\theta$ | $\sin\theta$ | $(0.092)(2)^2$ Primary copper loss | (3) − (7) At no load iron and friction loss | (8) + (7) Total pri. loss with friction | (3) − (9) Secondary input syn. watts | $\frac{(10)}{255.6}$ Rotor external torque in lb.-ft. | $(2)\times(4)$ Power comp. pri. amp. $I\cos\theta$ |
| 108.0 | 19.3 | 690 | 0.331 | 70° 30′ | 0.943 | 34.3 | 656 | 690 | 000 | 0.00 | 6.4 |
| 107.7 | 20.6 | 980 | 0.442 | 63° 45′ | 0.896 | 39.0 | 656 | 695 | 285 | 1.12 | 9.1 |
| 106.9 | 24.3 | 1,600 | 0.616 | 52° 0′ | 0.788 | 54.3 | 656 | 710 | 890 | 3.48 | 15.0 |
| 105.5 | 31.0 | 2,500 | 0.763 | 40° 30′ | 0.646 | 88.4 | 656 | 744 | 1,756 | 6.86 | 23.7 |
| 105.1 | 35.4 | 3,000 | 0.805 | 36° 30′ | 0.595 | 115.1 | 656 | 771 | 2,229 | 8.74 | 28.7 |
| 104.5 | 39.6 | 3,400 | 0.822 | 34° 45′ | 0.570 | 144.1 | 656 | 800 | 2,600 | 10.18 | 32.6 |
| 103.2 | 49.0 | 4,300 | 0.848 | 32° 0′ | 0.530 | 222.0 | 656 | 878 | 3,422 | 13.40. | 41.6 |
| 103.0 | 51.0 | 4,650 | 0.885 | 27° 40′ | 0.463 | 239.5 | 656 | 895 | 3,755 | 14.70 | 45.2 |

$R_1$ = 0.092 ohm, $R_2$ = 0.15 ohm. Synchronous speed 1,800 r.p.m.

| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|---|
| (2) × (6) Quadrature comp. pri. amp., $I \sin \theta$ | (12) − 6.4 Power comp. sec. amp., $I_2 \cos \theta_2$ | (13) − 18.2 Quadrature comp. sec. amp., $I_2 \sin \theta_2$ | $\sqrt{(14)^2+(15)^2}$ Total secondary current | (0.15) $(16)^2$ Secondary copper loss, $I_2^2R_2$ | $\frac{(17)}{(10)}$ Rotor slip, $S$ | (10) − (17) Motor output watts, $W_o$ | $\frac{(19)}{746}$ Motor output, hp. | $\frac{(19)}{(3)}$ 100 Motor efficiency, per cent. | 100[1 − (18)] Rotor speed, per cent. syn. | Rotor resistance |
| 18.2 | 0.0 | 0.0 | 0.00 | 0.0 | 0.000 | 000 | 0.00 | 0.0 | 100.0 | 0.15 |
| 18.5 | 2.7 | 0.3 | 2.72 | 1.1 | 0.004 | 284 | 0.38 | 29.0 | 99.6 | 0.15 |
| 19.2 | 8.6 | 1.0 | 8.66 | 11.3 | 0.013 | 879 | 1.18 | 55.0 | 98.7 | 0.15 |
| 20.2 | 17.3 | 2.0 | 17.40 | 45.5 | 0.026 | 1,710 | 2.29 | 68.4 | 97.4 | 0.15 |
| 21.1 | 22.3 | 3.0 | 22.50 | 76.0 | 0.034 | 2,153 | 2.88 | 71.6 | 96.6 | 0.15 |
| 22.6 | 26.2 | 4.4 | 26.55 | 106.0 | 0.041 | 2,494 | 3.35 | 73.3 | 95.9 | 0.15 |
| 26.0 | 35.2 | 7.6 | 36.00 | 198.0 | 0.058 | 3,224 | 4.34 | 75.0 | 94.2 | 0.15 |
| 23.9 | 38.8 | 5.7 | 39.63 | 236.0 | 0.063 | 3,520 | 4.72 | 75.7 | 93.7 | 0.15 |

direct-current motor is required. The armature winding is short-circuited through a pair of brushes and the position of the brushes, relative to the field, determines the direction of rotation and the magnitude of the torque and starting current. When the motor reaches full speed the brushes are automatically removed from the commutator and simultaneously all the commutator bars are short-circuited, thus converting the repulsion motor to a squirrel-cage induction motor at full

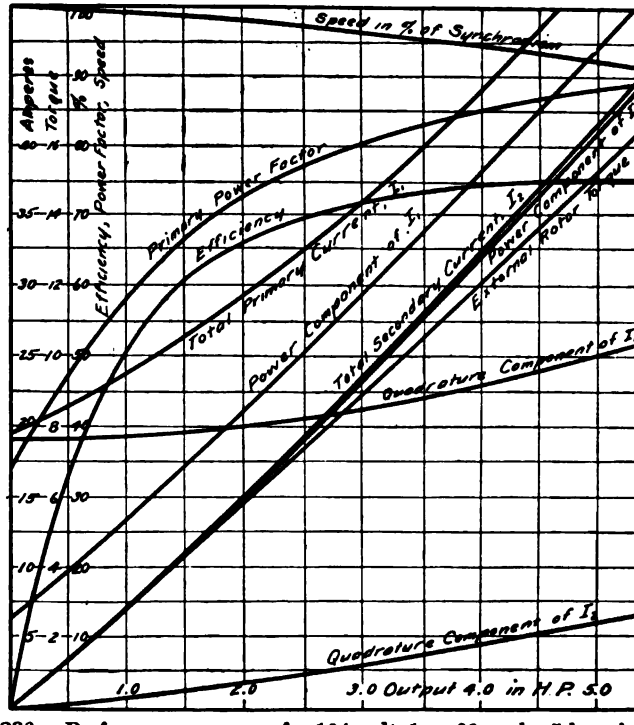



FIG. 236.—Performance curves of a 104-volt, 1-$\phi$, 60-cycle, 5-h.p., induction motor.

speed. Except for small motors this method of starting is the best. The wound rotor and commutator with the automatic device for converting the winding at full speed to the squirrel-cage type greatly increase the cost of manufacture of single-phase induction motors as compared to polyphase motors of the same rating. The ideal simplicity of the polyphase induction motor is lost in providing a starting torque, and more care is required in operating single-phase motors.

*Split-phase.*—The requirement of two magnetic fields differing both in time and space phase for producing a revolving field, may be provided from a single-phase circuit by means of the *split-phase* device. It consists essentially of two parallel circuits, as shown in Figs. 237 and 238, one of which, *B*, has resistance and the other, *A*, either inductive or condensive reactance. As the current is in phase with the voltage in circuit *B*, and either lags or leads in circuit *A*, the fields of magnetic flux are similarly out of phase. Hence by connecting the two circuits to windings differing in space phase the desired revolving field is obtained at standstill and consequently the required starting torque. Due to high first cost and to excessive maintenance, condensive reactance devices are not in

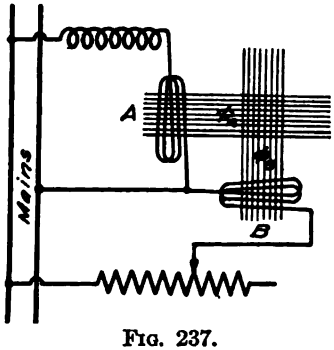



FIG. 237.

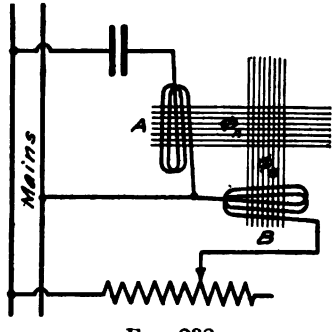



FIG. 238.

commercial use. The inductive reactance produces a low power factor on the line and gives only a weak starting torque, and hence is undesirable except for small motors. The split-phase device, using an inductive reactance, has a low first cost and small maintenance. It is used extensively for small motors requiring only a small starting torque.

*Shading Coil.*—Another device frequently used on small fan motors is the *shading coil.* The motor field has salient poles like a direct-current motor but made of laminated iron. Each pole is divided into two unequal parts as shown in Fig. 239. Around the small part is wound a short-circuited coil. In this coil induced currents flow, thus changing the time phase of the flux inside the shading coil as compared to the flux in the main part of the pole. The combination of the two fluxes differing in both time and space phase produces a

revolving field and consequently a starting torque. Necessarily this torque is small and the device can be used only with small motors requiring small starting torques.

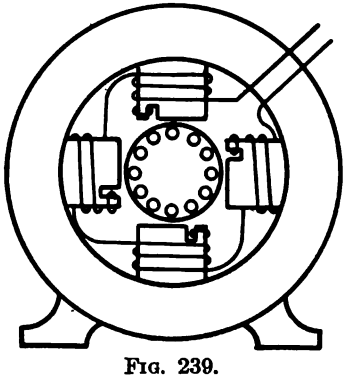


FIG. 239.

(*o*) **Starting Polyphase Induction Motors.**—The polyphase motor has inherently a torque at standstill and no special device is required. However, at standstill the motor acts like a short-circuited transformer and necessarily takes comparatively large currents from the mains. With small motors the starting current is not sufficient to disturb the system from which the

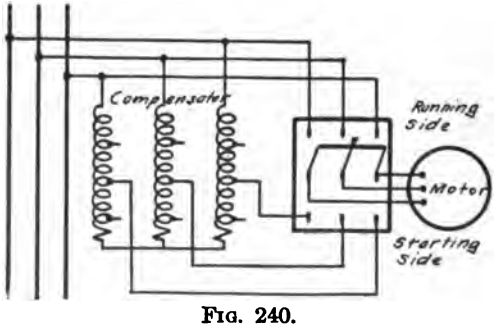



FIG. 240.

motor receives power and hence the small motor is started by throwing the supply switch directly on the mains. For overload protection the small motor generally has a double-throw switch; one side for starting directly on the line, and the other side a running position with the circuits properly fused to protect the motor under continuous operation. With large motors the starting current at full voltage may be larger than can

safely be supplied by the distributing system. To prevent excessive currents while starting, a *starting compensator* is placed between the mains and the motor. This apparatus is merely an autotransformer with circuit diagrams as shown in Fig. 240. The number of steps in the compensator depends on the type of motor, the relative size of the motor and the carrying capacity of the line. In most cases the autotransformer has one position for starting and one for running in addition to the *off* position of the lever, when the circuit is open. In the running position the motor is connected directly across the mains.

(*p*) **Induction Meters.**—The fundamental principle upon which induction-type wattmeters are designed is that rotating fields can be produced by two alternating currents of the same frequency passing through circuits differing in space phase. Essentially the instruments are small induction motors with cylindrical or disk rotors. The load for the integrating wattmeter consists of a metal disk rotating between the poles of a permanent magnet. In the indicating meter the torque produced by the rotating field is counterbalanced by spiral springs. The torque at any instant is proportional to the product of the voltage (current in the shunt coil) and the current (series coil), and hence for the complete cycle the average torque is proportional to the watts. In the integrating wattmeter the speed of the rotating element is proportional to the torque, and the instrument measures the energy delivered to the load.

## PROBLEMS

**1.** In a test on a 5-hp. 60-cycle, 6-pole, three-phase induction motor the following data were obtained:

| Volts, $E$ | Kilowatts | | Ohms, $r_1$ | Ohms, $r_2$ |
|---|---|---|---|---|
| | $W_1$ | $W_2$ | | |
| 130 | 1.36 | −0.90 | 0.265 | 0.18 |
| 130 | 1.45 | −0.85 | (Equiv. 1$\phi$) | (Equiv. 1$\phi$) |
| 130 | 1.55 | −0.75 | | |
| 129 | 2.07 | −0.20 | | |
| 128 | 2.40 | 0.00 | | |
| 129 | 3.05 | 0.55 | | |
| 128 | 3.60 | 0.95 | | |
| 128 | 4.25 | 1.42 | | |

Calculate and plot performance curves as in Table IV and Fig. 235.

**2.** In a test on a 5-hp., 4-pole, 60-cycle, single-phase motor the following data were obtained:

| Volts, $E$ | Amperes, $I$ | Kilowatts, $W$ | Ohms, $r_1$ | Ohms, $r_2$ |
|---|---|---|---|---|
| 110 | 20.7 | 0.70 | 0.09 | 0.18 |
| 110 | 21.5 | 0.98 | | |
| 110 | 25.6 | 1.67 | | |
| 110 | 33.3 | 2.91 | | |
| 110 | 37.5 | 3.46 | | |
| 110 | 41.2 | 3.90 | | |
| 110 | 48.6 | 4.69 | | |
| 110 | 52.3 | 4.95 | | |

Calculate data as in Table V and plot performance curves similar to Fig. 236.

**3.** A test on a 440-volt, 3-$\phi$, 60-cycle, 6-pole induction motor gives the following data: No-load line current = 4.2 amp.; no-load power factor = 0.32; locked-rotor line current (reduced to full-load voltage) = 55.2 amp.; locked-rotor power factor = 0.51; primary resistance = 2.28 ohms, between terminals.

Draw the circle diagram, Fig. 220, and plot the performance curves as in Fig. 221.

**4.** On test a 500-volt, three-phase, 50-cycle induction motor gives the following data: At no load the input is 5.35 kw. and the current is 146 amp. With blocked rotor and for 250 volts impressed, the motor takes 56.1 kw. and 802 amp. The motor is star-connected in both the stator and rotor. Stator resistance between mains = 0.03 ohm.

Draw the circle diagram and plot the performance curves.

**5.** Let the resistance of the rotor in problem 4 be increased until the starting torque is a maximum. Plot the corresponding performance curves for the motor.

For Experiments see page 539.

## CHAPTER XIV

## ALTERNATORS

One of the notable features in the development of the electrical industry during the past fifteen years has been the enormous increase in the size of the central stations. This has been due both to an increase in consumption of electric energy and to the combination of a large number of small plants into comparatively few large systems. Simultaneously with the growth of the central station has come the increase in the size of the generating units. At the Columbian Exposition "Jumbo," the largest generator in the world, had a rating of 750 kv.a., while today single units deliver 50,000 kv.a. continuous load. Any discussion of the historical development of the modern alternator is outside the scope of this volume, and only a few of the more important features in the design are mentioned. The increase in speed, largely due to the invention of the steam turbine, is the chief factor in making possible economical designs of large units. By placing the armature winding in slots the reluctance of the field circuit is greatly reduced and also made to a large degree independent of the size of the armature conductor. With the field placed on the rotating spider, only the exciting current need pass through brushes and collector rings, while the generated power is transmitted through stationary and well-protected copper conductors. To provide adequate voltage regulation for rapid changes in load, used to be a very difficult problem for the designers of alternators. The regulation is now secured, to a much more satisfactory degree than was possible in the self-regulating alternator, by means of regulating devices, outside the generator, such as the Tirrill regulator.

(*a*) **The Field.**—The number of poles is determined by the speed and the frequency. Except for small machines the field forms the rotating part. The poles are built up of punchings of sheet steel, about 0.025 in. thick, and riveted together between cast-steel end plates. The exciting current is lead to the pole windings through brushes bearing on cast-iron or cast-brass slip rings. The exciting current comes from a separate exciter

entirely independent of the generator. The excitation voltage is usually 120 volts, and hence the fields have few turns and take a comparatively large current. Except for small machines the winding is made from copper strips bent on edge forming a short rectangular-shaped coil.

(*b*) **The Armature.**—The armature conductors are placed on the inside of the stator surrounding the rotating field. In order to produce a magnetic circuit with low reluctance the armature conductors are placed in slots, thus leaving only a small air gap between the iron cores of the field and armature The armature core is made from punchings of sheet steel of about half the thickness of the sheet used for the fields. Ducts are provided in the iron core through which the air circulates, keeping the machine cool.

As already noted, the armature winding determines whether the machine is a single-phase, two-phase or three-phase alternator. The two-phase alternator has two single-phase windings spaced 90 electrical degrees apart, and similarly the three-phase machine has three single-phase windings spaced 120 electrical degrees. Very few alternators of the strictly two-phase and three-phase design have ever been built. The so-called two-phase and three-phase alternators are fundamentally designed as four-phase and six-phase machines with their windings spaced 90 and 60 electrical degrees, respectively. In the same manner as already explained for the induction motor, the four-phase and six-phase designs are converted into the so-called two-phase and three-phase alternators, respectively, by merely changing the connections inside the machine.

The three windings on the three-phase alternator may be connected in star or in delta as explained in Chap. IX. Alternators are generally wound star connected as this requires fewer turns, eliminates the 3rd harmonic and provides a convenient neutral point. The windings for any of the above alternators may further differ by being placed in several slots for each pole and for each phase, forming the so-called *distributed* winding. Moreover, the arrangement of the wires between each pair of poles may be of the wave or lap (chain) system, or a combination of the wave and lap form of winding. The windings may be in single or double layer, and the pitch may cover the full 180 electrical degrees or only a fraction of that distance. All of these factors influence the shape and magnitude of the generated voltage wave.

(*c*) **Wave Forms.**—The voltage wave produced by each armature conductor depends directly upon the distribution of the

lines of force in the magnetic field. The distribution of the lines depends upon the reluctance of the magnetic paths, and hence is largely affected by the shape of the field pole.

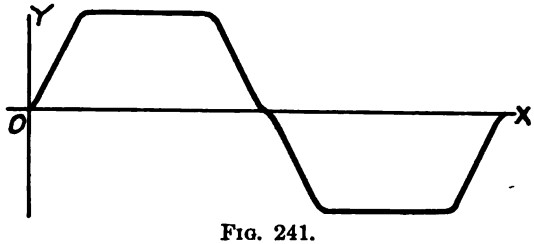



FIG. 241.

As a rule the distribution is not such as to give a simple sine wave of voltage but more of a rectangular shape as in Fig. 241. If the winding is placed in three slots spaced $\gamma$ electrical degrees each one of the coils produces the same shaped waves

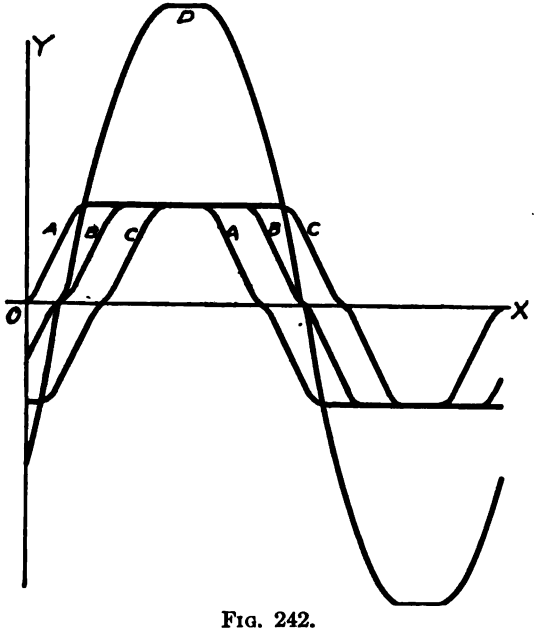



FIG. 242.

but differing in phase position by $\gamma°$. Hence the resultant wave at each point is the sum of the instantaneous values of the component waves, as shown in Fig. 242. In general, armature windings are distributed and placed in several

slots per phase per pole. The number of slots and the distribution of the field flux have a direct bearing on the harmonics in the voltage wave as explained in Chap. XXIII. The magnitude of the total generated voltage per phase depends upon the number and spacing of the slots per phase per pole, Fig. 243. Assuming that the voltage wave for each conductor can be expressed by an equivalent sine wave then the relation of the total voltage per phase to the e.m.f. generated in each conductor may be found by taking the vector sum of the component voltages as shown in Fig. 244. Let *A*, *B*, *C* and *D* be voltage vectors equal in magnitude and displaced in position to the same extent as the spacing of the slots on the

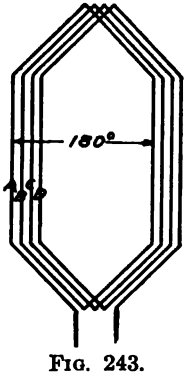



FIG. 243.

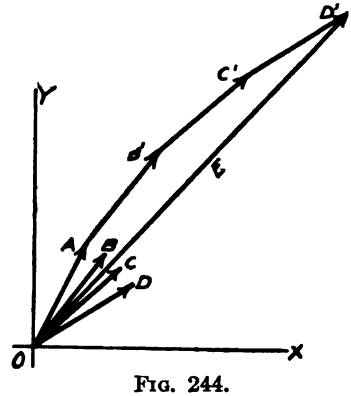



FIG. 244.

armature. Then the resultant vector, *E*, represents the total voltage per phase both in magnitude and phase position.

The ratio of the magnitude of the total voltage per phase to the product of the number of slots and the voltage generated in the conductor in each slot is known as the *distribution factor* and is represented by the letter *k*. The values of the distribution factor for several slots per pole are given in Table VI for two- and three-phase generators.

TABLE VI

| Slots per pole | *k*, Two-phase | *k*, Three-phase |
|---|---|---|
| 1 | 1.000 | 1.000 |
| 2 | 0.924 | 0.966 |
| 3 | 0.911 | 0.960 |
| 4 | 0.906 | 0.958 |
| 6 | 0.903 | 0.956 |

The value of the total voltage per phase is also reduced if the winding is *short-pitch;* that is, the two sides of the armature coil are less than 180 electrical degrees apart, Fig. 245, and hence the voltage generated under the north pole is not quite in phase with the voltage wave produced simultaneously by the corresponding south pole in the other side of the same coil. If the two sides of the coil are out of phase by $\delta$ electrical degrees the total voltage per phase is reduced by the factor $\cos \frac{\delta}{2}$ as may be seen from Fig. 246. Let the distributed winding be in four slots and the voltages on one side under a north pole be represented in phase and magnitude by the lines *A*, *B*, *C* and *D*, Fig. 246; and let the corresponding voltages on the other side, under a south pole be represented in phase and magnitude

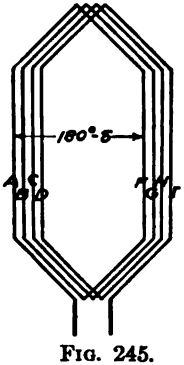



FIG. 245.

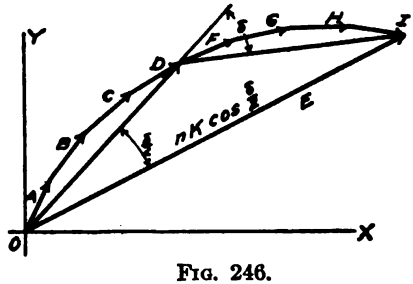



FIG. 246.

by *F*, *G*, *H*, and *I*. In each coil the two voltage waves are displaced by $\delta$ electrical degrees and hence the resultant voltage is equal to the product of the voltage generated per conductor, the number of conductors, the distribution factor and $\cos \frac{\delta}{2}$. Hence the terminal voltage may be expressed in terms of the constants of the design, $n$ the number of turns, $k$ the distribution factor, $\cos \frac{\delta}{2}$ the pitch factor, and $\frac{\pi}{a}\Phi$ the number of lines per pole cut by each conductor. Equation (42) in Chap. II gives the fundamental relation for the production of voltage by induction per turn. This is also the expression for a full-pitch concentrated winding; that is, all the turns placed in one slot per phase, per pole.

Let $n$ = the number of turns, or half the number of conductors.
$f$ = the frequency.
$\overset{m}{u}\Phi$ = the maximum useful flux per pole
$k$ = the distribution factor.
$\cos\frac{\delta}{2}$ = the short pitch factor.

The generated voltage, per phase per pole, therefore equals:

(*a*) For concentrated winding, full-pitch, $4.44nf\overset{m}{u}\Phi 10^{-8}$ volts (421)

(*b*) For distributed winding, full-pitch, $4.44knf\overset{m}{u}\Phi 10^{-8}$ volts (422)

(*c*) For distributed winding, short-pitch, $4.44knf\overset{m}{u}\Phi\cos\frac{\delta}{2}10^{-8}$ volts. (423)

(*d*) **Armature Reaction, Reactance and Resistance.**—For a given machine running at constant speed the factors $k$, $n$, $f$, and $\cos\delta/2$ are constant. The only variable factor in equations (421), (422) and (423) for the generated voltage is the number of lines cut by the armature conductors. The m.m.f. producing useful flux depends first, upon the field excitation and second, upon the *armature reaction.* The field excitation is supplied from some independent source and is usually varied with the load requirements by means of an automatic regulator. The currents in the armature also produce magnetic fluxes that combine vectorially with the main field to form the useful flux, $\overset{m}{u}\phi$, in equation (423). This factor is called the *armature reaction*, and it affects both the magnitude and time-phase of the generated voltage wave. Since the armature flux is in time-phase with the current, the armature reaction depends both on the magnitude of the load current and the load power factor. At no load the armature reaction is zero.

For any load the armature flux is proportional to the current and also its effect upon the field flux depends upon the power factor. With unity power factor the armature current reaches its maximum at the instant the conductor is directly under the pole as shown in Fig. 247. The flux in each pole is increased on the left half and decreased on the right half of the pole face. The total number of lines is approximately the same, the chief effect is the shifting of the distribution of the flux and a consequent change in the voltage wave form. However,

if the load takes a lagging current then the armature current reaches a maximum value in a position between the two poles, as indicated in Fig. 248. The armature flux in this case is in opposition to the field flux on both sides and it thereby decreases the useful flux for the same field excitation. With a load taking

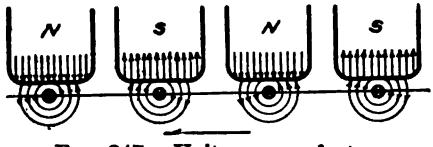



FIG. 247.—Unity power factor.

leading current the armature flux is in the same direction as the field flux as shown in Fig. 249. Hence for armature currents of the same magnitude, the leading current increases the useful flux and hence the generated voltage.

A small part of the armature flux, not affecting the field flux,

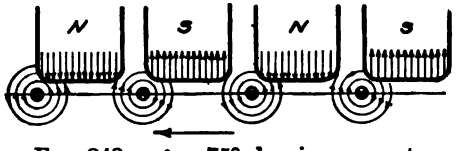



FIG. 248.— $\theta = 75°$, lagging current.

surrounds the conductors in the same way as the leakage flux in transformers. Most of this leakage flux is on the ends of the armature; that is, around portions of the coils that do not pass under the fields. This portion, known as the *leakage flux*, appears in the circuit as an *armature leakage reactance*. Evidently

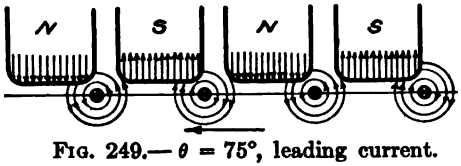



FIG. 249.— $\theta = 75°$, leading current.

the magnitude of the leakage flux depends directly upon the current and is not affected by the lag or lead of the load current. In modern alternators the effect of the armature reaction is three or four times as large as the armature leakage reactance. The armature reactance like the armature resistance consumes voltage and hence the terminal voltage is less than the

generated voltage in magnitude and differs in phase position. These relations are readily seen from a vector diagram. Referred to the armature windings the field flux pulsates, although of constant value in space. Let the wave shape of the fluxes, voltages and currents be of the equivalent simple sine form in order that their phase relations may be represented by the crank vector diagram. Let the diagram in Fig. 250 be drawn for the flux, voltage and current relations in one phase of the armature winding.

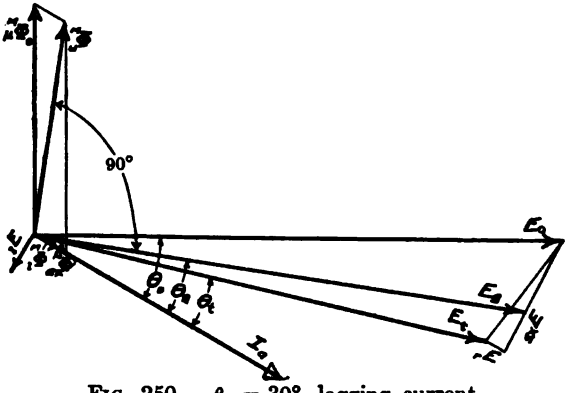



FIG. 250.— $\theta_0 = 30°$, lagging current.

$\overset{\nu}{u}\Phi_0$ = maximum flux due to the field excitation and passing through the armature windings; the useful flux at no load.
$E_0$ = voltage produced by the flux $\overset{\nu}{u}\Phi_0$.
$I_a$ = armature current (lagging in this case).
$\overset{\nu}{ax}\Phi$ = flux representing armature reaction.
$\overset{\nu}{l}\Phi$ = leakage flux producing armature reactance.
$\overset{\nu}{u}\dot{\Phi} = \overset{\nu}{u}\dot{\Phi}_0 + \overset{\nu}{ax}\dot{\Phi}$ = useful flux.
$E_g$ = voltage generated per phase.
$_xE$ = voltage absorbed by armature reactance.
$_rE$ = voltage absorbed by armature resistance.
$E_t$ = terminal voltage.
$\theta_t$ = terminal phase angle.
cos $\theta_t$ = load power factor.

With a lagging current the terminal voltage is less than the voltage generated in the alternator. In Fig. 251 the phase relations of the same quantities are shown for a load having a leading current. The terminal voltage is increased as the

armature reaction adds to the field producing a larger useful flux than at no load.

In Fig. 252 is shown the corresponding vector diagram for

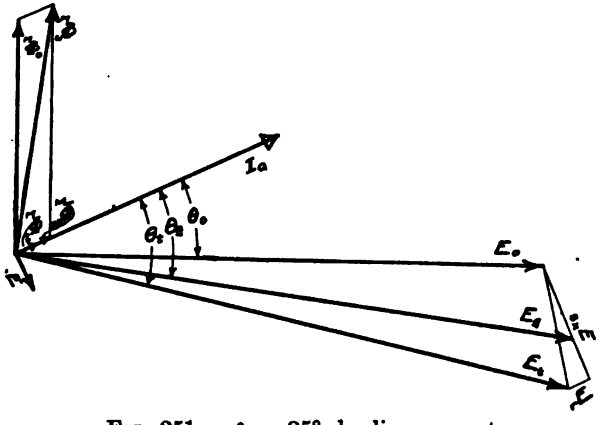



FIG. 251.— $\theta_0 = 25°$, leading current.

unity power factor, and in Figs. 253 and 254 the diagrams for currents of equal magnitude but lagging and leading the voltage, respectively, by 90°. For a given armature current the locus of the terminal voltage is a circle as shown in Fig. 255.

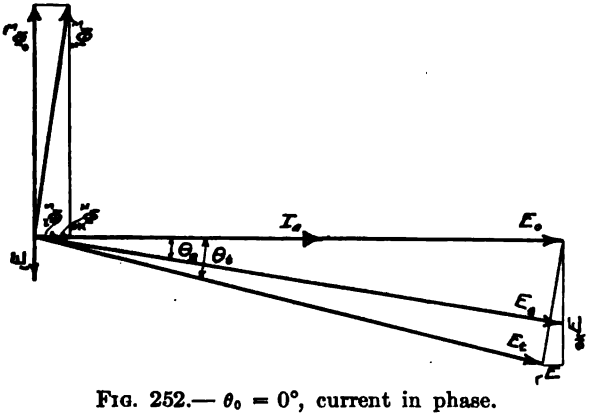



FIG. 252.— $\theta_0 = 0°$, current in phase.

The above vector diagrams refer to constant-current conditions. With constant load the current increases with the decrease of the power factor, and as the drop due to armature reaction and reactance are proportional to the armature current, the effect

at low power factors is greatly increased and the locus of the terminal voltage is no longer a circle. However, in commercial systems the voltage is kept constant or even caused to rise at

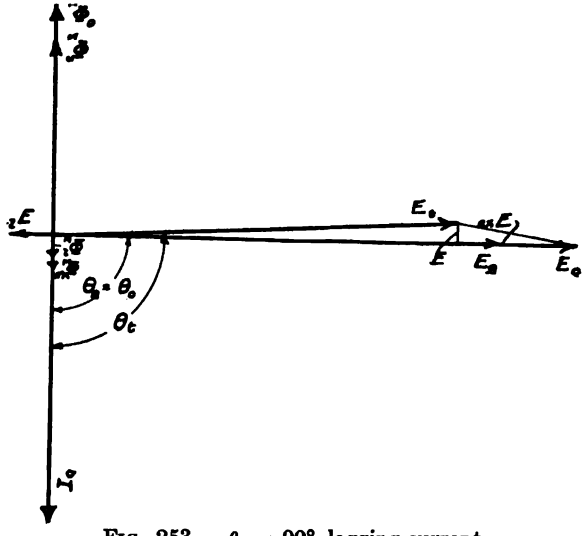


FIG. 253.— $\theta_0 = 90°$, lagging current.

the terminals of the generator with increase of load by increasing the field excitation. To keep the terminal voltage constant the field excitation must be increased so as to compensate for the

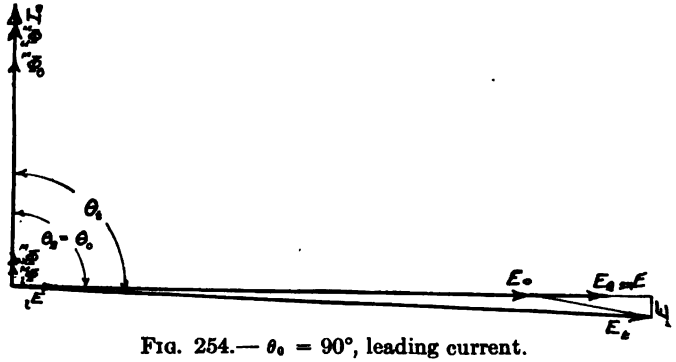


FIG. 254.— $\theta_0 = 90°$, leading current.

effects of armature reaction, reactance and resistance. The number of ampere-turns required also depends upon the previous magnetization or at what point on the magnetization curve the generator is operating.

(*e*) **Magnetization or Saturation Curve at No Load.**—As in direct-current generators, the magnetization curve is merely the relation between the terminal volts at no load and the field current. Usually the field currents are plotted as abscissæ and the corresponding volts as ordinates, as shown in Fig. 256. Experimentally the data for the curve are found by running the alternator at the rated speed and taking a series of readings of field currents and the corresponding volts at the terminals of the machine. For predetermining the saturation curve, when

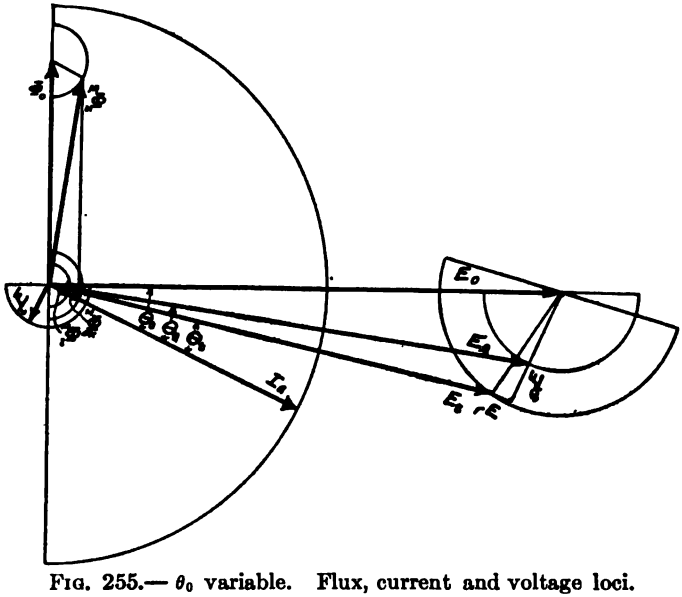



FIG. 255.— $\theta_0$ variable. Flux, current and voltage loci.

designing an alternator, accurate values of the reluctance of the magnetic circuit for the increasing flux densities must be known.

(*f*) **Synchronous Reactance.**—Both the armature reaction and reactance drop are proportional in magnitude to the armature current, and the voltages consumed by both are in time phase for all power factors as seen in Figs. 250 to 255. Both factors may therefore be represented by one equivalent reactance. This is called the *synchronous reactance*, $_sx_a$, and is measured experimentally in the following manner: The armature is short-circuited through an ammeter and the generator is brought up to synchronous speed without field excitation. Field current is then applied and a series of readings taken of the field currents and the corre-

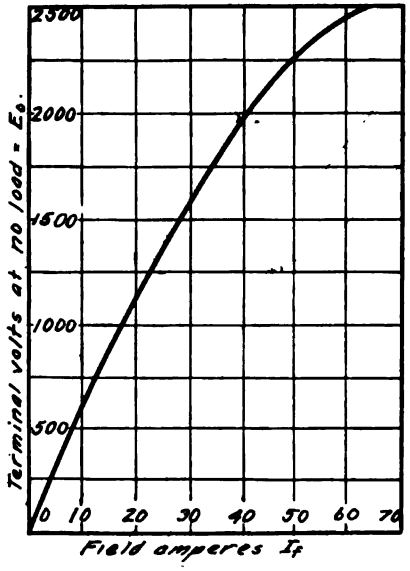



FIG. 256.

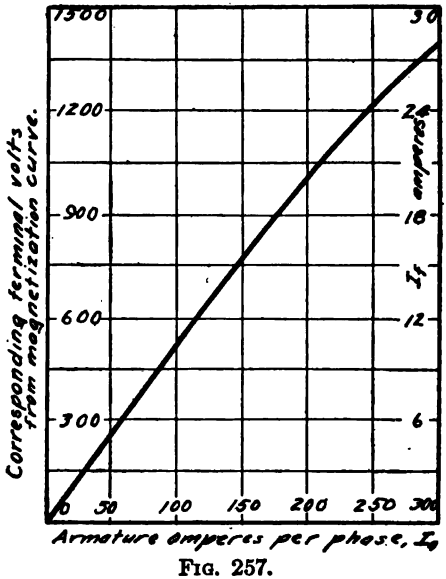



FIG. 257.

sponding armature currents, until the armature current reaches about 50 per cent. above normal full-load value. From the magnetization curve the voltages corresponding to the field-current readings are obtained. A curve is plotted as in Fig. 257 with armature currents as abscissæ, and the terminal volts corresponding to the field currents as ordinates. The curve is nearly a straight line, drooping slightly as the iron approaches saturation. Under short-circuit conditions the armature power factor is nearly zero and the vector relations are as shown in Fig. 253. The voltage consumed by the synchronous reactance is practically equal to the total voltage as the part taken by the resistance is small and almost in time quadrature with the induced voltage.

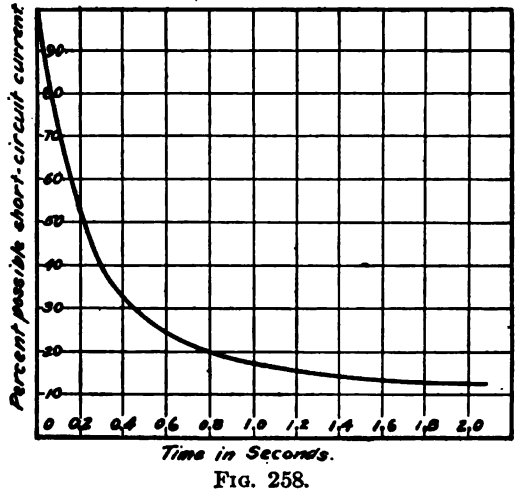



FIG. 258.

From the synchronous reactance curve:

$$E_0 = I_a\sqrt{r_a^2 + {}_sx_a^2},$$

or

$${}_sx_a = \frac{E_0}{I_a}, \text{ approximately} \qquad (424)$$

The value obtained for the synchronous reactance by this method is, however, considerably larger than the actual synchronous reactance of the alternator when operating under full load. Moreover, the synchronous reactance, and hence the synchronous impedance, vary with the load. While the data gained by the saturation and short-circuit reactance tests are of

importance when studying alternator characteristics, the calculated regulation is only an approximation to the actual value. In modern alternators from 10 to 25 per cent. of the total synchronous reactance is due to armature reactance while the remainder is caused by armature reaction. This is illustrated by Fig. 258. The ordinates represent the short-circuit current of a large alternator and the abscissæ the time in seconds after the short-circuit began. The *instantaneous short-circuit current* taken as 100 per cent., gives the ratio of the voltage to the armature impedance. After two seconds, the current reached the *sustained short-circuit* value which represents the ratio between the voltage and the synchronous impedance. The quantitative values of the synchronous reactance and armature leakage reactance are usually given in terms of the voltage consumed by full-load current and expressed in per cent. of the rated terminal voltage. A synchronous reactance of 40 per cent. means that for full-load current the voltage consumed in the synchronous reactance of the armature is 40 per cent. of the rated terminal voltage.

(*g*) **Field Excitation at Any Load and for Any Power Factor.**—From the saturation curve at no load, the synchronous reactance

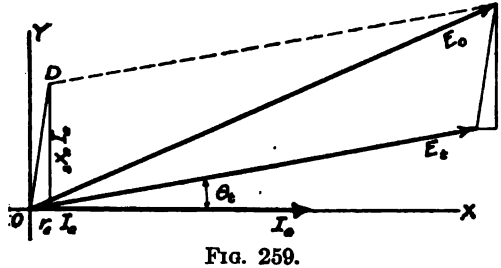



FIG. 259.

curve and the armature resistance, the approximate value of the required field excitation, for any load and at any power factor, may be determined. In the vector diagrams in Figs. 250 to 255, it is shown that the voltage loss caused by armature reaction, reactance and resistance may be represented by a triangle so drawn that the resistance drop is in phase, and synchronous reactance drop in quadrature, with the current. Let the conditions be a constant-terminal voltage, constant-load current and variable-load power factor. In Figs. 259 and 260 the current, $I_a$, is drawn on the $X$-axis, and the terminal voltage, $E_t$,

is leading by $\theta^\circ_t$. The vector for the armature resistance drop is parallel to the current, and the corresponding vector for the armature synchronous reactance drop in quadrature with the current or perpendicular to the $X$-axis. The vector sum of $E_t$, $r_a I_a$ and $_s x_a I_a$ is the required voltage $E_0$. This is the voltage that appears at the terminals at no load, and hence is proportional to the main field flux. From the no-load saturation curve the field excitation corresponding to $E_0$ may be found.

By varying the power factor but keeping the terminal voltage and armature current constant the loci of $E_t$ and $E_0$ are circles, with $E_t$ as radius and points $O$ and $D$ as centers, as shown in Fig. 260. The locus of the generated voltage may be found by separating the synchronous reactance drop into the parts proportional to the armature reaction and the armature reactance. Under ordinary operating conditions the load power factor, the terminal voltage and the armature current are all variable.

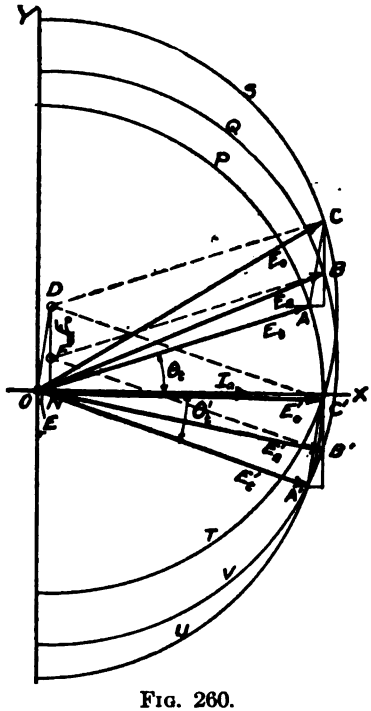



FIG. 260.

Assuming two factors as constant and the third variable, three combinations are possible. The corresponding loci are shown in Fig. 260.

First Case.—

$E_t$ and $I_a$ constant; $\theta_t$ variable.

$I_a$ = armature current, assumed constant.

$E_t$ = the terminal voltage, assumed constant.

$r_a$ = armature resistance.

$x_a$ = armature reactance.

$_s x_a$ = synchronous reactance of armature, equivalent to both the armature reactance and reaction.

$E_g$ = the generated voltage, proportional to the useful flux.

$E_0$ = the no-load voltage corresponding to the same field excitation, and proportional to the field flux.

$\theta_t$ = angle of lead or lag, variable.

$OA = E_t$

$ON = r_a I_a = {}_1E_a$

$NF = x_a I_a = {}_2E_a$

$ND = {}_3x_a I_a = {}_3E_a$

$OB = E_g$

$OC = E_0$

$$\dot{E}_g = \dot{E}_t + {}_1\dot{E}_a + {}_2\dot{E}_a = \dot{E}_t \cos\theta_t + {}_1\dot{E}_a + j(\dot{E}_t \sin\theta_t + {}_2\dot{E}_a) \quad (425)$$

$$\dot{E}_0 = \dot{E}_t + {}_1\dot{E}_a + {}_{23}\dot{E}_0 = \dot{E}_t \cos\theta_t + {}_1\dot{E}_a + j(\dot{E}_t \sin\theta_t + {}_{23}\dot{E}_a) \quad (426)$$

Circle $PAU$ = locus of $E_t$, ($O$ center, $E_t$ radius).

Circle $QBV$ = locus of $E_g$ ($F$ center, $E_t$ radius).

Circle $SCT$ = locus of $E_0$ ($D$ center, $E_t$ radius).

$\cos\theta'_t$ = power factor of locus for perfect regulation.

Since the sides of the triangle $OND$ are directly proportional to the armature current similar constructions may be drawn for any load current and the value of the field excitation determined.

Second Case.—

$E_t$ and $\theta_t$ constant; $I_a$ variable.

For constant power factor and terminal voltage the locus of the no-load voltage, $E_0$, is along the line $CA$; for with a change in $I_a$ the point $D$ moves along the line $OD$ and as $E_t$ and $\theta_t$ are assumed constant the point $C$ must move along the line $CA$.

Third Case.—

$\theta_t$ and $I_a$ constant; $E_t$ variable.

For constant power factor and current with variable terminal voltage, the triangle $OND$ remains constant and the no-load voltage is the resultant of $OD$ and the several values of $E_t$, and the locus of $E_0$ is along the line $DC$. The required field excitation may therefore be found from curves in Figs. 256, 257, and 260 for any load at any power factor and for any terminal voltage.

(*h*) **Regulation. TA Regulator.**—By the regulation of an alternator is meant the per cent. increase in voltage when full load is thrown off the machine, the frequency and field excitation remaining constant. Using the same notation as in Fig.

259 the regulation is $\frac{E_0 - E_t}{E_t} \times 100$, in per cent. From the previous discussion it is evident that the regulation of any given alternator depends upon the power factor of the load as shown in Fig. 260, and hence, a statement of the regulation of an alternator should include the power factor of the load. For close regulation the voltage lost in the armature must be relatively small. The resistance may readily be reduced to the required value but not the synchronous reactance. Fundamentally the separately excited alternator is not self-regulating and none of the internal automatic devices have proved satisfactory. As an example the diagram in Fig. 261 shows a composite excitation having a series field producing a compounding

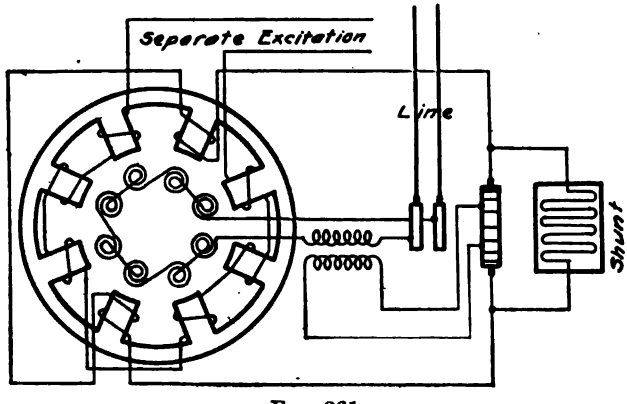



FIG. 261.

effect as in the direct-current compound-wound generator. Part of the alternating current transmitted by the transformer, is rectified by the commutator while the larger portion passes through the shunt.

Since the invention of automatic regulators entirely separate from the alternator, all the earlier methods of self-regulation have become obsolete. By proper adjustment the automatic regulator insures highly satisfactory regulation on the system, although the inherent regulations of the alternators may vary widely. With the use of an automatic regulator, like the TA type manufactured by the General Electric Co., inherent regulation of the alternator makes very little difference as the regulator so adjusts the excitation that constant voltage is obtained under all conditions of load (Fig. 262).

"An elementary diagram of the type TA, form A-2 regulator connections with an alternating-current generator and exciter is shown in Fig. 263.[1] The regulator has a direct-current control magnet, an alternating-current control magnet, and a relay. The direct-current control magnet is connected to the exciter busbars. This magnet has a fixed stop-core in the bottom and a

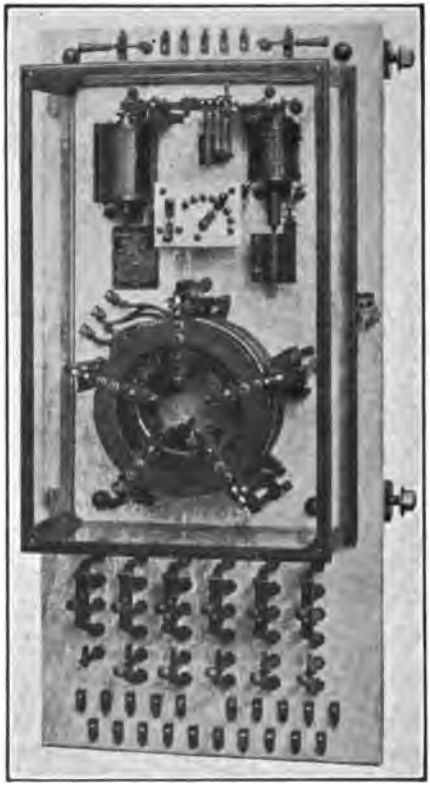


FIG. 262.—Type TA, form F-5, voltage regulator. (*General Electric Co.*)

movable core in the top which is attached to a pivoted lever having at the opposite end a flexible contact pulled downward by four spiral springs. For clearness, however, only one spring is shown in the diagram. Opposite the direct-current control magnet is the alternating-current control magnet which has a potential winding connected by means of a potential transformer

[1] Descriptive Circular, General Electric Co.

to the alternating-current generator or busbars. There is an adjustable compensating winding on the alternating-current magnet connected through a current transformer to the principal lighting feeder. The object of this winding is to raise the voltage of the alternating current busbars as the load increases. The alternating-current control magnet has a movable core and a lever and contacts similar to those of the direct-current control magnet, and the two combined produce what is known as the *floating main contacts*. The relay consists of a U-shaped magnet core having a differential winding and a pivoted armature controlling the contacts which open and close the shunt

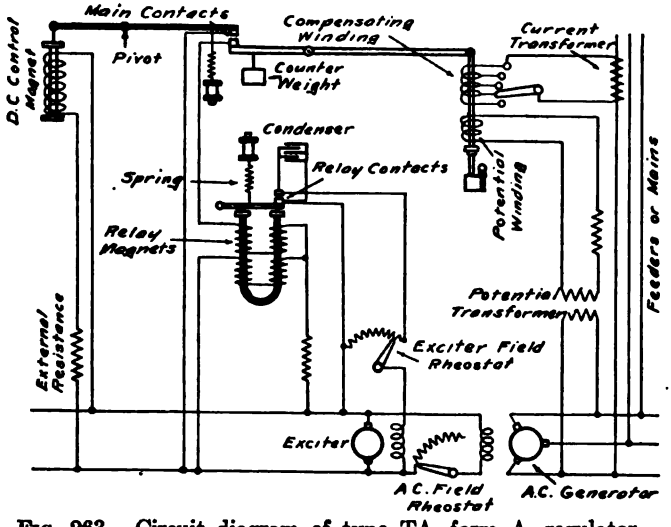



FIG. 263.—Circuit diagram of type TA, form A, regulator.

circuit across the exciter field rheostat. One of the differential windings of the relay is permanently connected across the exciter busbars and tends to keep the contacts open; the other winding is connected to the exciter busbars through the floating main contacts, and when the latter are closed neutralizes the effect of the first winding and allows the relay contacts to short-circuit the exciter field rheostat. Condensers are connected across the relay contacts to prevent severe arcing and possible injury. The circuit shunting the exciter field rheostat through the relay contacts is opened by means of a single-pole switch at the bottom of the regulator panel and the rheostat turned in until the

alternating-current voltage is reduced 65 per cent. below normal. This weakens both of the control magnets and the floating main contacts are closed. This closes the relay circuit and demagnetizes the relay magnet, releasing the relay armature, and the spring closes the relay contacts. The single-pole switch is then closed and as the exciter field rheostat is short-circuited the exciter voltage will at once rise and bring up the voltage of the alternator. This will strengthen the alternating-current and direct-current control magnets and at the voltage for which the counterweight has been previously adjusted the main contacts will open. The relay magnet will then attract its armature and by opening the shunt circuit at the relay contacts, will throw the full resistance into the exciter field circuit tending to lower the exciter and alternator voltage. The main contacts will then be again closed, the exciter field rheostat short-circuited through the relay contacts and the cycle repeated. This operation is continued at a high rate of vibration due to the sensitiveness of the control magnets and maintains, not a constant, but a steady exciter voltage."

(*i*) **Rating.**—While the rating of an alternator includes a statement of the normal terminal voltage, whether wound for one, two or three phases, speed and frequency, the term essentially applies to the capacity or guaranteed output of the machine. With the other factors given, the output depends upon the temperature rise on continuous operation at full load. This increase of temperature is usually specified, in degrees above room temperature, for continuous full load and short period of overload. The temperature rise depends directly upon the rate of heat generation by the losses in the alternator, and the rate of heat dissipation from the surface of the machine. The temperature rises until the dissipation equals the heat production. Since the copper losses vary as the square of the current, and for a given load the current varies inversely as the power factor, the temperature rise, and hence the rating can not be given for any specified load in kilowatts without stating the power factor. Therefore alternators are rated in *kilovolt-amperes* (*kv.a.*). This is necessary, as it is evident, that the manufacturer can have no control over the power factor of the load that the machine will be required to carry in service. As the iron losses also change somewhat for different power factors the rating in kv.a. is often supplemented by a statement of temperature rise for two power factors, usually unity and 85 per cent., lagging current.

(*j*) **Losses.**—The losses in an alternator are similar to those in the direct-current generator, and may be grouped as friction, windage, copper and iron losses. The friction losses are in the bearings and under the brushes. These and the windage are usually determined by empirical formulæ or measured by means of a calibrated motor. The brush and windage losses are small and may be neglected except in high-speed machines like turbo-alternators. The following empirical formula for bearing friction gives a close approximation:

$$\text{Bearing friction loss} = 0.81dl\left(\frac{V}{100}\right)^{\frac{3}{2}} \text{ in watts} \qquad (427)$$

$d$ = diameter of bearing in inches.
$l$ = length of bearing in inches.
$V$ = rubbing velocity of the bearing in feet per minute.

The copper loss is partly in the field and partly in the armature. The field or excitation loss is readily determined by taking volt and ampere readings. In the armature the resistance and the current are measured to determine the $r_aI_a^2$ losses. In large machines the resistance is very small and difficult to determine by direct measurements. If the dimensions of the armature conductors are given the resistance may be calculated. The leakage flux produces eddy currents in the armature conductors which cause an additional copper loss, sometimes called the *load losses*. The best way to determine these losses is by means of a calibrated driving motor. The alternator is run at normal speed with the armature short-circuited through an ammeter. The field excitation is adjusted until full-load current flows through the ammeter. The input of the driving motor is recorded and the power required to drive the alternator determined from the efficiency curve of the motor. For a second set of readings the alternator is run at the same speed with the armature circuits open and without field excitation. The power required to drive the alternator is again determined from the motor input and motor efficiency. The difference in the two cases is the $r_aI_a^2$ loss and the *short-circuit core loss*, including the load loss and iron loss due to the core flux. As the latter is also included when measuring the iron losses, it is necessary to separate the two parts in the short-circuit core loss. This is usually done empirically as recommended in the standardization rules of the A. I. E. E.: "Stray load losses are to be determined by operating the machine on short circuit and at rated-load

current. This, after deducting the windage and friction and $I^2R$ loss, gives the stray load loss for polyphase generators and motors." Having determined the friction, windage and load losses the iron loss may be found by taking a third set of readings for the input of the driving motor, the alternator running at normal speed, with full field excitation and open armature circuits.

(*k*) **Efficiency.**—The efficiency of an alternator is the ratio of the power output to the power input, and is usually expressed in per cent.

$$\text{Efficiency in per cent.} = \frac{\text{power output} \times 100}{\text{power input}} = \frac{(\text{output in kw.}) \times 100}{\text{output in kw.} + \text{losses in kw.}} \qquad (428)$$

A typical efficiency curve for large generators is shown in Fig. 264.

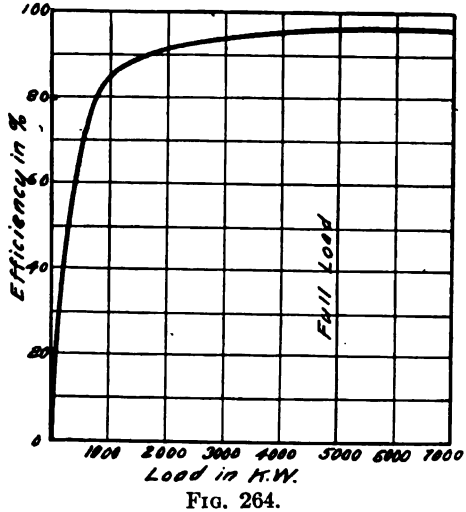



FIG. 264.

For Table VII the losses are given separately for several alternators. The relative amounts of energy changed into heat in the armature and field vary considerably for different designs. The ratios between the field amperes for no load and normal terminal voltage (taken from the no-load saturation curve) to the field amperes for full-load current with the terminals short-circuited (taken from short circuit saturation curve —straight line) are given in the S.C. ratio column. Armature resistance is between terminals at 25°C. Synchronous reactances are given in per cent. of terminal voltage at full-load current.

TABLE VII

| Full load, kv.a. | $E_t$ Volts | $f$ Cycles | Speed, r.p.m. | Phase | S.C. ratio | Iron loss, kw. | Friction and windage, kw. | Field $RI^2$ loss, kw. | Armature resistance across terminals, ohms | $x_s$ Per cent |
|---|---|---|---|---|---|---|---|---|---|---|
| 500 | 2,300 | 60 | 360 | 2 | 1.4 | 18.7 | 6.1 | 5.0 | 0.250 | 45 |
| 500 | 2,300 | 25 | 300 | 3 | 1.8 | 8.1 | 3.2 | 5.4 | 0.273 | 44 |
| 750 | 2,300 | 60 | 400 | 3 | 2.2 | 17.0 | 7.1 | 6.8 | 0.111 | 40 |
| 1,000 | 2,300 | 25 | 500 | 3 | 1.6 | 12.5 | 9.5 | 7.3 | 0.084 | 42 |
| 1,500 | 2,300 | 60 | 200 | 3 | 2.2 | 24.6 | 10.0 | 7.8 | 0.037 | 44 |
| 1,500 | 2,300 | 25 | 750 | 3 | 1.5 | 31.4 | 15.3 | 7.5 | 0.043 | 45 |
| 1,800 | 2,300 | 60 | 360 | 3 | 1.6 | 28.1 | 11.8 | 7.8 | 0.023 | 47 |

(*l*) **Parallel Operation of Alternators.**—In most power plants it is necessary to connect several generators to the same dis-

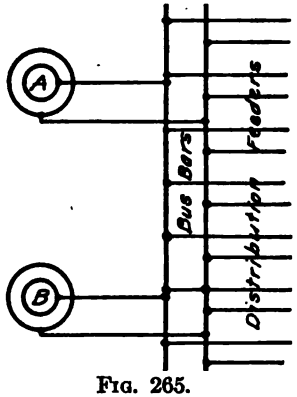



FIG. 265.

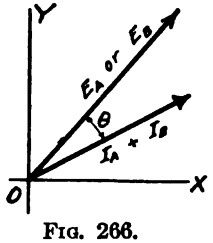



FIG. 266.

tribution system as this arrangement gives the best operating economy. In constant-potential systems the alternators must therefore be operated in parallel. The alternating nature of the voltage wave and the constant speed required for constant frequency make the operation of alternators fundamentally different from similar arrangements of direct-current generators. Consider the two alternators in Fig. 265, operating in parallel and delivering energy to the same busbars. If both machines produce simple sine voltage waves, it is possible to adjust the field excitation and the phase position of the two machines so that the voltages from the two alternators shall at every instant be equal. At the busbars the two voltages arrive in phase, are of equal magnitude and may be represented by one vector, as in Fig. 266. The load current is the sum of the currents coming from the two machines. With respect to the busbars the

two machines are therefore in parallel and the voltages and current waves are in phase. However, between the machines, the two armature windings and the connections to the busbars form a series circuit, and therefore relatively to each other the two voltage waves are in opposition, and may be represented by the diagram in Fig. 267. The two diagrams in Figs. 266 and 267 represent the same voltages; the difference in phase position merely depends on whether the parallel circuits for the load or the series circuit through the two armatures are under discussion. The conditions for parallel operation, with no cross-currents flowing between the alternators in the series circuit and for simple sine voltages are:

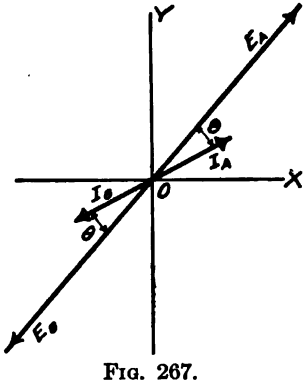


FIG. 267.

1. The machines must be in synchronism; that is, generate voltage at the same frequency.

2. The voltages must be equal in magnitude.

3. The voltages must be in phase (referred to the parallel circuit) or the machines must be *in step*.

(*m*) **Synchronizing Current.**—Given the two alternators operating in parallel and the voltages represented by vectors 180° apart as in Fig. 267. Let the power supplied by the prime-mover driving machine *A* be increased by a small amount. This tends to increase the speed of *A* and bring the voltage vectors less than 180° apart, as indicated in Fig. 268. The voltages in this phase relation will not neutralize completely but form a resultant voltage, $_{s}E$ causing a current $_{s}I$ to flow in the series circuit through the armatures of both alternators. The synchronizing current lags behind the resultant voltage, $_{s}E$, by an angle $_{s}\theta$, depending upon the total resistance and reactance of the series circuit, therefore including the resistance and reactance of both armatures.

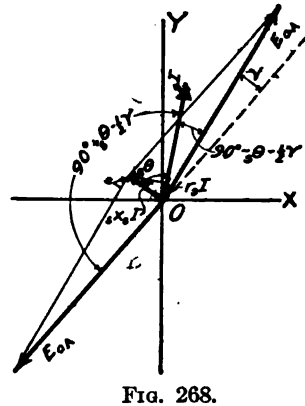


FIG. 268.

29.6
74.4/0



$r$ = total resistance in the series circuit.
$_{s}x$ = total synchronous reactance in the series circuit.
$E_{0A}$ = open-circuit voltage in alternator $A$.
$E_{0B}$ = open-circuit voltage in alternator $B$.
$_{s}E$ = resultant voltage causing the current $_{s}I$ to flow in the series circuit.
$_{s}I$ = synchronizing current.

$$\cos {}_{s}\theta = \frac{r}{{}_{s}z} = \frac{r}{\sqrt{r^2 + {}_{s}x^2}} \tag{429}$$

With the two voltages $E_{0A}$ and $E_{0B}$ 180°-$\gamma$ apart, as in Fig. 268 the synchronizing current leads $E_{0A}$ by $90° - ({}_{s}\theta + \frac{1}{2}\gamma)$ and lags behind $E_{0B}$ by $90° + ({}_{s}\theta - \frac{1}{2}\gamma)$. With ${}_{s}\theta$ large compared to $\gamma$ the angle of lag of the synchronizing current with respect to $B$ is greater than 90° and hence the cosine negative, while for generator $A$ the angle of lead is less than 90° and hence the cosine is positive. This means that alternator $A$ gives out power; that is, acts as a generator, while machine $B$ receives power from the series circuit. Hence the synchronizing current transmits power from $A$ to $B$ and therefore tends to retard $A$ and accelerate $B$ and thus bring the two machines back into phase opposition, or to keep them in synchronism. In the series circuit between the two alternators:

$$\text{The power supplied by } A = E_{0A}{}_{s}I \cos (90° - {}_{s}\theta - \tfrac{1}{2}\gamma) \tag{430}$$
$$\text{The power received by } B = E_{0B}{}_{s}I \cos (90° + {}_{s}\theta - \tfrac{1}{2}\gamma) \text{ or}$$
$$\text{(Using the negative sign)} = -E_{0B}{}_{s}I \cos (90° - {}_{s}\theta + \tfrac{1}{2}\gamma) \tag{431}$$

The difference in the power supplied by $A$ and the power received by $B$ is equal to the power lost in the series circuit, or ${}_{s}E{}_{s}I \cos {}_{s}\theta$.

$$E_{0A}{}_{s}I \cos (90° - {}_{s}\theta - \tfrac{1}{2}\gamma) + E_{0B}{}_{s}I \cos (90° + {}_{s}\theta - \tfrac{1}{2}\gamma) = {}_{s}E{}_{s}I \cos {}_{s}\theta$$

or

$$E_{0A}{}_{s}I \cos (90° - {}_{s}\theta - \tfrac{1}{2}\gamma) - E_{0B}{}_{s}I \cos (90° - {}_{s}\theta + \tfrac{1}{2}\gamma) = {}_{s}E{}_{s}I \cos {}_{s}\theta \tag{432}$$

$$E_{0A} = E_{0B};\ {}_{s}E = 2E_{0B} \cos (90° - \tfrac{1}{2}\gamma) \tag{433}$$

If several alternators are connected to one set of busbars the synchronizing current tends to keep all the machines in step. If the prime mover of any one machine tends to increase its speed, a resultant voltage is generated, causing a synchronizing

current to flow. The machine that leads delivers power, is thus retarded, and thereby tends to hold the machine in synchronism and in step. If, on the other hand, the prime mover of one of the alternators gives less than its share of the load the machine lags behind and receives power by the synchronizing current and therefore is in phase relation similar to machine $B$ in Fig. 268. The power supplied by the synchronizing current accelerates the lagging alternator, thus tending to keep the machine in synchronism. The maximum value of the synchronizing torque is proportional to the maximum power supplied or received through the series circuit. For the alternator tending to run too fast the expression for power is given in equation (430) and for the lagging machine in equation (431).

From (430):

$$
\begin{aligned}
{}_{s}P_{A} &= E_{oA}\,{}_{s}I \cos (90^\circ - {}_{s}\theta - \tfrac{1}{2}\gamma) \\
&= 2E_{oA}^{2}\,\frac{({}_{s}x \cos \tfrac{1}{2}\gamma + r \sin \tfrac{1}{2}\gamma)\ (\sin \tfrac{1}{2}x)}{r^{2} + {}_{s}x^{2}} \qquad (434)
\end{aligned}
$$

Considering ${}_{s}r$ and $\gamma$ constant, differentiate with respect to ${}_{s}x$ to find the value of the synchronous reactance in terms of the resistance and phase angle for maximum torque.

$$
\frac{d_{s}P_{A}}{d_{s}x} = 0. \quad \text{Hence } (r^{2} + {}_{s}x^{2}) \cos \tfrac{1}{2}\gamma - ({}_{s}x \cos \tfrac{1}{2}\gamma + r \sin \tfrac{1}{2}\gamma)\ 2{}_{s}x = 0 \qquad (435)
$$

Or,

$$
{}_{s}x = \frac{(1 - \sin \tfrac{1}{2}\gamma)r}{(\cos \tfrac{1}{2}\gamma)} \qquad (436)
$$

For small values of $\gamma$, that is, with the machines nearly in the desired 180° phase position, $\cos \frac{1}{2}\gamma$ is very nearly unity and $\sin \frac{1}{2}\gamma$ very small, and therefore the condition for maximum torque with the machines in step is:

$$
{}_{s}x = r \qquad (437)
$$

For the machine tending to lag, the value of the synchronous reactance:

$$
{}_{s}x = \frac{(1 + \sin \tfrac{1}{2}\gamma)}{(\cos \tfrac{1}{2}\gamma)}\,r \qquad (438)
$$

Hence, the condition for maximum torque is the same when the value of $\gamma$ is very small whether the machine tends to go faster or slower than synchronous speed. With the alternators in

step the maximum synchronizing force is obtained when the synchronous reactance and resistance are equal, or $_{s}\theta = 45°$. For larger values of $\gamma$, the value of $_{s}x$ is less than $r$, to give maximum synchronizing torque. It is, however, neither necessary nor desirable to design the machines for a maximum synchronous torque, as a considerably smaller value is ample for practical operation. Other factors entering into the design of alternators make it necessary to let the synchronous reactance be several times as large as the armature resistance.

(*n*) **Synchronizing. Synchroscopes.**—In order to connect an alternator to a system already in operation the conditions for parallel operation must be met, or, the alternator must be synchronized before it is connected to the busbars. The incoming machine must be adjusted to comply with the three required conditions:

(1) The machine and busbar voltages must be approximately equal in magnitude.

(2) The voltages must have the same frequency.

(3) The voltage waves must be in phase.

The first condition is met by adjusting the field excitation, using the switchboard voltmeter as indicator. The voltage is usually kept a little higher on the incoming machine. Conditions (2) and (3) are of much greater importance than a careful adjustment of the voltages. To determine the speed and phase relations for conditions (2) and (3) special indicators or synchronizing devices are used. These may be grouped as indicating and automatic. The indicating synchronizing device, or synchroscope, should show three things:

(1) Whether the speed of the incoming machine is too fast or too slow.

(2) The amount of the difference in speed from synchronism.

(3) The time of coincidence of phase relations of the incoming machine and the busbars.

The simplest of all synchroscopes is the incandescent lamp; when used for this purpose it is called a *synchronizing lamp*. In Fig. 269 let machines $A$ and $C$ be in operation while $B$ is being synchronized. The synchronizing lamps connected as indicated in the diagram, will pulsate in brightness as the machine $B$ approaches synchronous speed. By the rapidity of the pulsations is indicated the difference in speed. When the machine runs in synchronism with the busbar voltage the pulsations

cease; and when the lamps are at their greatest brilliancy, the phase relations are correct and the machine may be thrown into circuit. By connecting the lamps as on machine *C*, Fig. 269,

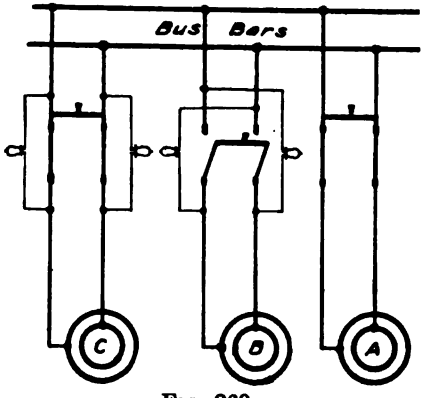



FIG. 269.

the lamps are dark when the machine and busbars are in phase. For machines having a higher voltage, transformers are used and the connections made so as to have the lamps either *dark* or *bright* when the machines are in phase, as shown in Figs.

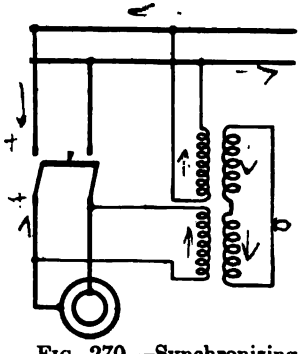


FIG. 270.—Synchronizing *bright*, using two transformers with the secondaries in series.

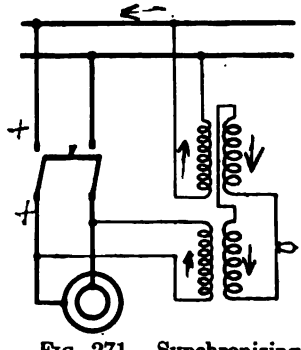


FIG. 271.—Synchronizing *dark*, using two transformers with the secondaries in opposition.

270 and 271. For three-phase machines, as in Fig. 272, the lamps become bright in a definite order while the speed of the incoming machine is too low, and in the reverse order when the speed is above synchronism. At synchronism the right-hand

lamp will be dark and the other two of equal brightness. For small machines the synchronizing lamps serve the purpose but for large alternators more accurate indicators are necessary.

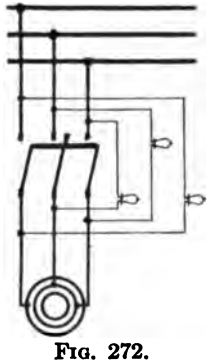


FIG. 272.

*The Lincoln Synchroscope.*—A pointer moving in front of a dial, like the hands of a clock, indicates the relative speeds and phase positions of the machines to be synchronized. The direction of rotation shows whether the speed of the incoming machine is too fast or too slow. The speed of the pointer indicates the relative speeds of the alternators. When the machines are running at the same speed and the voltage waves are in phase, the pointer remains stationary in an upright position. The circuit diagram of the instrument is shown in Fig. 273. A stationary laminated iron core $MM'$ is magnetized by a current

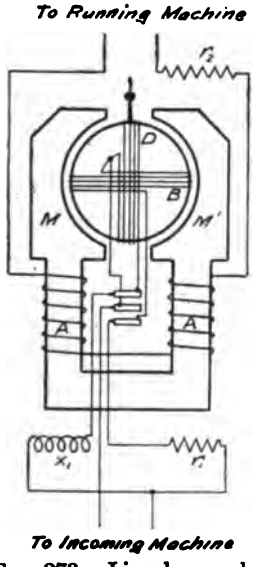



FIG. 273.—Lincoln synchroscope.

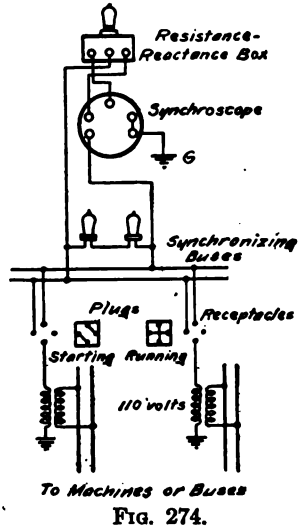



FIG. 274.

from the busbars flowing through coil $A$ and resistance $r_2$. The rotating part carrying the pointer consists of two coils, $B$ and $D$, rigidly attached to the axle, in space quadrature to each other.

Both coils are connected to the terminals of the incoming machine; $B$ through a resistance, $r_1$, and $D$ through an inductive reactance, $x_1$. The currents flowing in $B$ and $D$ therefore differ in time phase by nearly 90°. Hence the resultant magnetic field produced by the currents in $B$ and $D$ rotates in synchronism with the frequency of the voltage from the incoming alternator. The magnetic flux in $MM'$ pulsates in synchronism with the busbar voltage. The reaction between the stationary pulsating flux produced by the current in coil $A$ and the constant rotating flux produced by the currents in coils $B$ and $D$ gives the required torque for turning the pointer. When the voltage from the incoming machine has the same frequency and is in time phase with the busbar voltage the torque during one-half cycle is balanced by an equal torque in the opposite direction during the next half wave.

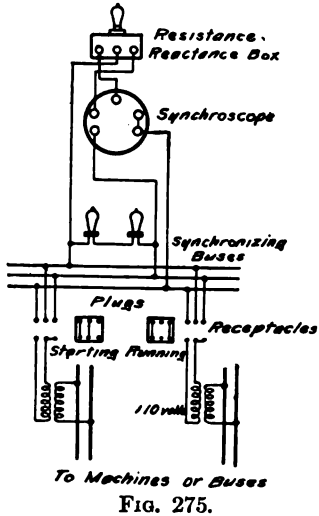



FIG. 275.

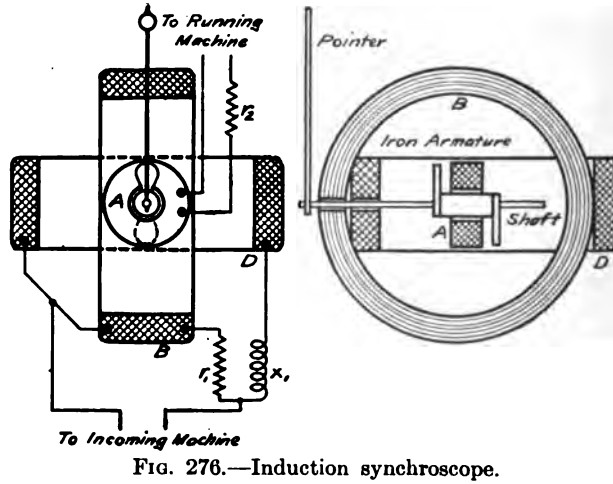



FIG. 276.—Induction synchroscope.

The pointer is fastened to the movable coils so as to take an upright position when the machines are in synchronism. Cir-

cuit diagrams showing the connections to the synchroscope in grounded and ungrounded systems are shown in Figs. 274 and 275.

*Induction Synchroscope.*—The mechanism consists of three stationary coils, *A*, *B* and *D*, Fig. 276, with a central movable iron core, to which is attached a pointer. Coil *A* is connected to the busbars through a resistance, $r_2$. The current in *A* produces a pulsating flux in the iron armature. Coils *B* and *D*, in space quadrature, are connected to the incoming machine; coil *B* through a resistance, $r_1$, and coil *D* through an inductive reactance, $x_1$. The currents in coils *B* and *D* produce a constant rotating field. The torque is produced, in the same manner as in the Lincoln synchroscope, by the reaction between the pulsating and rotating fields. If the speed of the incoming machine is too slow, the pointer rotates in a counterclockwise direction; if too fast, the direction of rotation is reversed. When the machines are in synchronism the pointer remains stationary in an upright position.

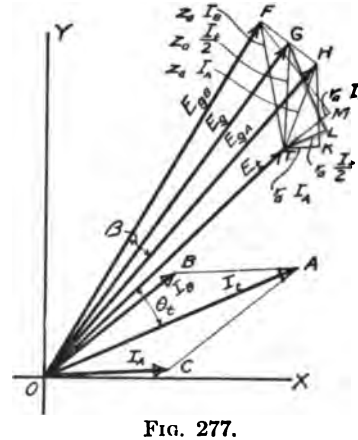



FIG. 277.

(*o*) **Division of Load, in Parallel Operation.**—The division of the load depends fundamentally upon the governors of the prime movers. Without entering into an extended discussion of the problem the essential principle involved may be shown by Fig. 227, assuming similar machines.

Let the diagram represent the current and voltage relations for one phase.

$OT = E_t$, constant busbar or terminal voltage.

$OA = I_t$, constant-load current.

$\theta_t$ = angle of lag for $I_t$ and $E_t$.

$r_a$ = armature resistance,

$x_a$ = armature reactance.

*First Condition.*—The two machines are in phase.

The voltage drop in each armature, per phase is:

$$z_a \frac{\dot{I}_t}{2} = r_a \frac{\dot{I}_t}{2} + jx_a \frac{\dot{I}_t}{2}$$ (each alternator supplies half the busbar current) (439)

$$\dot{E}_0 = z_a \frac{\dot{I}_t}{2} + \dot{E}_t \qquad (440)$$

$OG = E_0$, generated voltage in each machine.
Angle $GTL = {}_s\theta$ = armature angle of lag.

The two machines are dividing the load equally.

Power generated in $A = E_{gA} \frac{I_t}{2} \cos GOA$.

Power output from $A = E_t \frac{I_t}{2} \cos \theta_t$.

Power generated in $B = E_{gB} \frac{I_t}{2} \cos GOA$.

Power output from $B = E_t \frac{I_t}{2} \cos \theta_t$.

*Second Condition.*—Let the governor for the prime mover of alternator $B$ be adjusted to slightly increase the amount of power supplied. This tends to increase the speed of $B$ and hence to make the voltage differ from $A$ by an angle $\beta$.

Keeping the same position for the terminal voltage $OT$, the generated voltage of $B$ is $\frac{1}{2}\beta$ ahead and of $A$ $\frac{1}{2}\beta$ behind $OG$. It is also assumed that

$OF = OH$.
$OF = E_{gB}$ generated voltage in $B$.
$OH = E_{gA}$ generated voltage in $A$.
$TF = z_a I_B$ armature drop in $B$.
$TH = z_a I_A$ armature drop in $A$.

Upon $TF$ and $TH$ construct right triangles similar to $TGL$.

$TM = r_a I_B$; $FM = x_a I_B$.
$TK = r_a I_A$; $HK = x_a I_A$.

From $O$ draw $OB$ parallel to $TM$, and $OC$ parallel to $TK$.

In length, let $OB : \frac{1}{2}OA :: TM : TL$, or $OB : \frac{1}{2}I_t :: r_a I_B : r_a \frac{I_t}{2}$ (441)

Hence $OB = I_B$, the current from machine $B$, in phase and magnitude.

Similarly, $OC = I_A$, the current from machine $A$, in phase and magnitude.

$$\dot{I}_t = \dot{I}_B + \dot{I}_A \qquad (442)$$

Power generated in $B = E_{gB} I_B \cos FOB$ (443)
Power output from $B = E_t I_B \cos TOB$ (444)
Power generated in $A = E_{gA} I_A \cos HOC$ (445)
Power output from $A = E_t I_A \cos TOC$ (446)

Under the conditions assumed for the diagram it is evident that $B$ delivers more power than $A$.

In the above discussion it is assumed that the generated voltages in the two machines are equal in magnitude. This need not be the case but the division of the load will in any event depend upon the power supplied by the prime mover to each alternator; and since the alternators must run in synchronism the amount of power supplied by the prime movers and therefore the division of the load depends upon the adjustment of the governors. In direct currents the generated voltage depends upon the product of the speed and the field excitation. For constant voltage an increase in one requires a corresponding decrease in the other. For alternators running in parallel both the speed and the terminal voltage are constant, while the field is separately excited and hence variable. Referring to Figs. 250 and 255 it will be recalled that the useful flux has two components, one from the field excitation and another from the armature reaction. The product of the useful flux into the speed determines the generated voltage and for constant speed and constant voltage the useful flux must also be constant. Hence if the field excitation is changed while the speed and voltage are constant, the armature reaction must make a corresponding counter change to keep the useful flux constant. This is done automatically by the synchronizing current as may be seen from Fig. 278. Let the two alternators be operated in parallel with equal field excitations and each taking half the load. If the excitation of $A$ is increased, the voltage $E_{gA}$ *rises* slightly and the machine tends to carry more load. As the power supply is regulated by the governor of the prime mover, the speed of the machine *decreases* slightly until the two voltage waves differ by a small angle, $\beta$, less than 180°. This produces a resultant voltage, $_sE$, in the series circuit and as a result a current, $_sI$ in both armatures. Since the armature reactance is inductive the synchronizing current $_sI$ lags with respect to $E_{gA}$ and leads $E_{gB}$. The armature reaction therefore decreases the field flux in

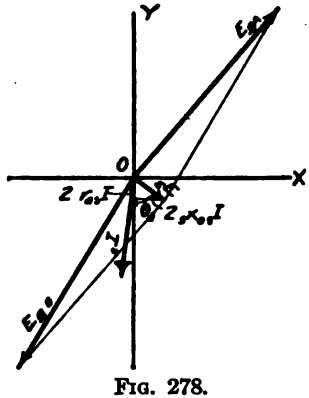


FIG. 278.

$A$ and correspondingly increases the field in $B$. The effect of the armature reaction automatically equalizes the resultant useful fluxes in the machines so as to produce equal terminal voltages.

## PROBLEMS

**1.** From the data given in Table VII find the efficiencies of the several alternators at 25 per cent., 50 per cent., 75 per cent., 100 per cent., 125 per cent. and 150 per cent. full load. Assume that the load losses vary with $I^2$ and at full load are equal to 50 per cent. of the iron losses. Neglect field rheostat losses.

(*a*) For $\cos \theta_t = 1.0$.

(*b*) For $\cos \theta_t = 0.85$.

**2.** With $E_t$ constant and $I_a$ constant and equal to the values for 50 per cent. full load, and $\cos \theta_t$ varying from 70 per cent. lagging current to 70 per cent. leading current, draw the loci for $E_0$ and $E_g$ for alternators No. 2 and No. 7 in Table VII. Let $X_a$ be 12.5 per cent. of ${}_sX_a$.

**3.** In problem 2 let $E_t$ be constant and $\cos \theta_t = .88$ per cent. (lagging current) and let $I_a$ vary from no load to full load.

Draw the loci for $E_0$ and $E_g$.

**4.** Given two alternators operating in parallel. The prime mover for $B$ tends to make it rotate faster than $A$.

(*a*) Draw vector diagram for the synchronizing circuit.

(*b*) With the same notation as in the diagram write the equations for the power delivered by $A$ and $B$.

(*c*) Prove that for the synchronizing circuit alone, machine $B$ is a generator and machine $A$ a motor.

**5.** Let $A$ and $B$ be alternators of the same type and rating as No. 3 and No. 5 respectively in Table VII. The prime mover of $A$ tends to run too fast so that the voltage wave from $A$ leads that from $B$ by 3°.

(*a*) Find the synchronizing current.

(*b*) Find the power in the synchronizing circuit flowing from $A$ to $B$.

**6.** In problem 4 let the load current lag 32° behind the busbar voltage.

(*a*) Draw the vector diagram for the load circuit.

(*b*) With the same notation as in the vector diagram, write the power equations for the output in the load circuit for $A$ and $B$.

(*c*) From the equations show that $B$ takes more than half the load.

**7.** Two alternators like No. 6 in Table VII operate in parallel. The voltage wave from $A$ leads that from $B$ by 4°. Let $X_a = 12.5$ per cent. of ${}_sX_a$. Assume that the voltages are adjusted as that machine $A$ takes half of the load plus the required synchronizing power, while machine $B$ takes half the load minus the synchronizing power. The load power factor at the busbars is 87 per cent., current lagging. Total load 1,700 kw.

(*a*) What is the load, in kw., for $A$? In kv.a.?

(*b*) What is the load in kw., for $B$? In kv.a.?

(*c*) If $B$ delivers full load, what is the load on $A$?

(*d*) What power is passing from $A$ to $B$ in the synchronizing circuit?

**8.** The following data apply to the test of a three-phase, 100-kv.a., 2300-volt generator when driven by a separately excited core loss motor at normal speed:

| Condition | Motor | | Generator | | |
|---|---|---|---|---|---|
| | $E$ | $I$ | $E_L$ | $I_f$ | $I_L$ |
| Belt off | 523.5 | 4.0 | | | |
| Belt on | 525.5 | 6.4 | 0.0 | 0.0 | 0.0 |
| Open circuit | 527.5 | 8.5 | 2000.0 | 7.0 | 0.0 |
| Open circuit | 528.0 | 9.5 | 2500.0 | 9.5 | 0.0 |
| Open circuit | 529.0 | 10.6 | 2900.0 | 12.2 | 0.0 |
| Open circuit | 530.0 | 11.7 | 3200.0 | 14.6 | 0.0 |
| Short circuit | 528.5 | 10.0 | 0.0 | 4.8 | 20.0 |
| Short circuit | 530.5 | 12.1 | 0.0 | 6.0 | 25.0 |
| Short circuit | 532.5 | 13.7 | 0.0 | 6.8 | 28.2 |

Star-connected ammeters used during short-circuit test.

Resistance of direct current motor armature, 0.87 ohm.

Resistance of generator between terminals, 8.62 ohms.

Calculate the efficiency and regulation of the above machine at full load on 85 per cent. power factor (lagging).

**9.** Alternators like No. 3 and No. 5 in Table VII are run in parallel. Before No. 3 is connected, No. 5 supplies 1500 kw. to a 2300-volt load at 85 per cent. power factor lagging. Alternator No. 3 is now connected and the governors of each driving turbine so adjusted that each machine carries its share of the load (500 kw. for No. 3 and 1000 kw. for No. 5). The field of No. 5 is not changed and that of No. 3 is adjusted to bring the line voltage to 2300 volts. What are the load currents and power factors of each machine? Give a complete vector diagram.

**1.** Two similar 1670 kv.a., $Y$-connected, 2300 volt, 25 cycle, 3 phase generators, designated as $A$ and $B$, are installed in a generating station and may be connected to the same set of busses. $A$ is running, and it is desired to synchronize $B$ with it. When synchronizing the operator carelessly throws the switch of machine in, when it is 15 degrees ahead of exact-phase opposition to $A$. (*a*) Which machine supplies power to bring the two machines into step? (*b*) How many kw. are supplied? (*c*) What is the value and phase position of the resultant voltage in the circuit of the two machines? (*d*) What is the value of the circulating current?

The synchronous reactance of each machine is 40 per cent., and the resistance 1.5 per cent.

**2.** The machines in problem (1) are driven by prime movers such that the speed regulation of machine $A$ is 4 per cent. and that of machine $B$ is 5.5 per cent. at full load of 1670 kv.a. and 85 per cent. power factor, and normal frequency. The rise or drop in speed is a constant amount per kw. change of load. (*a*) What is the combined kv.a. output of the two

machines when $A$ is delivering 125 per cent. of its rated kv.a. output? ($b$) What is the frequency at this load?

**3.** In problem (2) the combined load of the two machines is reduced to 2400 kv.a., load power factor remaining constant. ($a$) What is the frequency? ($b$) How many kw. does $A$ deliver? ($c$) How many kw. does $B$ deliver?

For Experiments see page 542.

## CHAPTER XV

## SYNCHRONOUS MOTORS AND SYNCHRONOUS CONDENSERS

Consider two similar alternators, connected in parallel to the same busbars as in Fig. 279, with equal field excitations, and running in synchronism carrying equal loads. In the series circuit between the alternators no cross-current is flowing and the voltage vectors are 180° apart. If the power supply of machine $M$ is reduced, the voltage vector $E_M$ lags behind the first position, relative to the voltage $E_G$ by an angle $\gamma$, as indicated in Fig. 280. The synchronizing current flowing in the series circuit transmits power from $G$ to $M$ as explained in Chap. XIV. If all power supply is cut off from the prime mover, machine $M$ still continues to run, receiving the necessary power from alternator $G$. If in addition a mechanical load is connected to $M$, additional power is transmitted from $G$ by an increased current in the series circuit. The difference in phase position of the voltage vectors for the two machines is larger with $M$ carrying a mechanical load than before, but the two machines continue to run in synchronism. Alternator $M$ is thus changed into a motor running in synchronism with an alternator $G$ by varying the phase position of the voltage waves. A synchronous motor is, therefore in principle, simply a reversed alternator. Instead of receiving mechanical energy and generating electric energy, it receives electric energy at synchronous speed and delivers mechanical energy. If the excitation of the motor is less than the generator, the resultant voltage and the synchronizing current change somewhat in phase position and magnitude but the motor stays in synchronism and power is transmitted from the generator $G$ to the synchronous motor $M$ under wide differences

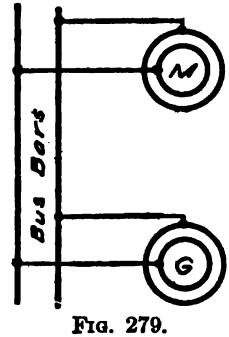


FIG. 279.

in field excitation and for a considerable range of load. For 80 per cent. field excitation of the motor the current lags behind the generator voltage, as shown in Fig. 281 and power is still transmitted from $G$ to $M$; or $G$ is a generator and $M$ a motor. For changes in load the generator and motor voltages vary in phase position, and for any given excitation of the motor, the locus of the motor voltage is a circle as shown in Fig. 282. For

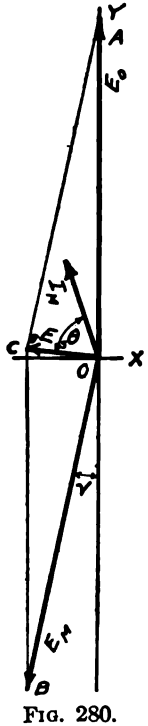



FIG. 280.

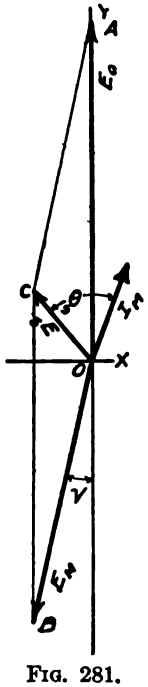



FIG. 281.

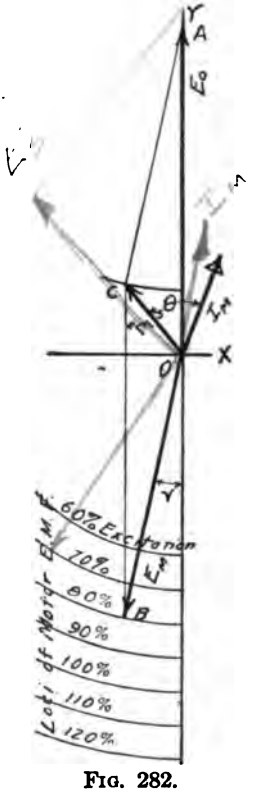



FIG. 282.

changes in the field excitation of the motor and for phase positions of the motor voltage, $E_M$, with respect to the generator voltage, $E_0$, the resulting voltage, $E$, changes in both magnitude and phase position. Hence the current flowing in the motor likewise changes both in magnitude and phase.

(*a*) **Current Loci. Power Input.**—The inter-relation of the motor voltage, current and power is best shown graphically by

vector diagrams. The loci of the voltages and motor current for different field excitations and loads are shown in Figs. 282 and 283.

$E_o$ = generator terminal or busbar voltage, assumed constant.

$E_M$ = motor-generated counter e.m.f., depending upon the motor field excitation, and given in per cent. of generator voltage, $E_o$.

${}_sE$ = resultant voltage causing current to flow in the motor armature.

$I_M$ = current in motor armature.

${}_sr$ = effective resistance of motor armature and leads at synchronous speed.

${}_sx$ = synchronous reactance of motor.

${}_sz = {}_sr + j{}_sx$ = synchronous impedance of motor.

${}_s\theta = \cos^{-1}\frac{{}_sr}{{}_sz}$ (assumed constant).

$$\dot{{}_sE} = \dot{E}_o + \dot{E}_M \tag{451}$$

$$\dot{I}_M = \frac{\dot{{}_sE}}{{}_sz} \tag{452}$$

The line $OF$ is drawn at an angle ${}_s\theta$ from $E_o$, and the distance $OF$ is equal to the quotient of $E_o$ and ${}_sz$.

$$OF = \frac{E_o}{{}_sz} = {}_oI_M \tag{453}$$

The current vector $I_M$ in Fig. 281 is drawn for 80 per cent. motor field excitation. The locus of the motor voltage, $E_M$, is a circle and hence the locus of the resultant voltage, ${}_sE$, is also a circle with $A$ as center and a radius equal to $E_M$. Therefore, the locus of the motor current, $I_M$, is a circle with $F$ as center and a radius equal to the motor voltage divided by the synchronous impedance. For 80 per cent. excitation the radius for the current locus is 80 per cent. of $FO$, Fig. 283.[1] With $F$ as a center a series of circles may be drawn with radii directly proportional to the motor excitation, expressed in per cent. of $E_o$. These circles are the current loci for different motor excitations and for varying load. The component of current in phase with $E_o$ represents power and hence the power component and the corresponding power input are measured directly by the projection of the current along the $Y$-axis. Likewise the quadrature com-

[1] McAllister, "Alternating-current Motors," p. 190.

ponent of the current and the corresponding reactive power are measured by the projection of the current along the $X$-axis. Thus the diagram shows directly the power input for any motor excitation and also the corresponding power passing to and fro between the motor and the generator.

(*b*) **Losses. Current Loci for "Mechanical Power."**—Since the synchronous motor is not self-exciting the energy consumed in the field need not be taken into consideration when discussing

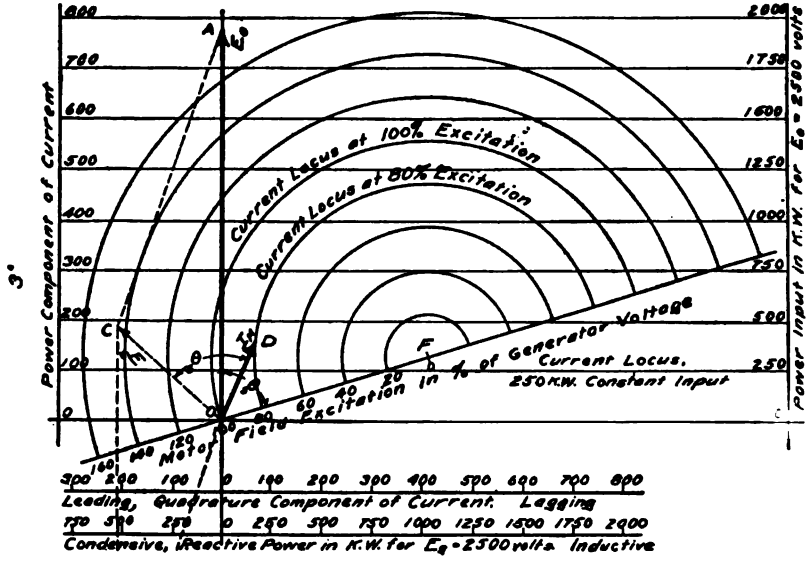



FIG. 283.

the characteristics of the machine, except for the total efficiency. The energy delivered to the rotor is therefore consumed by the rotor copper and iron losses, windage, friction and the outside load. Since the power absorbed by the iron losses, windage and friction is practically constant for all loads, it is convenient to combine it with the outside load into one term in the equations. "Mechanical power" (as used in this chapter) = outside load + (windage, friction, rotor iron losses), or

"Mechanical power" = outside load + a constant.

When discussing the synchronous-motor characteristics it may therefore be assumed that:

$$\text{Power input} = {}_srI_u^2 \text{ losses} + \text{"mechanical power"} \qquad (454)$$

In Fig. 284, $E_o$, the generator busbar voltage, the line $OF$ and the broken lines for the current loci are the same as in Fig. 283. For zero motor field $OF$ represents the motor current both in magnitude and phase and the projection on the $Y$-axis, $OB$, the power input. As no mechanical power is produced the input is consumed by the copper losses.

$$OB = {}_sr\,{}_0I_M^2 \tag{455}$$

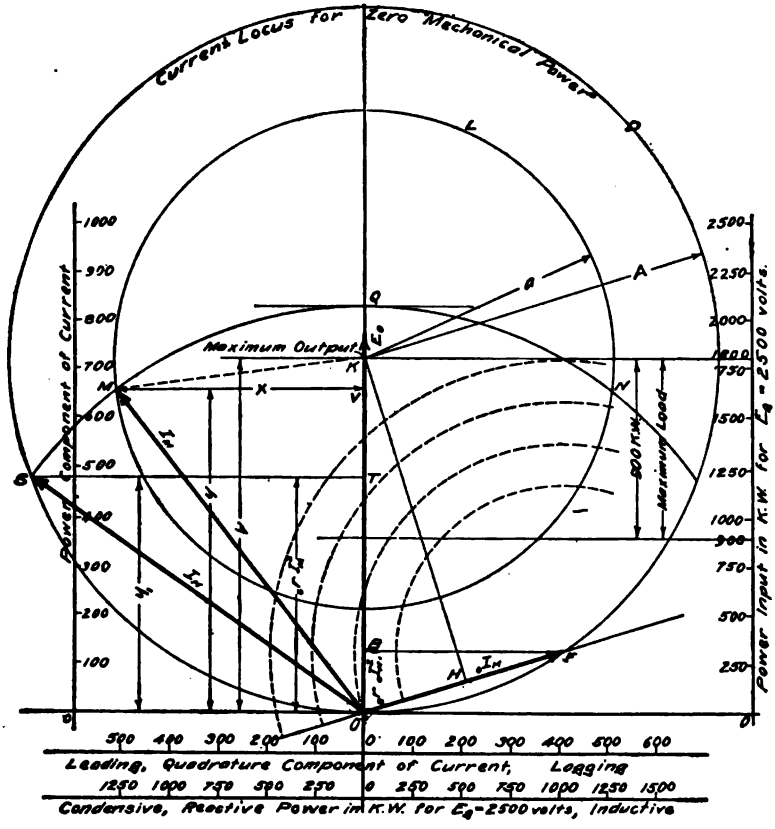



FIG. 284.

With its center along the $E_o$ vector, a circle is drawn through $O$ and $F$. This is the locus of currents for zero mechanical power. From similar triangles,

$$OB : OF :: OH : OK, \text{ or } OB : {}_0I_M :: \tfrac{1}{2}{}_0I_M : OK \tag{456}$$

Hence

$$OB = \frac{{}_0I_u^2}{2OK} \tag{457}$$

Similarly, for any point $S$ on the circle $OFD$ the projection on $E_0$ represents the power input from (455) and (457),

$$OT = \frac{\overline{OS}^2}{2OK} = \frac{I_u^2}{2OK} = {}_sr\, I_u^2 \tag{458}$$

and hence zero mechanical power. Therefore, in order to produce any mechanical power the current vector must fall inside the circle $OFDS$. As the mechanical load at the shaft includes all but the ${}_sr\, I_u^2$ losses, it is evident that for any power input the losses will be least when $E_0$ and $I_u$ are in phase, or for unity power factor. The power input for a current in phase with the voltage, as $OQ$, is given directly from the diagram. The copper loss may be found by drawing a circle with $OQ$ as radius and $O$ as center. The intersection with the locus for no mechanical load determines the ${}_sr\, I_u^2$ loss equal to the projection $OT$. The mechanical load is therefore given by the difference of $OQ$ and $OT$ and is equal to $TQ$.

The *maximum mechanical power* that the motor theoretically can carry may be shown to be one-half of the power represented by $OK$, the radius of the current locus for zero mechanical power. The term *maximum mechanical power* in this discussion refers to conditions which can not actually be met by the motor. It is based on the assumption that the resistance and the counter e.m.f. are the only factors opposing the flow of the current, as in a direct-current motor. The synchronous impedance limits the armature current to values much smaller than indicated in the diagram. Hence the *maximum power possible* is fictitious as it can not be produced by the machine.

That the current loci for any constant mechanical power output of the motor are circles concentric with the locus for zero mechanical power may be proved as follows:

In Fig. 284 draw any circle $LMN$ with $K$ as center and of radius $a$ less than $A$, the radius of the circle $OFD$. Let the circular arc $SMQ$ intersect the circle $LMN$ at $M$. Draw $MV$ perpendicular to $OK$. Then $TV$ represents the mechanical power delivered by the motor since $OV$ represents the total input and

$OT$ the losses. Hence, if the distance $TV$ is proved constant for point $M$ on the circle $LMN$ corresponding to any point $S$ on $OFD$, then the circle $LMN$ must be the current locus for the constant mechanical power $TV$.

Let $OT = y_1$; $OV = y$; $MV = x$; $OS = OM = I_M$

From the equation of the circle $OFD$, or from equation 458,

$$I_M = \sqrt{2Ay_1} \tag{459}$$

From the triangle $OMV$,

$$I_M = \sqrt{x^2 + y^2} \tag{460}$$

From the triangle $MVK$,

$$a^2 = x^2 + (A - y)^2 \tag{461}$$

Eliminating $I_M$ and $x$ and solving for $TV$,

$$y - y_1 = TV = \frac{A^2 - a^2}{2A}, \text{ a constant} \tag{462}$$

For maximum mechanical load $a$ equals zero, and the corresponding current locus is the center of the circle $OFD$.

The efficiency at maximum load

$$= \frac{y - y_1}{y} = \frac{\frac{A}{2}}{A} = 50 \text{ per cent.} \tag{463}$$

The larger the load, the smaller is the radius of the current locus. For the maximum load the circle of the current locus has contracted into a point, at the center of the set of current loci for the loads between zero mechanical output and the largest load the motor can carry. A convenient way of graphically determining the locus for any given load is shown in Fig. 285, in which the generator voltage, the current locus for zero mechanical power and the scales for current and power are the same as in Fig. 284. The maximum power is represented by half the radius, and the corresponding current by the radius of the locus for zero mechanical power. With $O$ as center and $OK$ as radius draw the arc $LKM$. The line $LM$ bisects $OK$ and hence the length $NK$ represents the maximum output or maximum mechanical power of the motor.

A scale for the output may be drawn as in Fig 285. The current locus for any load, as for 500 kw., may be determined by

drawing a line parallel to the $X$-axis from the output scale and intersecting the arc $LKM$ at $P$. The circle drawn with $K$ as center and $KP$ as radius is the required current locus for an output of 500 kw. mechanical power. Similarly, the locus for any other output may be found as shown in Fig. 285 for 350, 800 and 150 kw. In practical operation the current vector must be within

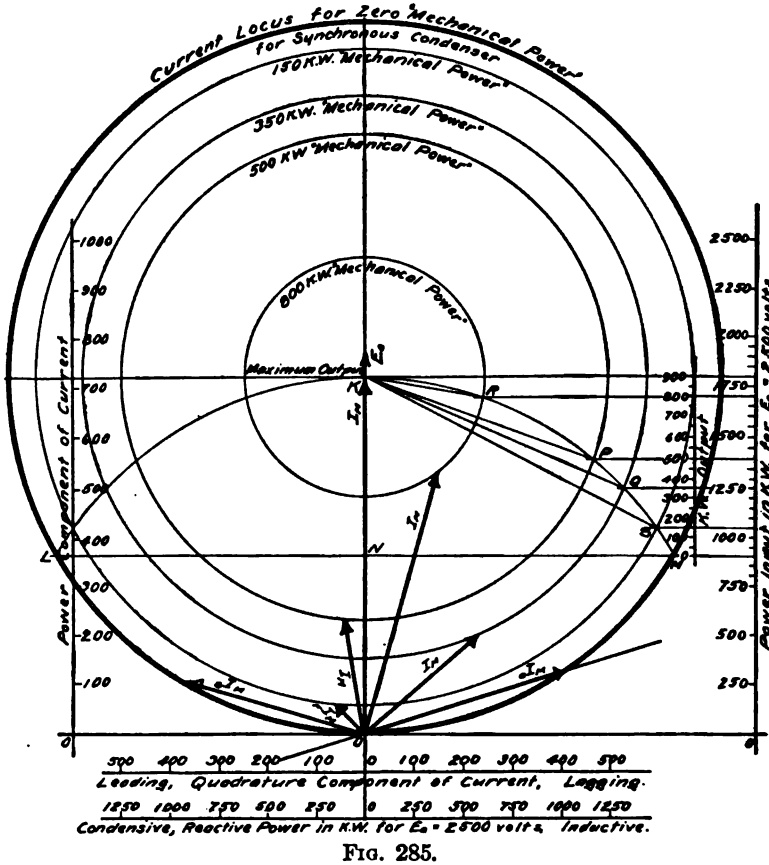



FIG. 285.

the locus for zero mechanical power, since even with no load some power is required to overcome friction, windage and iron losses. The current locus for the machine operating as a synchronous condenser (no mechanical load) is shown in Fig. 285.

The copper losses are excessive unless the motor operates near unity power factor and in the lower part of the diagram.

At maximum load the copper loss at unity power factor is 50 per cent. Hence for an efficiency greater than 50 per cent. the current must be within the arc *LKM*. The ordinary operation of the synchronous motor for the conversion of electric to mechanical power is within much narrower limits. This may be seen from the diagram, as 150 kw. is full load for this motor. To obtain the best efficiency for any load the power factor must be near unity and hence the current vector lies near the $E_o$ vector and varies with the load from zero to the value corresponding to the maximum load. In Figs. 283, 284 and 285, the power component is represented by the projections of the current vector on the *Y*-axis in phase with $E_o$. The product of the power component of the current and the voltage gives the power input to the motor.

$$E_o I_u \cos \theta_i = \text{motor input, or generator power output} \quad (461)$$

Similarly the quadrature current and the reactive power are represented along the *X*-axis, in quadrature with $E_o$

$$E_o I_u \sin \theta_i = \text{reactive power} \quad (462)$$

From Fig. 285 it is seen that the quadrature component of the current may be either lagging or leading with respect to the impressed voltage, $E_o$; and hence the reactive power may be either inductive or condensive, depending upon the excitation of the motor field. The fact that the current taken by a synchronous motor may be made either leading or lagging by merely adjusting the field excitation, and hence under control of the operator, is of great commercial importance. With a leading current the excitation of the motor field is under most conditions, as may be seen from Fig. 285, over 100 per cent. of the generator voltage. This has led to the statement that an "over-excited synchronous motor acts like a condenser." With a leading current the corresponding reactance is condensive and in that respect, *and that only*, the over-excited synchronous motor acts like a condenser. However, the ability to receive a leading current and hence to store power in time-phase opposition to magnetic inductance, is the important commercial feature of a condenser, and for the same reason the chief advantage of the over-excited synchronous motor. The commercial importance of an adjustable condensive reactance may be seen in the design of power distribution net-works and especially in long-distance transmission

systems. A simple illustration of improving the power factor of the load by operating a synchronous motor in parallel with an induction motor will be given.

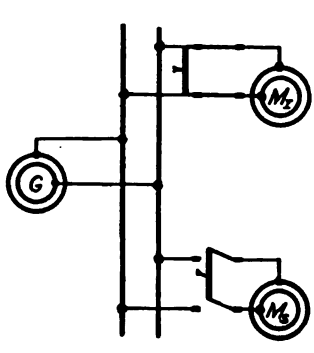



FIG. 286.

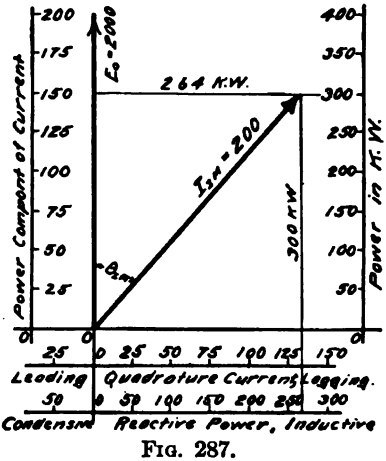



FIG. 287.

In Fig. 286 let a generator $G$ supply power to an induction motor, $M_1$. Let the rated capacity of the generator be 400 kv.a.,

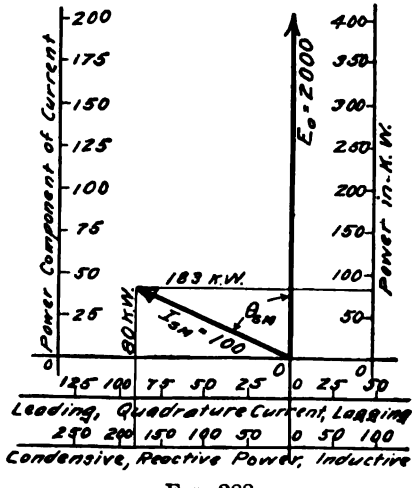



FIG. 288.

the motor load 300 kw. at 75 per cent. power factor. The vector diagram of the voltages and currents with scales showing the real and reactive power is shown in Fig. 287. It is required to carry

an additional load of 80 kw. by the same generator. This evidently can not be done by using an induction motor as the 400 kv.a. taken by the first motor is equal to the rated capacity of the generator. It is, however, possible to improve the power factor by using an over-excited synchronous motor for the additional load, thereby reducing the kv.a. so that the larger load takes less current from the generator. In Fig. 288 is shown the vector diagram of voltage and current for a 200-kv.a. synchronous motor taking 80 kw. at 40 per cent. power factor, leading current. Combining the vector diagrams for the induction and synchronous motors in Fig. 289 gives the relative magnitudes and

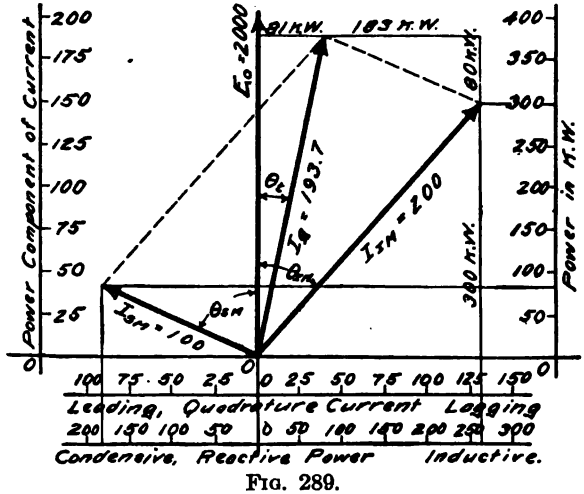



FIG. 289.

phase relations of the currents in each of the motors and in the generator.

Power taken by the induction motor:

$$E_o I_{IM} \cos \theta_{IM} = 2{,}000 \times 200 \times 0.75 = 300 \text{ kw.}$$

Power taken by the synchronous motor:

$$E_o I_{SM} \cos \theta_{SM} = 2{,}000 \times 100 \times 0.40 = 80 \text{ kw.}$$

Power delivered by the generator to both motors:

$$E_o I_t \cos \theta_t = E_o I_{IM} \cos \theta_{IM} + E_o I_{SM} \cos \theta_{SM} = 380 \text{ kw.}$$

Reactive power in the induction motor:

$E_o I_{IM} \sin \theta_{IM} = 2{,}000 \times 200 \times 0.66 = 264$ kw. (lagging current).

Reactive power in synchronous motor:

$E_g I_{SM} \sin \theta_{SM} = 2{,}000 \times 100 \times 0.915 = 183$ kw. (leading current).

Reactive power in generator:

$E_g I_{IM} \sin \theta_{IM} - E_g I_{SM} \sin \theta_{SM} = 264 - 183 = 81$ kw. (lagging current).

Current in generator:

$$\begin{aligned} \dot{I}_g &= \dot{I}_{IM} + \dot{I}_{SM} \\ &= \dot{I}_{IM} (\cos \theta_{IM} - j \sin \theta_{IM}) + \dot{I}_{SM}(\cos \theta_{SM} + j \sin \theta_{SM}) \\ &= \dot{I}_{IM} \cos \theta_{IM} + \dot{I}_{SM} \cos \theta_{SM} - j(\dot{I}_{IM} \sin \theta_{IM} - \dot{I}_{SM} \sin \theta_{SM}) \\ &= 190 - j40, \text{ or } \dot{I}_g = 193.7 \text{ amp.} \end{aligned}$$

$\cos \theta_i = 0.98$ and $\theta_i = 11° 28'$.

Hence, although 80 kw. additional load is supplied by the generator with both motors in operation, the current in the generator is reduced from 200 to 193.7 amp. As the loss in the generator copper varies as the square of the current, the heat is reduced in the proportion of $\overline{200^2} : \overline{193.7.^2}$ From the diagram in Fig. 289 it is evident that by means of a synchronous motor the power factor of the load at the generator may be changed at will. Aside from the economy effected by operating at or near unity power factor, as illustrated above, the control of the power factor is a useful method for regulating the voltage in long-distance transmission systems as explained in Chap. XXII.

(*c*) **Synchronous Condenser.**—In large power systems, and particularly in connection with long-distance transmission lines, over-excited synchronous motors are kept *floating on the line*, that is, running at synchronous speed but not carrying any load, for the purpose of supplying the necessary leading or lagging current to control the power factor and thereby regulate the voltage. Under such conditions the synchronous machine is called a *synchronous condenser*. The derivation of the term is plain, for the synchronous condenser produces the same effect in causing a leading current as a stationary condenser of equivalent condensance. The mechanical power is expended in overcoming friction and windage and hence is constant in value. The current locus is therefore a circle for mechanical power equal to the losses caused by friction, windage, and losses in the iron, as shown in Fig. 285.

(*d*) **Power Equations.**—From the vector diagrams used in the above graphical analysis of the synchronous motor the algebraic

expressions for power are readily obtained. Let the diagram in Fig. 290 represent the general case.

$E_o$ = the impressed voltage on the motor terminals.
$E_M$ = the voltage corresponding to the motor counter e.m.f.
$I_M$ = the current in motor armature.
${}_s x$ = synchronous reactance of motor.
${}_s r$ = effective resistance of motor armature at synchronous speed.

$$\cos {}_s\theta = \frac{{}_s r}{\sqrt{{}_s r^2 + {}_s x^2}}$$

$\cos \theta_t$ = terminal power factor.
$\gamma$ = time-phase angle between $E_o$ and $E_M$.

Power input $= E_o I_M \cos \theta_t$ (466)

"Mechanical power" $= E_M I_M \cos (\theta_t - \gamma)$ (467)

Copper losses $= {}_s r I_M^2$ (468)

Power output = "mechanical power" − friction and windage and rotor iron loss (469)

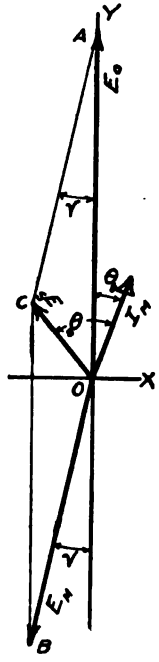


FIG. 290

It is sometimes desirable to have the power input and the "mechanical power" expressed in terms of $E_o$, $E_M$, ${}_s\theta$, $\gamma$, ${}_s r$ and ${}_s x$.

These may be derived directly from equations (466) and (467) by substituting for $I_M$ and $\theta_t$.

From Fig. 290:

$$I_M = \frac{{}_s E}{\sqrt{{}_s r^2 + {}_s x^2}} \qquad (470)$$

$$E_o \cos ({}_s\theta - \theta_t) = E_o - E_M \cos \gamma \qquad (471)$$

and

$${}_s E \sin ({}_s\theta - \theta_t) = E_M \sin \gamma \qquad (472)$$

Substituting in (466):

$$\text{Power input} = \frac{-E_o E_M}{\sqrt{{}_s r^2 + {}_s x^2}} \cos (\gamma + {}_s\theta) + \frac{E_o^2}{\sqrt{{}_s r^2 + {}_s x^2}} \cos {}_s\theta \qquad (473)$$

$$\text{"Mechanical power"} = \frac{E_o E_M}{\sqrt{{}_s r^2 + {}_s x^2}} \cos (\gamma - {}_s\theta) - \frac{E_M^2}{\sqrt{{}_s r^2 + {}_s x^2}} \cos {}_s\theta \qquad (474)$$

(*e*) **Test Data. Rating. Efficiency.**—The saturation and synchronous reactance curves are obtained by running the synchronous motor as a generator and taking the data as explained for alternators. From the synchronous reactance and armature resistance, ${}_s\theta$ may be found. The value of $E_M$ is determined from the current in the field and the saturation curve. $E_o$, $I_M$ and $\theta_t$ may be taken directly by voltmeter, ammeter and wattmeter readings for any given load and field excitation. The fixed losses (windage, friction and iron losses) may be found by measuring the input with the motor running without load. Adjusting for unity power factor keeps the copper losses at a minimum. These must be subtracted from the input to obtain the fixed losses. The rating of a synchronous motor is based on the same principle as already discussed for alternators. The capacity is given in hp. or kv.a. with a statement of temperature rise at unity power factor. The efficiency is the ratio of output to input, usually expressed in per cent. As with alternators the efficiency may be found by measuring input and output or by measuring either the input or the output and the losses. For large machines the latter method is preferable.

(*f*) **The V and O Curves.**—In a synchronous motor giving constant mechanical power the armature current is a minimum if the power factor is unity. By varying the field excitation of the motor the power factor may be varied with corresponding change in the armature current. Curves plotted with the field excitation as abscissæ and the armature current as ordinates for a constant load, assume a form like the letter *V*, as seen in Fig. 291, and are known as the *V curves* of a synchronous motor. A series of curves may be obtained for a number of loads, as in Fig. 291.

The *V* curves may be found graphically from the current loci diagram in Fig. 285 in a simple manner, as shown in Fig. 292. The voltage or field excitation of the motor is laid off on a line from the origin to the intersection of the current locus for a constant load with the loci for different field excitations and this distance used as ordinate in obtaining a point on the *V* curve. By plotting the values of the current for all points on the circular load loci, the *V* curves are converted into closed curves, sometimes called the *O curves* of the synchronous motor. In Fig. 292 the parts that may be found experimentally are drawn in heavy lines.

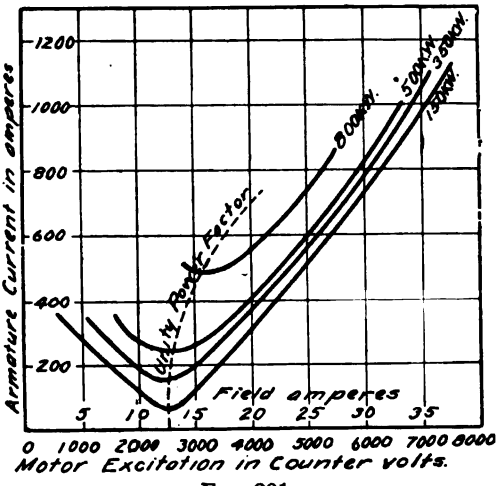



FIG. 291.

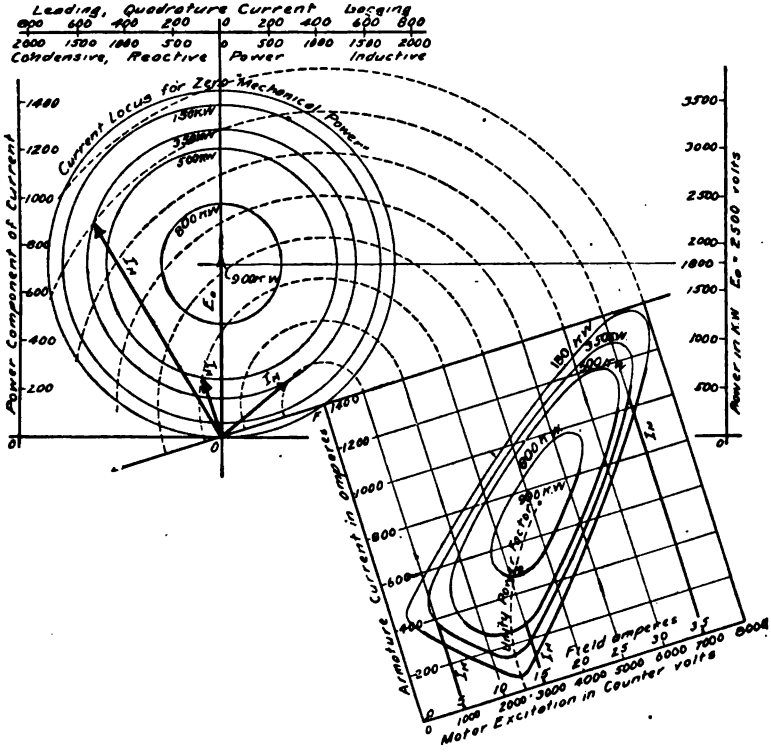



FIG. 292.

The broken line in Fig. 292 indicates the field excitations and corresponding minimum currents for unity power factor. In attempting to extend the experimental observations further by either increasing or decreasing the motor field excitation the machine is thrown out of synchronism. Curves drawn as in Fig. 292 are necessarily only approximations to the actual characteristic curves of commercial machines, since several assumptions were made that are only partly fulfilled in practice.

1. It was assumed that the motor voltage $E_M$ could be varied indefinitely; while in the actual motor the field flux is limited by magnetic saturation. Although sufficient field excitation might be applied so as to obtain readings, during a short test, as high as indicated by the heavy lines in the diagram, the limits are far beyond the voltage that may be applied continuously to the motor.

2. The synchronous reactance and impedance were assumed as constant, while these vary in the commercial motor both with the saturation of the iron and the position of the armature with regard to the field. These variations in the reactance and impedance are most pronounced at low power factors.

3. The characteristic curves in Figs. 291 and 292 represent conditions for constant electric power, including all but the copper losses. The loss due to friction and windage is practically constant, but the iron losses depend upon the field excitation, and increase with the magnetic induction. Hence for constant mechanical output the whole system of curves is shifted somewhat toward the lower field excitation.

(*g*) **Load Curves.**—The curves for current, power factor and efficiency, as a function of the output at constant excitation, are often called *load curves.* These may be derived directly from Figs. 285 and 292, if the same assumptions be made, that the core loss, friction, windage and synchronous reactance remain constant. By plotting the curves in rectangular coördinates, with power output as abscissæ, the characteristics of the synchronous motor may readily be compared to those of the induction motor. For the latter one set of curves determines the performance of the machine for any given impressed voltage. In the synchronous motor the field excitation can be varied and hence an infinite series of load curves may be obtained for any given impressed voltage. The power factor curves are of special interest By inspection of Fig. 283, it is readily seen that unity power factor

is obtained only at points of intersection of the current locus for the given field excitation with the voltage vector along the *Y*-axis. The load curves for a synchronous motor at three field excitations are illustrated in Figs. 293, 294 and 295. For a field excitation of 80 per cent., Fig. 293, the current locus does not intersect the voltage vector and the power factor never reaches unity. Under these conditions the load curves are similar to those of the induction motor. At an excitation of

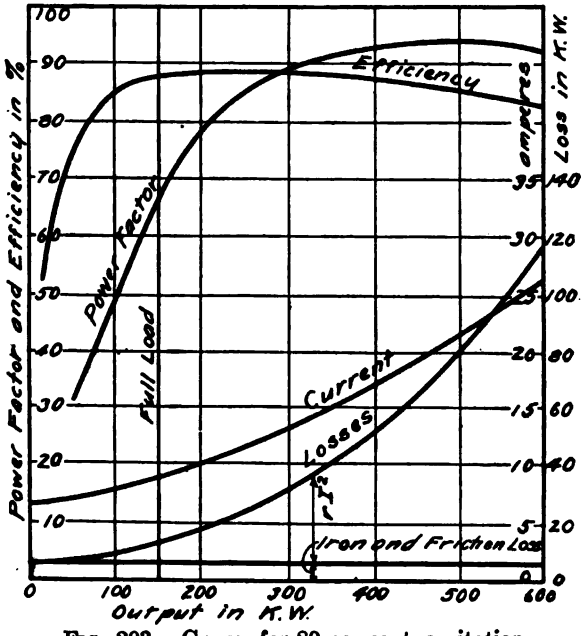



FIG. 293.—Curves for 80 per cent. excitation.

99 per cent. the power factor is unity at two loads as shown in Fig. 294.

By referring to Figs. 283 and 285 it is evident that the limits within which the power factor will be unity for two loads as illustrated in Fig. 294, are approximately, 98 per cent. and 100 per cent. field excitation. For excitations above 100 per cent. the power factor becomes unity for one load only as illustrated in Fig. 295.

It should be noted for a field excitation slightly below 100 per cent. the power factor is near unity from zero to full load. As the losses are a minimum at unity power factor the

most desirable field excitation for economical operation is about 99 per cent. when running simply as a motor. In most cases the synchronous machine is required to supply a leading current; that is, to act partly as synchronous condenser, in order to bring the power factor of the system nearer unity. In general the economical operation of the whole system determines the per cent. of field excitation of the synchronous machines.

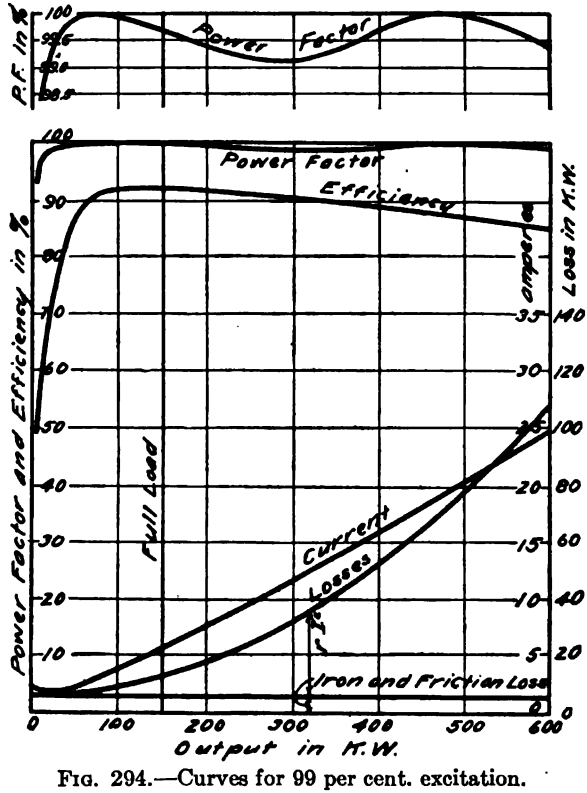



FIG. 294.—Curves for 99 per cent. excitation.

(*h*) **Polyphase Synchronous Motors. Equivalent Single-phase.**—The above discussion is based on single-phase circuits and single-phase apparatus. The same treatment may be applied without modification to polyphase apparatus, provided *equivalent single-phase* values are used for the current, resistance, synchronous current, synchronous reactance and synchronous impedance of the armature and supply circuits.

*For two-phase motors* the *equivalent single-phase* resistance, syn-

chronous reactance, and synchronous impedance equal one-half the respective values as measured in one of the phases; and the *equivalent single-phase* current is twice the current in one phase.

*For three-phase motors* (whether star- or delta-connected) the *equivalent single-phase* resistance, synchronous reactance and synchronous impedance equal one-half the respective values as measured between two terminals; and the equivalent single-phase current equals $\sqrt{3}$ times the current in one of the mains. The equivalent single-phase voltage is the same as measured between the mains for two-phase or three-phase.

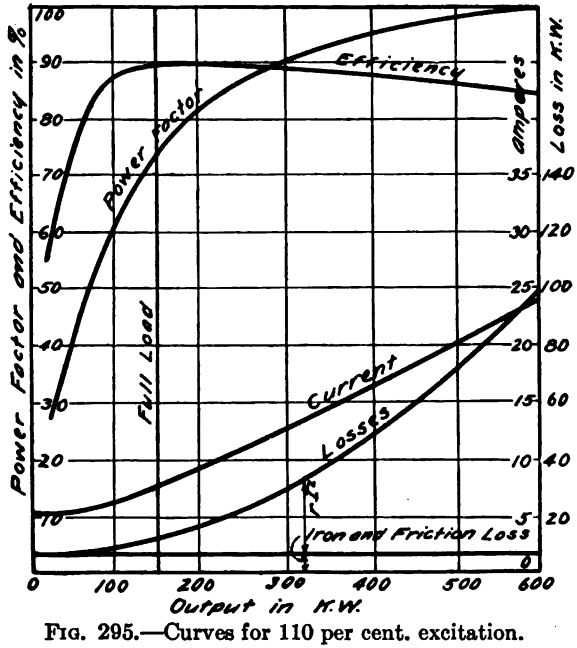



FIG. 295.—Curves for 110 per cent. excitation.

It should also be noted that while the synchronous motors discussed above operate on a constant-potential system it is seldom that the impressed voltage is constant at the terminals of the motor. If the busbars are kept at constant voltage the resistance and reactance of the leads from the busbars to the motor should be combined with the resistance and synchronous reactance of the armature to obtain the data from which the performance of the motor may be calculated. In some cases the synchronous motor may be located at a considerable distance from

the busbars or the generator where the voltage is kept constant. The resistance and reactance of the line from the point of constant voltage must in all cases be included with the resistance and synchronous reactance of the armature in preparing the data from which the performance curves of the synchronous motor may be calculated. Necessarily, if the constants of the leads to the busbars or a considerable length of transmission line are included, the performance curves differ from the set obtained from the same motor operating under the condition of constant potential at the terminals of the motor. Hence, in order to determine the operating characteristics of a synchronous motor the constants of the line from the point of constant potential to the terminals of the motor must be known, as well as the resistance and synchronous reactance of the machine.

(*i*) **Surging or Hunting of a Synchronous Motor.**—In Chap. XIII it is shown that the torque of an induction motor is a function of the speed; that a decrease of speed increases the torque. Hence, if the load is suddenly increased on an induction motor the speed decreases until the torque becomes sufficient for the new load. Conversely, if the load is decreased the motor reacts by an increase in speed. This change in torque and speed is gradual and without any oscillation.

In the synchronous motor the torque is *not a function of the speed* but of the *relative phase positions of the rotor and the impressed voltage*. Hence, for a change of load the rotor is required to change its relative phase position without change of speed. Strictly the two requirements are incompatible and the change is affected in a series of oscillations which may produce serious disturbances in the system. Suppose the load is decreased. The torque is then in excess of the amount required by the load and thus causes an acceleration of the armature until the phase position is reached at which the torque is just sufficient for the load. However, upon reaching this position the speed of the armature is above synchronism and hence moves past the desired position. This further change of phase position also decreases the torque which becomes less than the load, and the difference must be supplied by a deceleration of the armature. When the armature has again reached synchronous speed its phase position is such as to give less torque than required by the load. The deceleration of the armature, therefore, continues and the phase position is retraced until the second

position, where the torque balances the load, is again reached. However, the armature rotates by this time below synchronous speed and hence the armature again passes the desired position. The change in the kinetic energy in the rotating mass therefore causes the armature to oscillate around the desired position, where the torque is just sufficient for the load.

Under certain conditions[1] the action is cumulative, the successive oscillations increase in magnitude, and as a result the

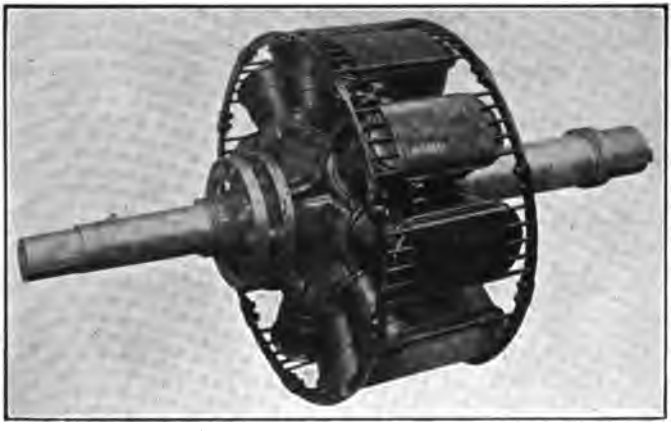


FIG. 296.—Amortisseur winding or grid. (*General Electric Co.*)

machine, in a short time, drops out of synchronism. In most cases the synchronous motor is designed so that the successive oscillations decrease in magnitude and rapidly become negligible. The most common and useful means for rapidly reducing hunting oscillations is the *amortisseur winding*. This consists simply of a short-circuited grid, similar to the squirrel-cage rotor of an induction motor, placed in slots at the surface of the field iron core, Fig. 296. The oscillations of the armature cause a corresponding motion in the magnetic flux which induces a voltage causing currents to flow in the amortisseur winding. The $rI^2$ losses from these currents act as a brake on the armature oscillations, and thereby the hunting of the synchronous motor is rapidly decreased. The amortisseur winding is sometimes used as a squirrel-cage rotor when starting the machine as an induction motor. When running in synchronism no lines of force cut the

[1] STEINMETZ, "Instability of Electric Circuits," *Trans.* A. I. E. E., vol. XXXII, 2007.

amortisseur winding and hence it does not affect the operation of the motor under normal conditions. Its chief purpose is to provide the means for a gradual adjustment of the motor-phase positions for changes in load, and thus to eliminate troublesome surging or hunting in the synchronous machine.

## PROBLEMS

**1.** From the data in Table VIII find the efficiency of the several motors for 25 per cent., 50 per cent., 75 per cent., 100 per cent., 125 per cent. and 150 per cent. full load. Assume that the load losses vary with $I^2$ and at full load are equal to 50 per cent. of the iron losses. Neglect losses in rheostats.

(*a*) For unity power factor.

(*b*) For 85 per cent. power factor.

TABLE VIII

| | Full load, hp. | $E_t$ volts | $f$ cycles | Speed, r.p.m. | Phase | S.C. ratio | Iron loss, kw. | Friction and windage, kw. | Armature resistance across terminals, ohms | $x$ per cent. |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 400 | 2,300 | 25 | 150 | 3 | 1.6 | 5.4 | 4.2 | 0.573 | 40 |
| 2 | 750 | 2,400 | 60 | 720 | 3 | 1.1 | 10.1 | 7.2 | 0.166 | 44 |
| 3 | 1,000 | 2,300 | 60 | 514 | 2 | 1.2 | 15.0 | 4.9 | 0.092 | 44 |
| 4 | 1,000 | 2,200 | 25 | 500 | 3 | 1.0 | 16.0 | 6.5 | 0.061 | 45 |
| 5 | 1,500 | 2,200 | 25 | 500 | 3 | 1.5 | 16.7 | 13.8 | 0.061 | 42 |
| 6 | 1,500 | 2,300 | 60 | 514 | 3 | 1.0 | 17.0 | 16.0 | 0.068 | 37 |

**2.** For one of the motors in Table VIII draw current loci as in Fig. 285 for zero mechanical power and for full load.

**3.** An induction motor and a synchronous motor are connected to the same 2,200 volt mains. The induction motor takes 137 kw. at 84 per cent. power factor. The synchronous motor takes 85 kw. The power factor of the total load in the mains is 98 per cent. (lagging current).

(*a*) Find the power factor of the synchronous motor.

(*b*) Draw the vector diagram showing the currents in the motors and in the mains and their time-phase relations to the voltage.

**4.** A synchronous motor is installed in a substation having the following load: Incandescent lamps 76 kw.; induction motor 35 kw. at 82 per cent. power factor; induction motor 120 kw. at 87 per cent. power factor. The load for the synchronous motor is 92 kw. The power factor at the busbars is 97 per cent. (current lagging). $E$ = 2,200 volts.

(*a*) Find the power factor of the synchronous motor.

(*b*) Find the rating of the synchronous motor if its load is 75 per cent. of full load.

(*c*) Draw the vector diagram showing the magnitude of the currents in the lamps and motors and their time-phase relation to the busbar voltage.

**5.** Synchronous and induction motors operate in parallel. The total

load is 385 kw. The power factor at the busbars is unity, for the induction motor 84 per cent. and for the synchronous motor 70.7 per cent.

(*a*) Find the load in kilowatts and in kilovolt-amperes for the two motors.

(*b*) Draw the vector diagram for the currents with the busbar voltage as the reference vector.

**6.** The machine given in problem 8, Chap. XIV, is run as a synchronous motor at a mechanical load of 90 horse power from a 2300-volt three-phase line. Find the "V" Curve for this load.

For Experiments see page 545.

## CHAPTER XVI

## ROTARY CONVERTERS

The synchronous or rotary converter consists of a direct-current generator and synchronous motor combined into one machine. The direct-current commutator at one end of the armature and the alternating-current slip rings at the other are connected to the same winding. Through the slip rings the converter receives power as a synchronous motor; and at the same time the commutator delivers energy as a direct-current generator. The motor and generator currents flow simultaneously in the same armature conductors and are acted upon by the same field. The synchronous converter is therefore equivalent to a motor-generator set, and is used extensively for converting alternating into direct currents, especially in electric railway systems. When the process is reversed and the machine converts direct into alternating currents it is known as an *inverted synchronous or rotary converter.*

(*a*) **Voltage Relations.**—In the preliminary discussion let the converter be separately excited and consider the losses negligible so that the same amount of energy is delivered through the brushes on the direct-current end as is received at the slip rings from the alternating-current circuit. The alternating-current side of the rotary converter has characteristics similar to the synchronous motor. As in the synchronous motor, the phase relations of the voltage and alternating current in the rotary converter may be changed by varying the field excitation. Let the excitation be of such value as to give unity power factor for the alternating current. Hence the maximum value of the current in the group of armature coils under consideration occurs when the maximum voltage is developed. If the fields are not distorted the maximum voltage is generated when the center of the group of coils is passing at right angles to the lines of force from the field; that is, opposite the center of the field pole. When the center of the same group of coils is midway between two poles no voltage is generated; and at intermediate positions the instantaneous

values are represented by $e = {}^{M}E \sin(\omega t)$. Diagrammatic representations of the commutator and slip-ring connections are shown in Figs. 297, 298 and 299 for a two-pole machine.

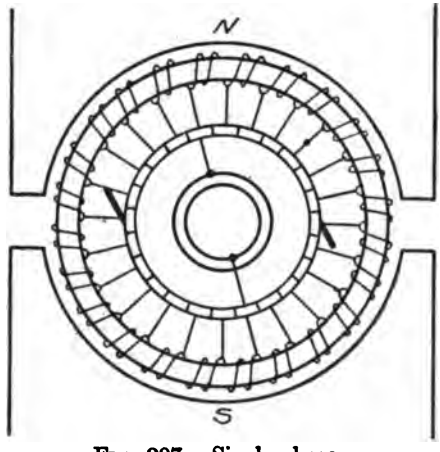



FIG. 297.—Single-phase.

The voltage between the direct-current brushes is the sum of the voltages of one-half of the coils on the armature. The instan-

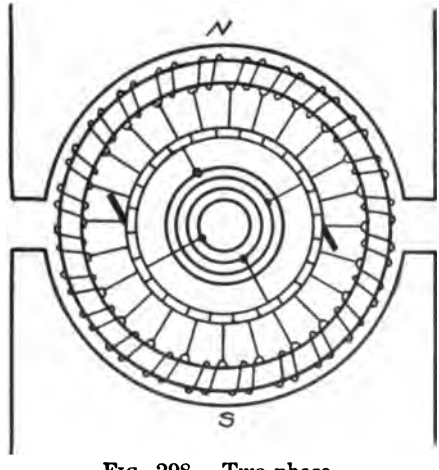



FIG. 298.—Two-phase.

taneous voltages in each coil may be represented by vectors drawn through some point *O* as in Fig. 300 or arranged in a polygon as in Fig. 301. The number of sides of the polygon

corresponds to the number of coils; and the vertices lie in a circle, as the coils are equal. When the armature revolves, the points connected to the slip rings in Fig. 297 pass under the commutator brushes at the point of maximum voltage of the

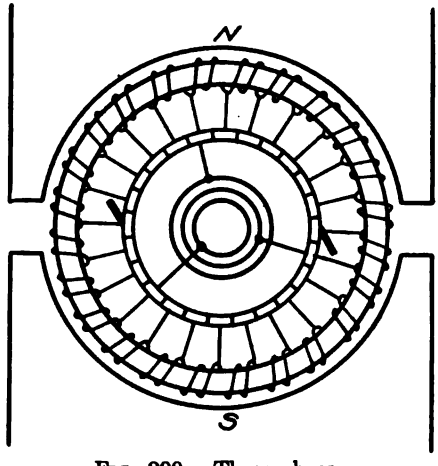



FIG. 299.—Three-phase.

alternating wave. Hence the direct-current voltage, $E_0$, must be equal to the maximum value of $E_2$, the alternating-current single-phase voltage.

$$E_2 = \frac{E_0}{\sqrt{2}} = 0.707E_0 \qquad (481)$$

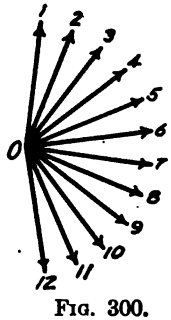



FIG. 300.

In the two-phase or quarter-phase converter, Fig. 298, the slip-ring connections are 90° apart, and the voltage relations for the single-phase and quarter-phase circuits are shown in Fig. 302 by the relative lengths of the diameter and the chord for 90°.

$$E_4 = E_2 \sin 45° = \frac{E_0}{\sqrt{2}} \sin 45° = 0.500E_0 \qquad (482)$$

Similarly for a three-phase converter, Figs. 299 and 302, the slip-ring connections are 120° apart and the relative length of the chord and diameter give the relation between the magnitude of the three-phase voltage $E_3$ and single-phase voltage $E_2$.

$$E_3 = E_2 \sin 60° = \frac{E_0}{\sqrt{2}} \sin 60° = 0.612E_0 \qquad (483)$$

In the same manner the voltage $E_6$ between the slip rings of a six-phase converter is:

$$E_6 = E_2 \sin 30° = \frac{E_0}{\sqrt{2}} \sin 30° = 0.354E_0 \qquad (484)$$

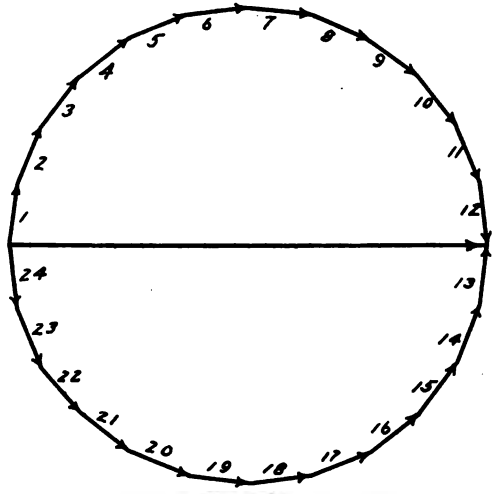



FIG. 301.

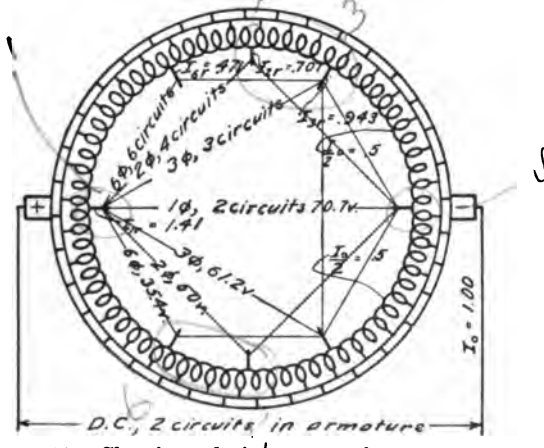



FIG. 302.—Showing relative current for same power.

These results may be expressed by one equation as in (485) by letting $n$ represent the number of slip rings in the converter.

$$E_n = \frac{E_0}{\sqrt{2}} \sin \frac{\pi}{n} \qquad (485)$$

For single-phase $n = 2$
For three-phase $n = 3$
For quarter-phase $n = 4$
For six-phase $n = 6$

(*b*) **Current Relations.**—Under the assumptions of unity power factor and no losses, the current relations are readily found by equating the alternating-current input to the direct-current output and substituting the voltage ratios found in the preceding paragraph.

Let $I_0$ = the direct current delivered through the brushes.
$I_2$ = the current in the single-phase winding.
$I_3$ = the current in the three-phase windings.
$I_4$ = the current for the quarter-phase windings.
$I_6$ = the current for the six-phase windings.

For single-phase converters:

$$E_0 I_0 = 2E_2 I_2 \therefore I_2 = 0.707 I_0 \tag{486}$$

For three-phase converters:

$$E_0 I_0 = 3E_3 I_3 \therefore I_3 = 0.544 I_0 \tag{487}$$

For quarter-phase converters:

$$E_0 I_0 = 4E_4 I_4 \therefore I_4 = 0.500 I_0 \tag{488}$$

For six-phase converters:

$$E_0 I_0 = 6E_6 I_6 \therefore I_6 = 0.471 I_0 \tag{489}$$

For a converter of $n$ rings, the current in the *armature circuits.*

$$I_n = \frac{\sqrt{2} I_0}{n \sin \frac{\pi}{n}} \tag{490}$$

Since two circuits are connected to each collector ring the currents in the mains are the vector differences of the currents in the circuits.

Letting the subscript $r$ indicate *at the rings*, the *currents in the mains* at unity power factor will be:

For a—

two-ring converter (single-phase) $I_{2r} = 2I_2 = 1.41 I_0$ (491)

three-ring converter (three-phase) $I_{3r} = \sqrt{3} I_3 = 0.943 I_0$ (492)

four-ring converter (two-phase) $I_{4r} = \sqrt{2}I_4 = 0.707I_0$ (493)

six-ring converter (six-phase) $I_{6r} = I_6 = 0.471I_0$ (494)

The general expression for current in the *mains* or collector rings:

$$I_{nr} = 2I_n \sin\frac{\pi}{n} = \frac{2\sqrt{2}I_0}{n} \quad (495)$$

In Table IX the respective values of the currents and voltages in the circuits and mains are given for 1.0 amp. and 100.0 volts in the direct-current mains.

TABLE IX.—VOLTS AND AMPERES FOR CONSTANT POWER

D. C. Mains. $E_0 = 100$ volts. $I_0 = 1.00$ amp. $W_0 = 100$ watts

| | No. of circuits in armature | No. of rings | Volts between mains | | Amperes per armature circuit or per phase | | Amperes per main |
|---|---|---|---|---|---|---|---|
| | $n$ | $n$ | $E_n = \frac{E_0}{\sqrt{2}}\sin\frac{\pi}{n}$ | ${}^{M}E_n = \sqrt{2}E_n$ | $I_n = \frac{\sqrt{2}I_0}{n\sin\frac{\pi}{n}}$ | ${}^{M}I_n = \sqrt{2}I_n$ | $I_{nr} = \frac{2\sqrt{2}I_0}{n}$ |
| Single-phase | 2 | 2 | 70.7 | 100.0 | 0.707 | 1.000 | 1.41 |
| Three-phase | 3 | 3 | 61.2 | 86.7 | 0.544 | 0.770 | 0.943 |
| Quarter-phase..... | 4 | 4 | 50.0 | 70.7 | 0.500 | 0.707 | 0.707 |
| Six-phase... | 6 | 6 | 35.4 | 50.0 | 0.471 | 0.666 | 0.471 |

In Fig. 302 the effective value of current in the leads and voltage per phase are indicated corresponding to 1 amp. at 100 volts, direct-current.

Thus, for a three-phase converter the current in the leads is 0.943 amp. and for a six-phase converter 0.471 amp. For the single-phase converter the current at the slip rings is 1.41 amp. while the voltage is only 70.7 volts.

(*c*) **Heat Losses at Unity Power Factor.**—From the previous discussion it is evident that the rotary converter may be used as a generator receiving power from a prime mover and delivering either direct or alternating currents, or both simultaneously. It may also be used as a synchronous or direct-current motor receiving the energy in the electric form and delivering mechanical energy by means of pulley and belt. As a rotary converter it receives energy from the alternating-current mains and delivers

all but the amount lost in the machine to the direct-current mains. As an *inverted converter* or *inverted rotary*, power is received from the direct-current mains and delivered to the alternating-current mains of phase and frequency depending upon the number of slip-ring connections and the speed. Under the several kinds of service the armature heating is not the same for equal power passing through the machine. Since the rating of the synchronous converter depends upon the permissible temperature rise for the same reasons as for alternators and synchronous motors, the difference in heat loss in the armature affects the capacity of the machine. The power rating of a

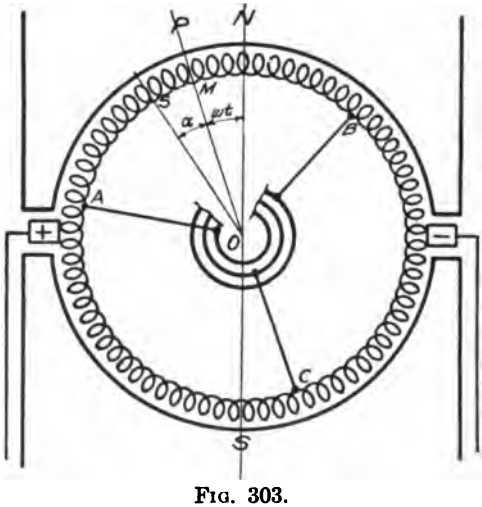



FIG. 303.

rotary converter is expressed in terms of the rating the same machine would have if used as a direct-current generator driven by a prime mover. The difference in rating when used as a rotary converter as compared to a direct-current generator is determined by the $RI^2$ losses in the armature, since the eddy-current and hysteresis losses are approximately the same in both cases. Since the slip rings and commutator bars are connected to the same armature, the instantaneous value of the current in any conductor will be the algebraic sum of the simultaneous values of the currents flowing in the direct-current and alternating-current circuits.

Consider a two-pole $n$-ring converter, as indicated by the diagram in Fig. 303. Let $M$ be the conductor midway between

the slip-ring connections $A$ and $B$, and let $S$ be the conductor at a distance of $\alpha$ from $M$. For the several conductors in the circuit between $A$ and $B$ the value of $\alpha$ varies from 0 to $\pm \frac{\pi}{n}$. The maximum value of the alternating current, $\sqrt{2}I_n$, flows when conductor $M$ passes under the center of the field pole (unity power factor assumed). Hence the instantaneous value for any other position of the armature is for conductor $M$:

$$i = \sqrt{2}\, I_n \cos \omega t - \frac{I_0}{2} \tag{496}$$

When conductor $S$ reaches the positive brush,

$$\alpha = \frac{\pi}{2} - \omega t$$

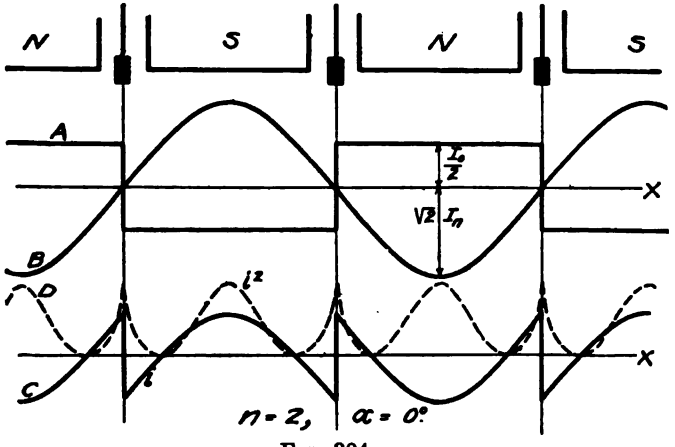



FIG. 304.

and at the negative brush

$$\alpha = -\left(\frac{\pi}{2} + \omega t\right) \text{ or } \frac{3\pi}{2} - \omega t.$$

The notation in equation (496) applies to multipolar machines by using electrical degrees and letting $I_0$ represent the direct current passing through each set of brushes. In Fig. 304 are shown the currents flowing through conductor $M$ in a single-phase converter, *i. e.*, $n = 2$, $\alpha = 0$. The direct current reverses direction at the brushes forming a wave of rectangular shape. The neutral points of the alternating current coincide

with the wave reversals of the direct current, and the maximum value is twice that of the direct current. The instantaneous value is the algebraic sum of the simultaneous values of the direct and alternating currents. Hence curve $C$, plotted as the sum of curves $A$ and $B$, represents the wave shape and magnitude of the resultant current flowing in the armature conductor. The broken line, curve $D$, is plotted to the square of the instantaneous values in curve $C$ and hence its ordinates are proportional to the heat

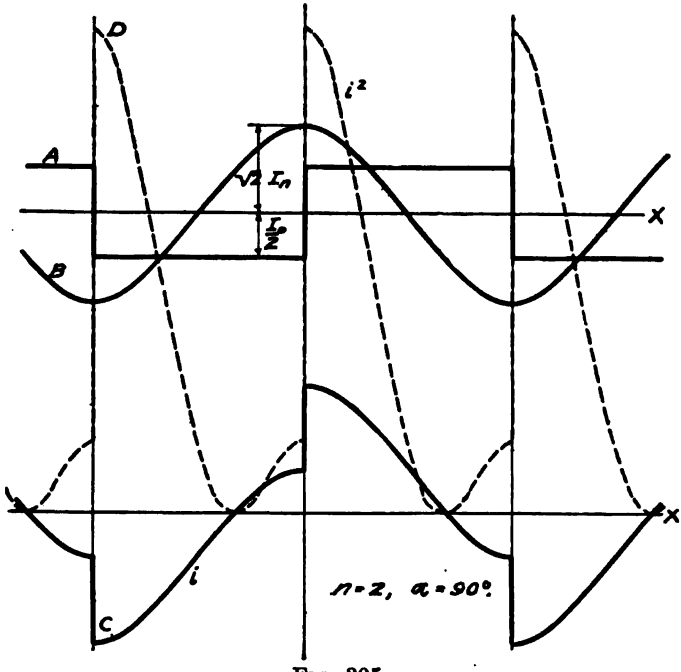



FIG. 305.

generated in the conductor due to the $RI^2$ losses. In Fig. 305 are shown the corresponding curves for the conductor at the point of connection to the slip ring, when $n = 2$, $\alpha = 90°$. By shifting the relative positions of the direct and alternating waves the instantaneous values, and hence the heating, become much greater for the conductor nearest to the slip-ring connection than for conductor $M$ midway between the slip rings. Similar curves for $n = 2$ and $\alpha = 45°$ are shown in Fig. 306.

The curves for a three-phase converter are like Fig. 304 for $n = 3$, and $\alpha = 0$ and like Fig. 305 for $n = 3$ and $\alpha = \frac{\pi}{3}$. From the figures and from the equation it is evident that in order to determine the heat loss in the armature it is necessary to find first the average loss in a conductor for a cycle, and second, the average of the loss in the several conductors between each pair of leads.

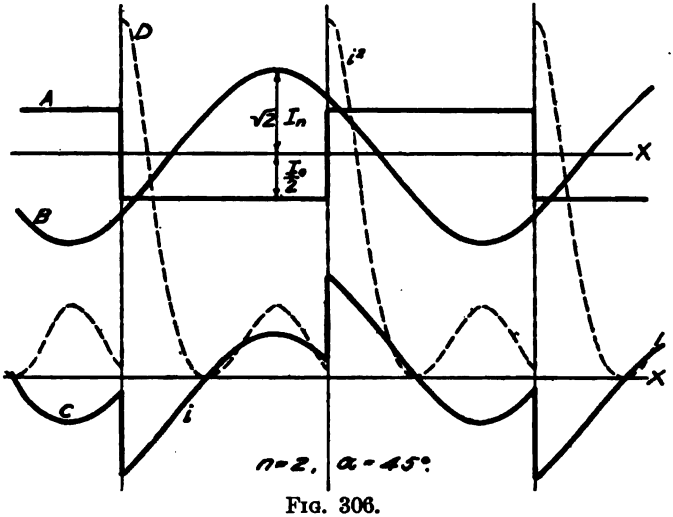



FIG. 306.

From equations (490) and (496):

$$i = \frac{2I_0 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} - \frac{I_0}{2} \tag{497}$$

The heat loss is proportional to the square of the current.

$$i^2 = \frac{4I_0^2 \cos^2(\omega t)}{n^2 \sin^2\left(\frac{\pi}{n}\right)} - \frac{2I_0^2 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} + \frac{I_0^2}{4} \tag{498}$$

$$= \frac{I_0^2}{4}\left[\frac{16 \cos^2(\omega t)}{n^2 \sin^2\left(\frac{\pi}{n}\right)} - \frac{8 \cos(\omega t)}{n \sin\left(\frac{\pi}{n}\right)} + 1\right] \tag{499}$$

The average heat loss per conductor in half a cycle

$$= r \int_{-(90°+\alpha)}^{(90°-\alpha)} \frac{i^2 d(\omega t)}{\pi}$$

in which $r$ is the resistance of one armature conductor between taps.

$$r\int\frac{i^2d(\omega t)}{\pi}=\frac{rI_0^2}{4\pi}\int_{-(90^\circ+\alpha)}^{(90^\circ-\alpha)}\left[\frac{16\cos^2(\omega t)}{n^2\sin^2\left(\frac{\pi}{n}\right)}-\frac{8\cos(\omega t)}{n\sin\left(\frac{\pi}{n}\right)}+1\right]d(\omega t) \quad (500)$$

$$=\frac{rI_0^2}{4}\left[1-\frac{16\cos\alpha}{\pi n\sin\frac{\pi}{n}}+\frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right] \quad (501)$$

To find the average heat in the armature the average value per conductor as given in equation (501) must be integrated between the limits of $\alpha=-\frac{\pi}{n}$ and $\alpha=\frac{\pi}{n}$ and the result divided by $\frac{2\pi}{n}$.

Average heat in armature:

$$=\frac{rI_0^2}{4\frac{2\pi}{n}}\int_{-\frac{\pi}{n}}^{\frac{\pi}{n}}\left[1-\frac{16\cos\alpha}{n\pi\sin\left(\frac{\pi}{n}\right)}+\frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right]d\alpha \quad (502)$$

$$=\frac{rI_0^2}{4}\left[1-\frac{16}{\pi^2}+\frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right] \quad (503)$$

One-half of the armature is taken into the calculations both for the alternating current and the direct current, and hence the ratio is the same as for the whole armature.

The average armature heating is therefore $\left[1-\frac{16}{\pi^2}+\frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}\right]$ times the heat generated by the same direct current. Hence for the same temperature rise an $n$-ring rotary converter transmits

$$\frac{1}{\sqrt{1-\frac{16}{\pi^2}+\frac{8}{n^2\sin^2\left(\frac{\pi}{n}\right)}}}$$

times as much current as when the machine operates as a direct-current generator.

From this ratio the power ratings of the synchronous converter are found in terms of the direct-current generator as stated in Table X.

TABLE X.—POWER RATINGS OF A SYNCHRONOUS CONVERTER

| As a direct-current generator | As a single-phase converter $n = 2$ | As a three-phase converter $n = 3$ | As a two-phase converter $n = 4$ | As a six-phase converter $n = 6$ | As a twelve ring converter $n = 12$ |
|---|---|---|---|---|---|
| 1.00 | 0.85 | 1.33 | 1.63 | 1.93 | 2.44 |

In the preceding discussion of the current relations, it was assumed that no energy was lost in the converter and that the

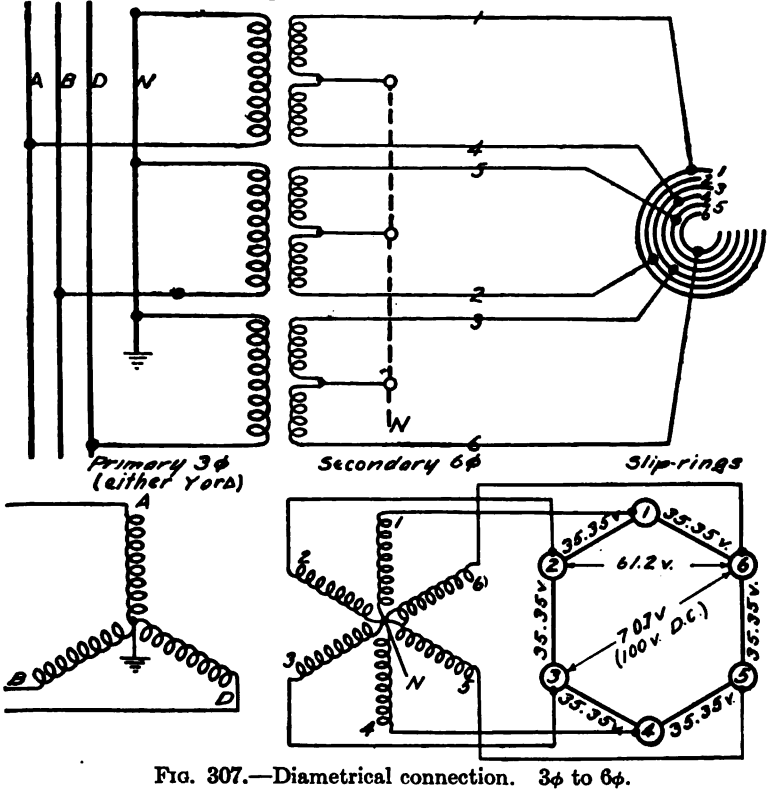



FIG. 307.—Diametrical connection. $3\phi$ to $6\phi$.

field excitation was adjusted for unity power factor. For power factors less than unity,[1] and taking into account the efficiency of the machine, the heat losses in the armature conductor are greatly increased. The load losses are also increased for power factors less than unity. Any variation of wave shape from the

[1] STAHL, *Electrical World*, Oct. 28, 1911, p. 1090.

sinusoidal form also affects the heat losses in the converter. The rating of the converter is computed on the basis of sine waves and unity power factor as given in Table X.

(*e*) **Transformer Connections for Rotary Converters.**—In most systems using rotary converters the energy is delivered to the converter substation in the three-phase system. From the relative ratings as given in Table X it is evident that the same

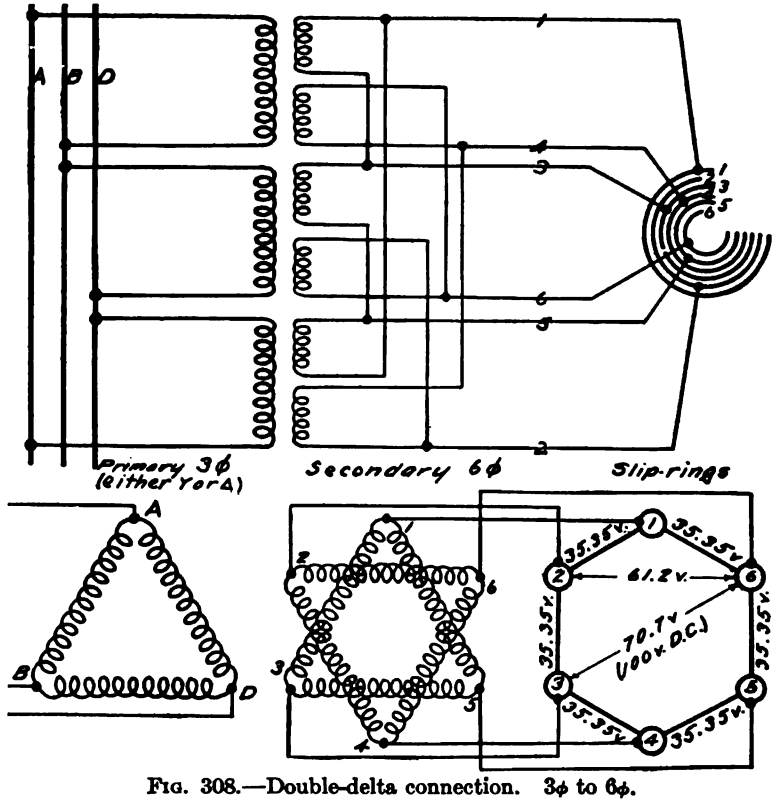



FIG. 308.—Double-delta connection. 3φ to 6φ.

machine will give a larger output for the same heating if connected as a six-phase converter than in the three-phase system. The ratio is 1.33:1.93, or 1:1.45; an increase of 45 per cent. in the capacity of the converter. The change from three-phase to six-phase is readily accomplished. The converter itself requires six slip rings instead of three with connections to the windings at intervals of 60 electrical degrees instead of 120. The transformation from a three-phase to a six-phase system may be

accomplished in several ways, and the following connections are in general use:

| | | |
|---|---|---|
| 1. Diametrical | 3$\phi$ to 6$\phi$ | Fig. 307. |
| 2. Double $\Delta$ | 3$\phi$ to 6$\phi$ | Fig. 308. |
| 3. Double $Y$ | 3$\phi$ to 6$\phi$ | Fig. 309. |
| 4. Double $T$ | 3$\phi$ to 6$\phi$ | Fig. 310. |
| 5. Double $T$ | 2$\phi$ to 6$\phi$ | Fig. 311. |
| 6. Ring | 3$\phi$ to 6$\phi$ | Fig. 312. |
| 7. Distributed $Y$ | 3$\phi$ to 3$\phi$ (with Edison three-wire system on d.c. side) | Fig. 313. |

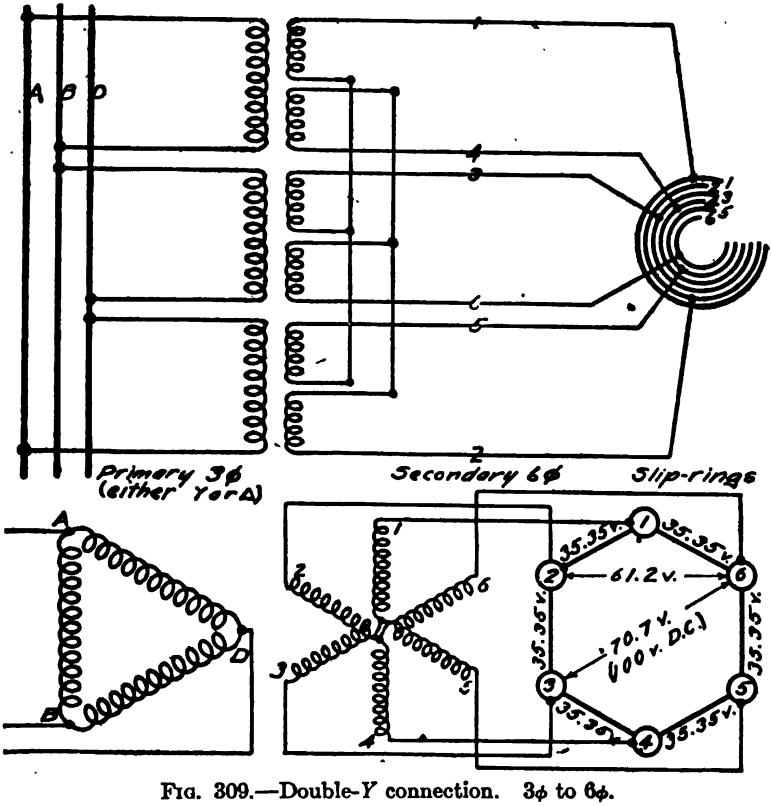



FIG. 309.—Double-$Y$ connection. 3$\phi$ to 6$\phi$.

The diametrical or double-delta connections are most commonly used with rotary converters. A neutral can conveniently be secured in the diametrical connection and this arrangement is generally preferred whether the direct-current side is two-wire or three-wire Edison system.

The choice of connections for any system often depends on factors outside of the rotary converter; such as the type of direct-current distribution system, minimum insulation stress, voltage ratios, operating three-phase in connection with six-phase, ground connections, etc. By means of the double-$T$ connection, Fig. 311, the transformation from two-phase is made directly to six-phase.

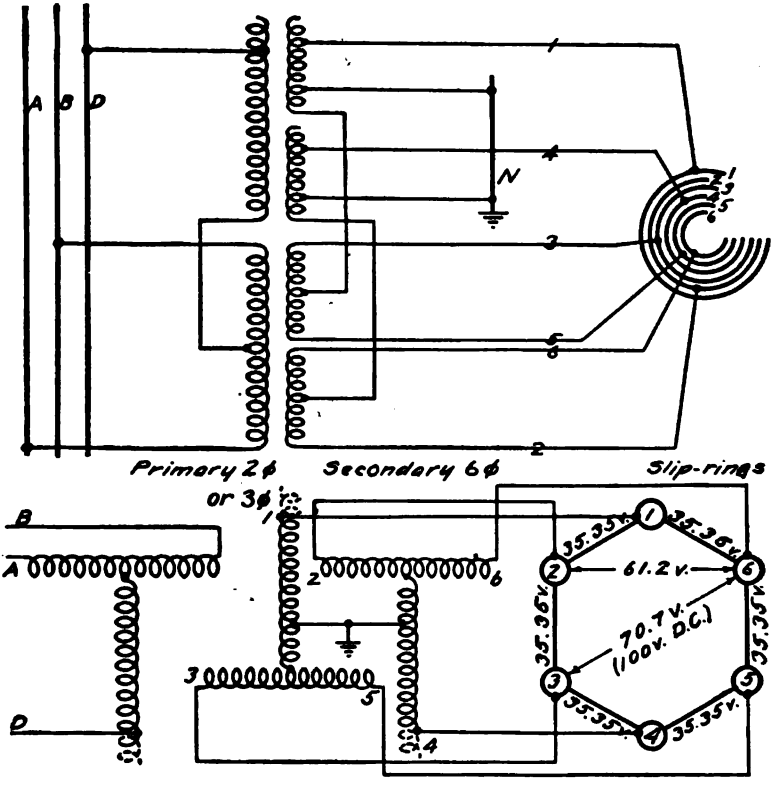



FIG. 310.—Double-$T$ connection. $3\phi$ to $6\phi$.

The *teaser* transformer must either have a tap at 86.7 per cent. of the full voltage or a proportionate number of turns on the secondary windings. The double-$T$ connection, Fig. 310, may also be used for three-phase to six-phase transformation in connection with the three-wire Edison system on the direct-current side. The distributed $Y$ eliminates flux distortions in the transformers when operating a three-phase rotary converter connected to a three-wire Edison system. The two windings

are interconnected as shown in Fig. 313. The unbalanced neutral current flows through the two coils in each transformer in opposite directions, and hence has no magnetizing effects. With the simple $Y$ connection even a 10 per cent. unbalanced neutral current magnetizes the transformer cores beyond the saturation point and thus causes abnormally large magnetizing currents to

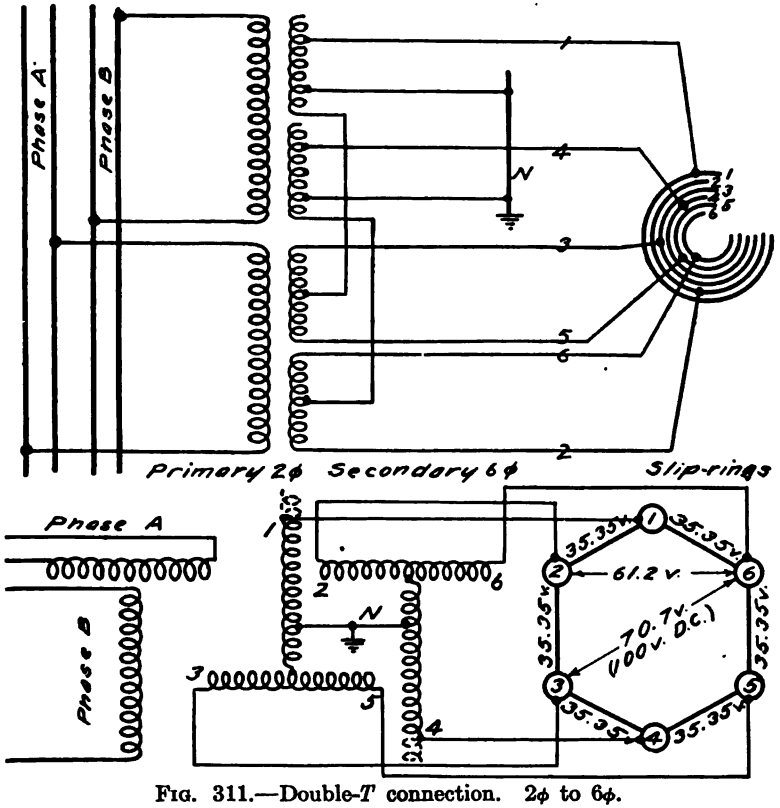



FIG. 311.—Double-$T$ connection. $2\phi$ to $6\phi$.

flow, with accompanying excessive losses. In the distributed $Y$ connection the primary may be connected in $\Delta$ and it is therefore often preferable to the double-$T$. The double-$T$ connection requires only two transformers, but has also the disadvantage that the triple harmonic exciting current is not short-circuited.

(*f*) **Synchronous Booster, Split-pole, Auxiliary-pole and Commutating-pole, Rotary Converters.**—As has already been shown, the ratio of the alternating-current to the direct-current voltage

in a rotary converter is a constant, depending upon the number of phases and the maximum value of the alternating-current wave. The ratio is not affected by changes in the field excitation, as the alternating-current part of the converter acts like a synchronous motor and hence changes in field excitation merely cause variation in the power factor without affecting the voltage.

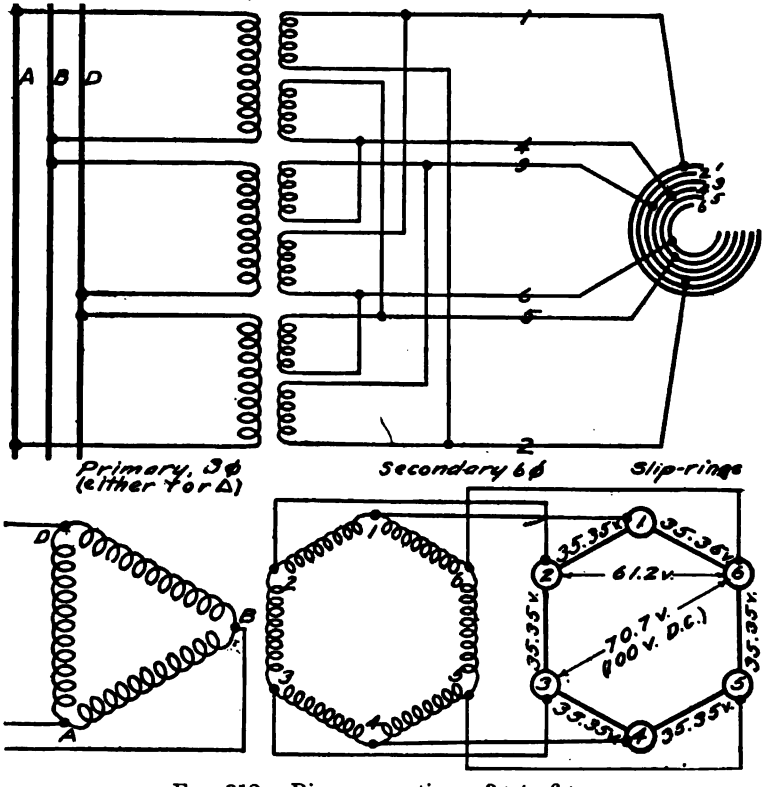



FIG. 312.—Ring connection. $3\phi$ to $6\phi$.

In the operation of the converter it is often necessary to have the direct-current voltage under control of the operator although the power is supplied from constant-potential alternating-current mains. The control of the direct-current voltage is usually secured in one of two ways:

(*a*) To vary the alternating-current voltage impressed on the rotary converter by means of a booster, or change in transformer ratio, or by using reactive leads, and,

(*b*) By varying the wave shape of the alternating-current voltage.

The synchronous booster rotary converter, as illustrated in Fig. 314, consists of a simple rotary converter in combination with an alternating-current generator mounted on the same shaft and having the same number of poles. By varying the

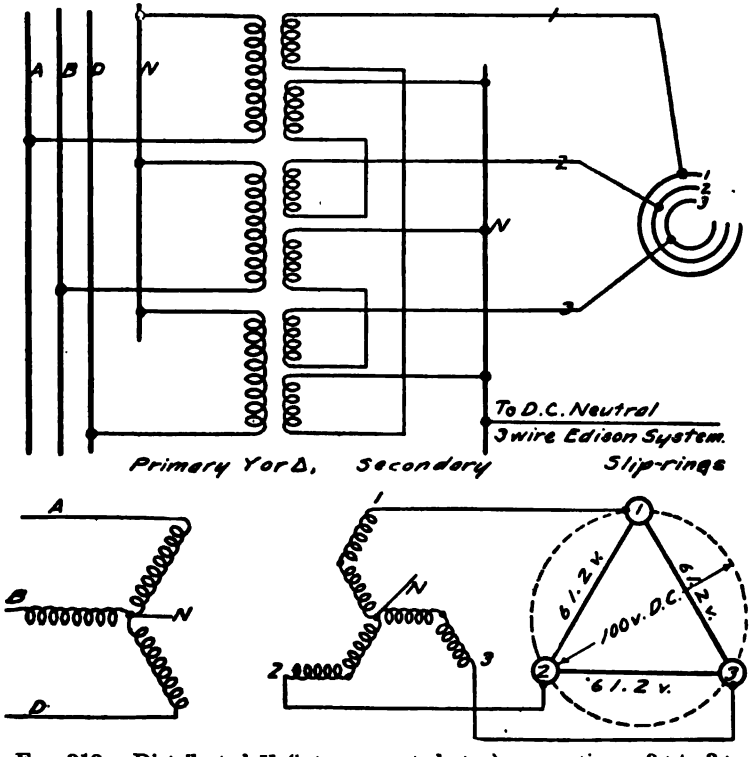



FIG. 313.—Distributed-*Y* (interconnected star) connection. 3$\phi$ to 3$\phi$.

field of the alternating-current generator, the voltage impressed on the rotary converter part of the machine may be increased or decreased at will. The direct-current voltage is thereby under direct control of the operator. In commercial machines the range of voltage regulation is usually from 15 per cent. above to 15 per cent. below the line voltage. The principle of the booster converter is therefore simple and easily understood.

The change in the alternating voltage impressed on the rotary

converter may likewise be obtained by changing the ratio of the voltage transformation between the machine and the constant-voltage transmission line by means of induction regulators. To a certain extent a similiar variation may be secured by using inductive reactances in the alternating current leads to the converter. The choice of apparatus depends upon many factors, but their action on the rotary converter is the same; namely, to vary the impressed voltage and thereby control the value of the direct-current voltage delivered by the converter.

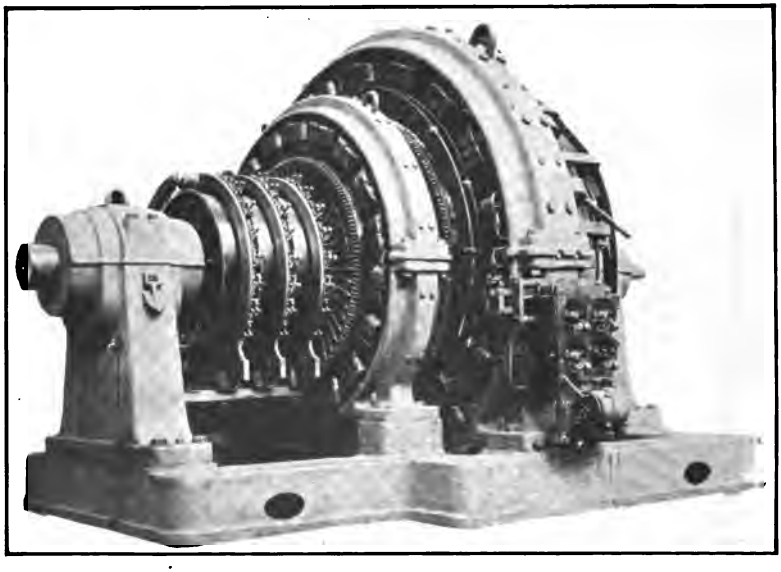


FIG. 314.—Synchronous booster rotary converter. (*Westinghouse Elec. & Mfg. Co.*)

The action of the split-pole converter is based on an entirely different principle. The direct-current voltage is equal to the maximum value of the impressed alternating-current voltage wave. Hence, by changing the wave shape the direct-current voltage may be varied several per cent. while the effective value of the alternating current remains constant. This change in wave shape may be secured by splitting the pole parallel to the shaft into three narrow sections, each having a separate winding. By making the field excitation of the middle section stronger than for the outside parts, the flux distribution and hence

the voltage wave, is peaked as in Fig. 315 giving a direct-current voltage higher than for the corresponding sine wave. On the other hand, if the side sections are stronger the wave shape is flattened, as in Fig. 316, and the direct-current voltage is less than for the sine wave of equal effective value.

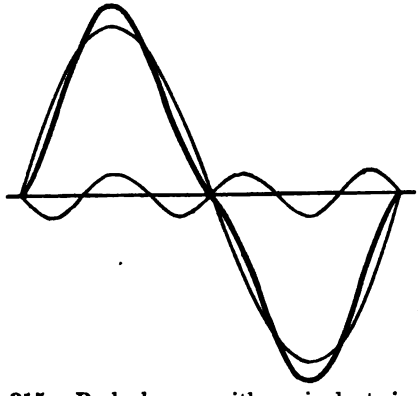


FIG. 315.—Peaked wave with equivalent sine wave.

The action of the sectional pole converter, Fig. 317, is to some extent dependent upon the change in wave shape, but depends chiefly on the shifting of the field flux relative to the direct current brushes. The direct current of the converter, as in any

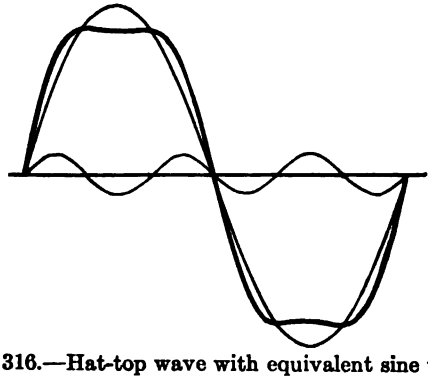


FIG. 316.—Hat-top wave with equivalent sine wave.

direct-current generator, varies with the shifting of the brushes relative to the field poles. Hence, by changing the relative field excitation of the main and auxiliary poles the magnetic flux is

shifted while the brushes remain fixed. Thereby the direct-current voltage may be raised or lowered without changing the impressed alternating-current voltage. It should be noted that

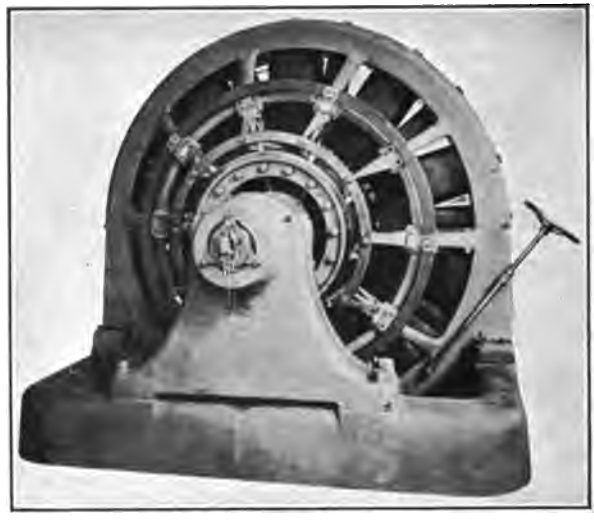


FIG. 317.—Sectional pole, rotary converter. (*General Electric Co.*)

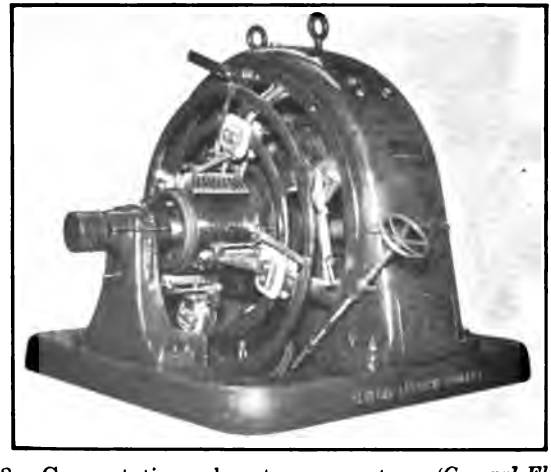


FIG. 318.—Commutating pole, rotary converter. (*General Electric Co.*)

the sectional pole of the rotary converter merely provides a means for varying the direct-current voltage and is not an interpole for commutation. In fact, the range over which the

voltage may be varied by means of the sectional pole is chiefly limited by commutation troubles. For this reason the sectional pole is placed nearer the trailing tip so as to leave the necessary space for commutation.

The commutating pole is used on many rotary converters, Fig. 318, for the purpose of improving the commutation, in the same manner as for direct-current generators. The commutating pole is placed midway between the main poles and the excitation is so designed as to automatically provide satisfactory commutation during rapid fluctuations of the load. With commutating poles it is necessary to raise the direct-current brushes in order to avoid sparking when starting a converter from the alternating side. A rigging is provided so that all the brushes may be raised by the movement of one lever. In other respects the starting of a rotary converter from the alternating-current end is similar to that of a synchronous motor. It may also be started from the direct-current side as a direct-current motor.

TABLE XI.—ROTARY CONVERTERS, SIX PHASE, 600 VOLTS DIRECT CURRENT

| | Rated full load | $f$ | Speed | Iron loss | Friction and windage | Field loss | Armature resistance from d.c. end |
|---|---|---|---|---|---|---|---|
| | Kw. | Cycles | R.p.m. | Kw. | Kw. | Kw. | Ohms |
| 1 | 500 | 60 | 900 | 8.0 | 9.2 | 5.2 | 0.011 |
| 2 | 500 | 25 | 750 | 5.0 | 3.0 | 5.1 | 0.014 |
| 3 | 1,000 | 60 | 600 | 19.0 | 18.0 | 7.2 | 0.005 |
| 4 | 1,000 | 25 | 500 | 12.0 | 15.0 | 7.3 | 0.009 |
| 5 | 1,500 | 60 | 600 | 24.0 | 27.0 | 8.1 | 0.004 |
| 6 | 1,500 | 25 | 375 | 19.0 | 12.0 | 8.4 | 0.005 |
| 7 | 2,000 | 60 | 450 | 30.0 | 22.0 | 9.0 | 0.003 |
| 8 | 2,000 | 25 | 300 | 28.0 | 19.0 | 9.5 | 0.004 |

## PROBLEMS

**1.** From the data in Table XI calculate the efficiency for 25 per cent., 50 per cent., 75 per cent., 100 per cent., 125 per cent. and 150 per cent. full load. Assume the load losses at full load to be equal to 50 per cent. of the iron losses. Neglect losses in rheostat. Field excitation in all cases adjusted for unity power factor.

**2.** For 600 volts on the direct current end what are the theoretical alternating voltages between adjacent slip rings for 2, 3, 4, and 6-ring converters? Neglecting losses, what alternating line currents would give 100 amperes on the direct current end for the above?

**3.** For a six ring rotary, problem 2, what is the voltage on the secondary terminals in each case?

(*a*) Diametrical connection.
(*b*) Double-delta connection.
(*c*) Double-star connection.
(*d*) Ring connection.
(*e*) Double-$T$, $2\phi$ connection.
(*f*) Double-$T$, $3\phi$ connection.
(*g*) Interconnected star (using three rings).

For Experiments see page 545.

## CHAPTER XVII

## SINGLE-PHASE COMMUTATOR MOTORS

Simplicity is the notable feature of constant-speed alternating-current machines, when compared to the corresponding direct-current apparatus; simplicity in design, in construction and in operation. Alternators, transformers and induction motors cost less to manufacture, have lower operating and maintenance expense and a lower depreciation than the corresponding direct-current machines. While the transformer is the foundation of the alternating-current system, the alternator and particularly the squirrel-cage induction motor comply almost perfectly with the requirements in their respective fields. For constant-speed service the alternating-current machinery offers many advantages.

However, for variable-speed loads, the reverse is true. Direct-current motors can operate efficiently under a wide range in speed, while the corresponding alternating-current motors are complicated and comparatively inefficient. To provide a strong starting torque the polyphase induction motor needs a wound rotor with an adjustable resistance; and if operated continuously at half synchronous speed its efficiency would be less than 50 per cent. The single-phase induction motor has inherently no starting torque and hence the devices necessary to bring it up to speed introduce complications, increase first cost and maintenance and lower the efficiency.

In electric railway service the speed variation is an important feature and the starting torque is very large. The requirements of street railway loads can be complied with, in a highly satisfactory manner, by the direct-current series motor. For inter-urban and trunk lines the problem of power transmission may make it more desirable to use a single-phase system although a comparison of the motors alone is decidedly in favor of the direct-current machine.

(*a*) **The Straight Series Motor.**—The basic principle of the single-phase series motor is the same as that of the direct-current

series motor; and the simplest method for explaining the characteristics of the former is by direct comparison with the known characteristics of the latter. The field and armature windings are connected in series and the circuit diagram, Fig. 319, is the same as for a direct-current series motor. Since the field flux and armature current reverse direction simultaneously, the torque continues in the same direction for successive half cycles. Superimposed upon the series-motor principle is the transformer action inherent in all alternating-current apparatus. A direct-current series motor could be operated as an alternating-current motor if it were not for excessive heating produced by the eddy currents and hysteresis losses in the poles, yoke and armature, and by the induced currents in the short-circuited armature coils, and for destructive sparking at the commutator.

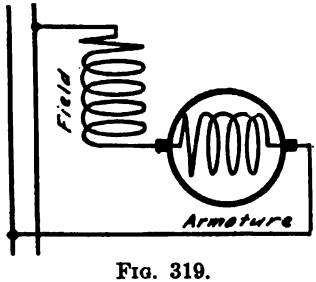



FIG. 319.

To reduce the eddy currents to a minimum the entire magnetic circuit of the alternating-current series motor is laminated; and the iron used must have low hysteresis loss. The alternating field flux necessitates other changes in the design, as the transformer action is the source of four undesirable features in the commercial operation of the motor, in comparison to the corresponding direct-current series motor.

(*a*) The commutator troubles are increased.

(*b*) The armature heating is greater.

(*c*) The starting torque is weaker.

(*d*) The power factor is always less than unity, and very low during the starting period.

The brushes cover two or more commutator bars and the corresponding armature coils are short-circuited in the same manner as for direct-current series motors. In the alternating-current motor these coils form a secondary circuit interlinked with the alternating field flux. The resulting induced voltage causes heavy currents to flow in the short-circuited coils, which increase the heating and produce excessive sparking. To improve commutation the voltage induced per coil must be made as small as possible; or the product of the number of turns in the short-circuited coil and the interlinking alternating flux must be

sufficiently small to give satisfactory commutation. This is obtained by several modifications in the design as compared to the standard direct-current motor.

(*a*) By reducing the number of turns per coil. Except for very small motors the least possible number is used, or one turn per coil.

(*b*) By using more poles, hence more paths, and therefore less flux per coil. This increase is limited by other features of the design; but in alternating-current railway motors six poles are usually used instead of four in the direct-current motor.

(*c*) By increasing the number of commutator segments.

(*d*) By reducing the magnetic flux density.

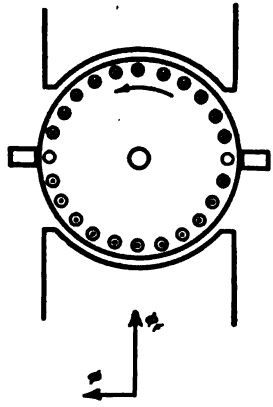


FIG. 320.

To compensate for the weaker field and fewer turns the armature current must be increased in order to have the same torque, and this in turn necessitates a larger armature diameter and a longer commutator. The low field densities require only a few field turns and hence only a small space for the field coils. Alternating-current series motors have, as a result, short stubby field poles of larger cross-section than the corresponding direct-current motors. The few field turns and large cross-section give a low reluctance in the magnetic circuit. For the same total field flux the inductance is directly proportional to the number of turns. In order to have a fairly good power factor the reactive power component must be small and hence the inductance in the circuit must be kept as low as possible. The relative value of the field and armature ampere-turns has an important bearing on the commutation of the motor. In a direct-current series motor the total flux in the armature is the resultant of the main field flux and the cross-flux produced by the armature current. Illustrating by a two-pole motor as in Fig. 320 the *main field* may be represented by the vector, $\phi_F$, and the armature *cross-field* by $\phi_A$. The conductors short-circuited by the brushes lie in the cross-field and hence when the armature rotates, the flux, $\phi_A$, induces a voltage which in turn produces a current in the short-circuited turns. Consequently, if the

armature ampere-turns are relatively large, the cross-field, $\phi_A$, is strong and the induced voltage and current in the short-circuit turns may create serious difficulties in commutation. Therefore, in order to secure satisfactory commutation, $\phi_F$ is made stronger than $\phi_A$, especially in direct-current motors without commutating poles. In motors supplied with commutating poles the ratio may approach unity although the distortion of the resultant field is undesirable as it causes larger core losses and higher maximum voltages between the commutator segments. While a strong field is desirable in the direct-current series motor, it is not permissible in the alternating-current series motor, as already explained, and the strength of the cross-field may be much larger than the main field as indicated in Fig. 321.

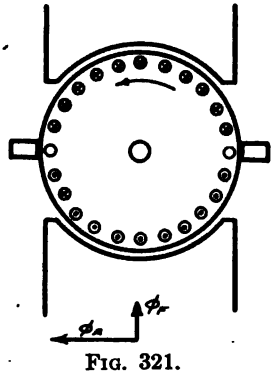


FIG. 321.

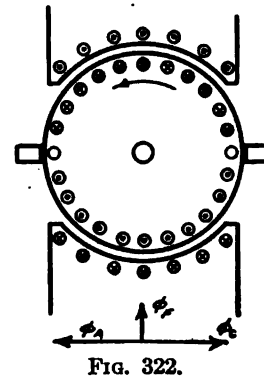


FIG. 322.

(*b*) **Compensated Straight Series Motor.**—In order to prevent destructive sparking on the commutator the cross-field is neutralized by means of an auxiliary or compensating winding, placed in slots in the pole faces as indicated in Fig. 322. The compensating winding, in the straight series motor, is connected in series with the armature and has the same number of turns, Fig. 323. The currents in the armature under the field pole and in the parallel conductors of the compensating windings flow in opposite directions and hence the magnetic fluxes in the two windings are of equal strength but opposite in direction, as indicated in Fig. 322. Compensation for the conductors under the brushes and the open space between the field poles may be secured by fractional pitch windings or by the use of commutating poles.

*Power Factor.*—The compensating winding improves the power factor as well as the commutation. In Fig. 324 is shown the vector diagram of a plain alternating-current series motor, and in Fig. 325 the corresponding vector diagram for a single-phase motor having a compensating winding but omitting the

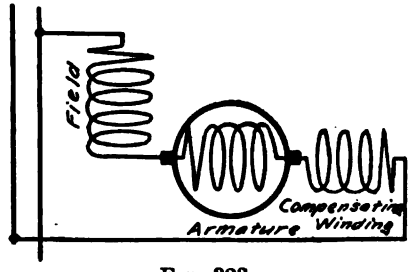



FIG. 323.

currents induced in the coils short-circuited by the brushes. The current is represented by a vector, $I$, along the $X$-axis and the respective volts consumed by the real and reactive power components in the circuit are shown both in magnitude and phase position by the voltage vectors.

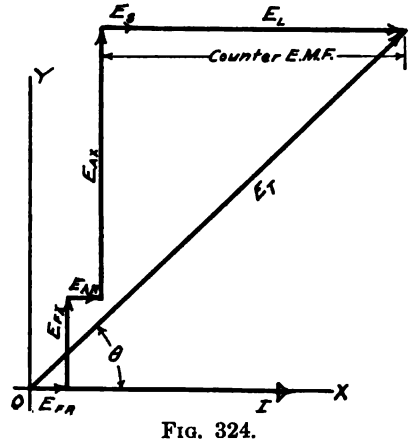



FIG. 324.

$E_{FR}$ = voltage for field resistance.
$E_{FX}$ = voltage for field reactance.
$E_{AR}$ = voltage for armature resistance.
$E_{AX}$ = voltage for armature reactance.
$E_s$ = voltage for speed loss (friction, windage and iron).

$E_L$ = voltage for output or mechanical load.
$E_S + E_L$ = the counter e.m.f.
$E_{CR}$ = voltage for resistance of compensating winding.
$E_{CX}$ = voltage for reactance of compensating winding.
$E_T$ = total impressed voltage.
$E_{AX} - E_{CX}$ = voltage for leakage between armature and compensating windings.

For straight series motor, Fig. 324:

$$\dot{E}_T = \dot{E}_{FR} + \dot{E}_{AR} + \dot{E}_S + \dot{E}_L + j(\dot{E}_{FX} + \dot{E}_{AX}) \quad (511)$$

$$E_T = \sqrt{(E_{FR} + E_{AR} + E_S + E_L)^2 + (E_{FX} + E_{AX})^2} \quad (512)$$

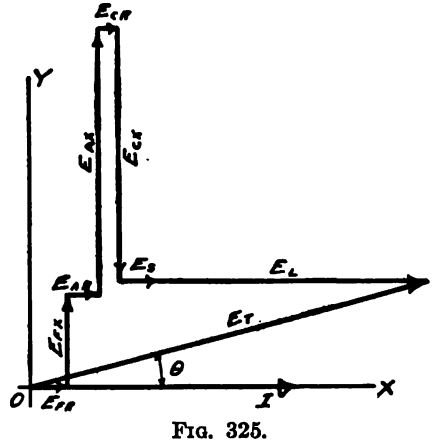



FIG. 325.

For compensated straight series motor, Fig. 325:

$$\dot{E}_T = \dot{E}_{FR} + \dot{E}_{AR} + \dot{E}_{CR} + \dot{E}_S + \dot{E}_L + j(\dot{E}_{FX} + \dot{E}_{AX} - E_{CX}) \quad (513)$$

$$E_T = \sqrt{(E_{FR} + E_{AR} + E_{CR} + E_S + E_L)^2 + (E_{FX} + E_{AX} - E_{CX})^2} \quad (514)$$

It is readily seen from the vector diagrams that the power factor is greatly improved by use of the compensating winding. In commercial designs the power factor without the compensating winding is about 35 to 40 per cent. while with a compensating winding it may reach 90 per cent. While the inductive reactance due to the main field can not be eliminated, it may be reduced to a comparatively small value, by providing a magnetic

circuit of low reluctance. This is, in part, gained by the larger cross-section of fields and armature necessary for obtaining good commutation, as already explained, but *to a larger degree by using a smaller air gap than in the direct-current motor.*

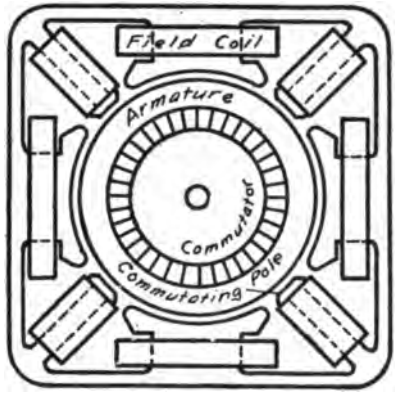



FIG. 326.—Direct-current series motor.

*Comparative Size.*—In Figs. 326 and 327 is shown an approximate comparison between a 600-volt direct-current railway motor

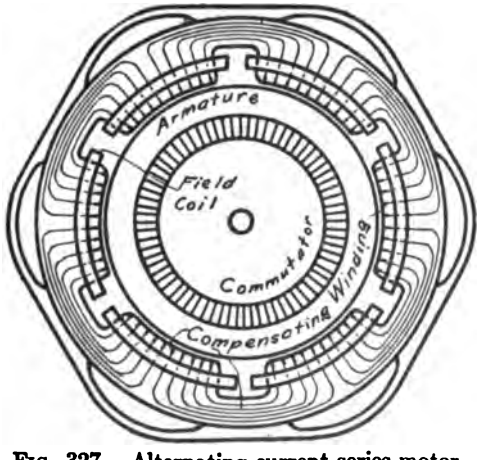



FIG. 327.—Alternating-current series motor.

and the corresponding 25-cycle alternating-current compensated motor The dimensions along the shaft are approximately the same. In general the 25-cycle motor is from 25 to 45 per

cent. and a 15-cycle motor from 15 to 30 per cent. heavier than the corresponding direct-current motor. For higher direct-current voltages the comparison is more favorable to the single-phase motor.

*Resistance Leads.*—As already stated, the alternating field flux induces heavy currents in the armature coils when short-circuited by the brushes. These currents produce two very undesirable effects:

(*a*) Destructive sparking on the commutator; and

(*b*) Excessive losses in the armature.

By inserting *resistance leads* between the commutator segments and the armature coils the commutation is improved

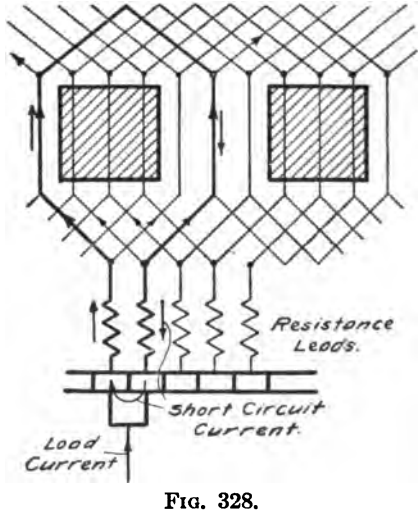



FIG. 328.

and the heat losses are reduced. From Fig. 328 it should be noted that two resistance leads are in series in the short-circuit carrying the induced currents, while the same two resistance leads are in parallel in the load-current circuit. By this means the short-circuit currents and the accompanying commutator sparking are reduced until satisfactory commutation is secured. With the decrease of the short-circuit current the $RI^2$ losses in the armature are also reduced, while the resistance leads add a new heat-loss factor to the circuit. In well-designed motors the sum of the losses in both the resistance leads and the armature windings is less than in the armature alone, if the resistance leads

were removed. The resistance leads also improve the starting torque and provide a certain protection against higher harmonics and other disturbing influences. The chief objection to the resistance leads is the increased complication in the design. The armature must needs be compact and the introduction of a resistance section between the commutator and the active portion of the armature winding under the field flux is a serious handicap to the designer. In motors operating on low frequencies, 15 cycles or less, satisfactory commutation can be secured without the use of resistance leads. At 25 cycles, the standard fre-

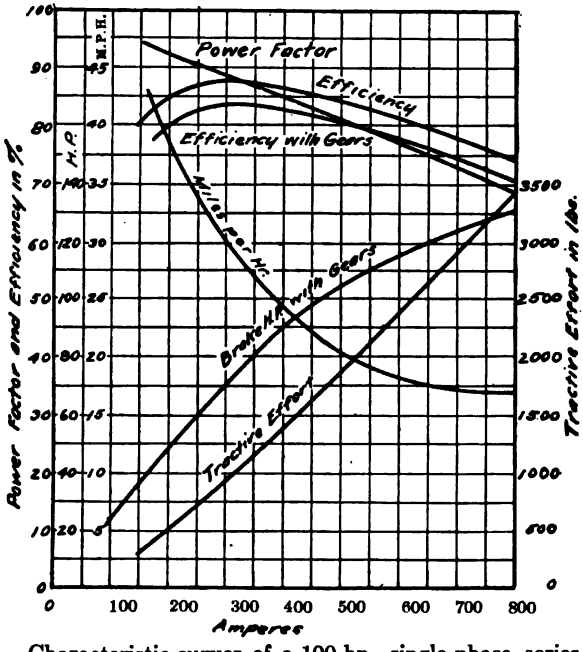



FIG. 329.—Characteristic curves of a 100-hp., single-phase, series, compensated, railway motor. 245 volts, 25 cycles. (*Westinghouse Elec. & Mfg. Co.*)

quency for electric railways, most motors are supplied with resistance leads, although some designers consider their use an unnecessary complication. For circuits having 40 or more cycles per second, resistance leads are necessary for successful operation.

Summarizing the more important features in the design of the compensated straight series motor as compared to the corresponding direct-current series motor we have:

(*a*) Both field and armature cores are laminated.

(*b*) Larger armature diameter.

(*c*) Field poles are short and stubby; more in number and of greater cross-section.

(*d*) Commutator longer, of larger diameter and of greater number of segments.

(*e*) Small air gap.

(*f*) Compensating winding.

(*g*) Resistance leads.

Typical characteristic curves of a compensated, series, alternating-current, railway motor are shown in Fig. 329. It should be noted that this motor will operate in a highly satisfactory manner on direct-current circuits. It is, in fact, a better but more expensive machine than the ordinary direct-current, series motor.

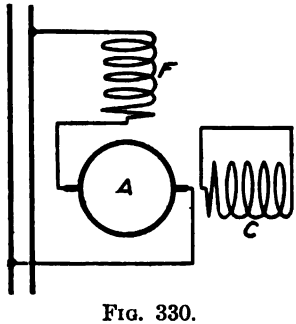



FIG. 330.

(*c*) **Inductively Compensated Series Motor. Primary Excitation.**—In the discussion of the induction motor it was shown that aside from delivering mechanical power the motor operated like a transformer. Similarly in a series motor the alternating fields induce voltages in the several circuits in the same manner as in the stationary transformer. With the compensating winding placed in grooves, parallel to the armature conductors, currents for neutralizing the armature cross-field may be obtained by short-circuiting the compensating winding as shown in Fig. 330. The inductively compensated series motor possesses practically the same operating characteristics on alternating-current circuits as the straight series motor.

(*d*) **Inductively Compensated Series Motor, Secondary Excitation.**—By connecting the field in series with the compensated winding as in Fig. 331 the motor field is excited by the secondary circuit. The armature only is in the primary connected to the supply mains, while the voltage induced in the compensating coil sends current through both the main field and compensating windings. The current in the armature leads the main field flux on account of the large inductive reactance in the secondary, and the power factor of the motor is very low. Obviously the power factor may be improved by inserting a shunt across the field as indicated in Fig. 332, but this arrangement

also causes a marked loss in efficiency. The time phase of the armature current with respect to the main field produces a commutating field, which at synchronous speed is in magnitude and phase position of the required value to give good commutation. This motor can not be operated on direct-current circuits and

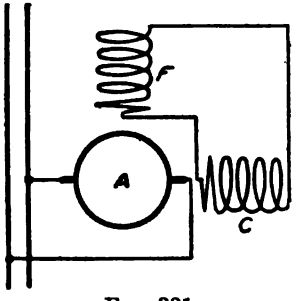



FIG. 331.

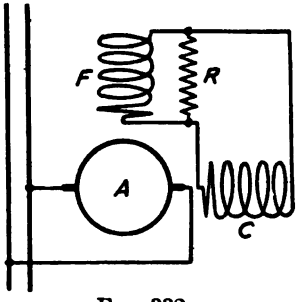



FIG. 332.

its commercial use seems to be limited to certain types of induction meters.

(*e*) **The "Repulsion" Motor.**—The main field or stator is connected to the supply mains while the wound armature is short-circuited through brushes placed 180 electrical degrees apart, as indicated by the circuit diagrams in Figs. 333 and 334;

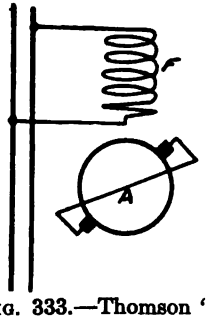



FIG. 333.—Thomson "repulsion" motor.

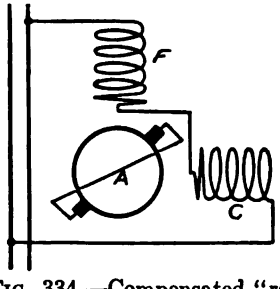



FIG. 334.—Compensated "repulsion" motor.

a compensating winding may be connected in series either with the stator or armature winding. The "repulsion" motor is fundamentally a transformer, with the primary winding and core stationary. The secondary is like an ordinary direct-current armature, in which the armature conductors and core rotate, while the short-circuited brushes are stationary. Hence,

while the conductors in the secondary winding rotate, the secondary circuit remains in a definite position with respect to the stator. In the "repulsion" motor as in the single-phase induction motor, there are transformer and speed

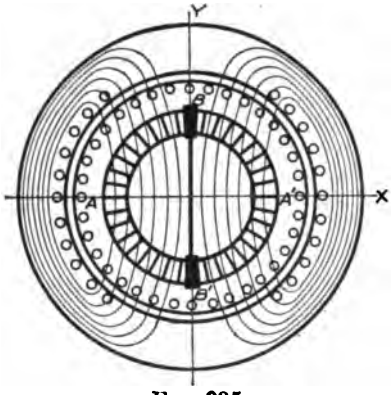



FIG. 335.

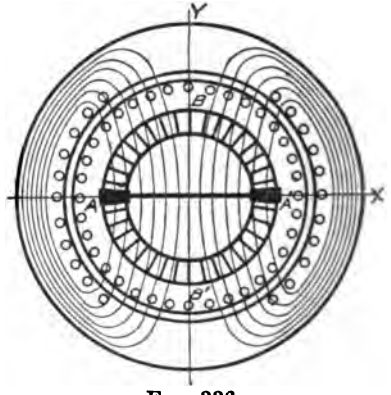



FIG. 336.

fields, whose reactions with the armature currents produce the desired torque.

At standstill the speed field is zero and the starting torque is produced by the reactions between the induced armature current and the stator field. The magnitude of the starting torque depends upon the relative position of the armature brushes and the direction of the stator field. If the brushes are in line or at right angles to the direction of the stator field flux, no torque is produced, as may be seen by inspection of Figs. 335 and 336. The magnetic poles are along the $Y$-axis and the maximum transformer action occurs at $AA'$ along the $X$-axis.

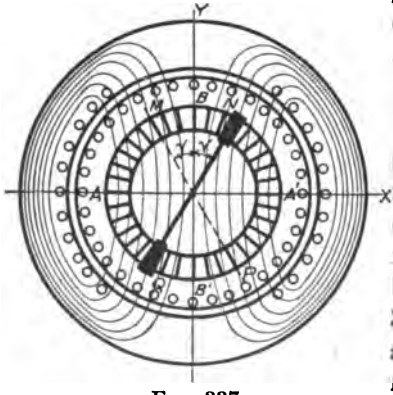



FIG. 337.

Any reaction between the field and the induced armature current in sector $AB$, Fig. 335, is at all instants balanced by the equal and opposite torque in the $BA'$ sector. Similarly, the torque in $AB'$ is at all instants equal and opposite to the reaction in the $B'A'$ sector. The voltage induced from $A$ to $B$, Fig.

336, is neutralized by the induced voltage from $B$ to $A'$. Similarly, $AB'$ is neutralized by $B'A'$. Hence there is no difference in voltage between the brushes, Fig. 336, no armature current flows and therefore there is no torque.

Let the short-circuited brushes be displaced by an angle $\gamma$ from the axis of the stator field, as in Fig. 337, with the line $NQ$ and the symmetrically placed line $MP$ $2\gamma$ apart. Comparing Fig. 337 with Figs. 335 and 336 it is seen that the conditions for the conductors in the areas $NA'P$ and $MAQ$ in Fig. 337 are the same as for all the conductors in Fig. 335; while the conductors lying in the areas $MBN$ and $PB'Q$ have the same conditions as all the conductors in Fig. 336. Hence in the arcs $NP$ and $MQ$ the transformer action of the primary field induces voltages that cause currents to flow through the armature and the short-circuited brushes. No torque is, however, produced as the reaction in the arc $NA'$ is balanced by the equal and opposite action in $PA'$, and similarly for $MA$ and $QA$. In the arcs $MBN$ and $PB'Q$ the resultant transformer action is zero since the voltage induced in arc $MB$ is neutralized by the equal and opposite voltage induced in $BN$; likewise $QB'$ is balanced by $B'P$. However, the currents flowing in the arcs $MBN$ and $QB'P$, Fig. 337, produced by the transformer action in $MAQ$ and $NA'P$ react upon the stator field and produce a starting torque. In the conductors between $MB$ and $BN$ the currents flow in the same direction and react upon the same field, while on the opposite side between $Q$ and $P$ the relative directions of both the current and field flux are reversed. Hence the starting torque produced at $B'$ is in the same direction as at $B$ and both tend to cause rotation in the same direction. For successive half waves of the impressed voltage both the primary field and the induced armature currents reverse in direction, hence producing a torque continuously in the same direction.

In analyzing the reactions between the armature and the field it is simpler to let the line joining the brushes be the reference axis and to separate the primary field into components in phase and in quadrature with the armature winding, as shown in Fig. 338. The component, $_t\phi_p$, of the primary field, $\phi_p$, in line with the brushes is the transformer field which induces voltages that cause currents to flow in the armature. The quadrature component, $_f\phi_p$, is the main field; the reactions between the armature currents and the main field $_f\phi_p$ produce the starting torque.

$$_t\phi_p = \phi_p \cos \gamma \tag{516}$$

$$_f\phi_p = \phi_p \sin \gamma \tag{517}$$

$$\frac{_t\phi_p}{_f\phi_p} = \cot \gamma \tag{518}$$

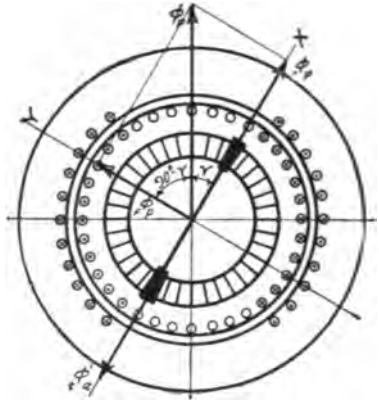


FIG. 338.

The induced armature currents also produce a field $_t\phi_a$ in opposition to the primary transformer flux $_t\phi_p$. If the transformer action were perfect with both resistance and leakage

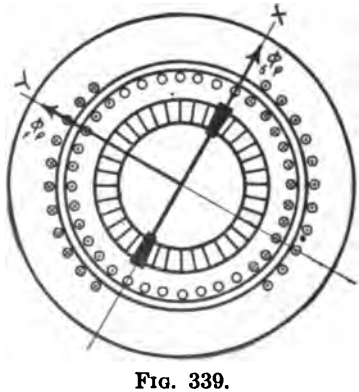


FIG. 339.

reactance zero, that is, neglecting all losses, then $_t\phi_a = {}_t\phi_p$ and the total flux in the motor would be $_f\phi_p$. Hence for an "ideal" repulsion motor at standstill the total voltage impressed on the primary circuit is balanced by the voltage induced by the

flux ${}_t\phi_p$ in the stator. In the actual motor the armature resistance, leakage reactance and iron losses absorb part of the impressed voltage and the flux ${}_t\phi_p$ is correspondingly larger than ${}_t\phi_a$. The armature current is in space-phase and practically in time-phase with the flux ${}_f\phi_p$, and the reaction produces a strong starting torque.

When the armature rotates, a speed field is produced in the same manner as explained for the single-phase induction motor in Chap. XIII. Neglecting losses and assuming an "ideal" motor, the speed field is in space quadrature to the primary field as in Fig. 339. Under the assumed conditions the speed field must produce a counter e.m.f. that just balances the armature speed voltage. At synchronous speed, $S$, the speed field is equal to the primary field. At any other speed, $s$, the speed field is:

$$ {}_s\phi_p = \frac{s}{S}\,{}_f\phi_p \qquad (519) $$

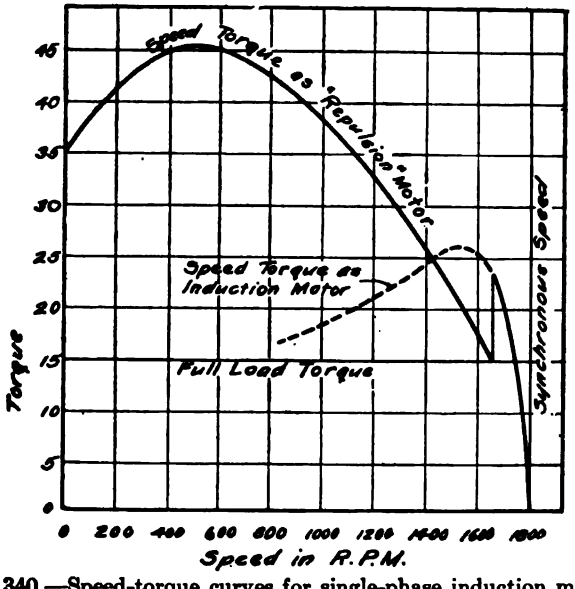



FIG. 340.—Speed-torque curves for single-phase induction motor ("repulsion" starting).

The speed voltage is in time phase with the flux ${}_f\phi_p$ and hence in time quadrature with ${}_s\phi_p$. Simultaneous values are therefore expressed by equations 520 and 521.

$$_f\phi_p = {}^{M}{}_f\phi_p \cos \omega t \tag{520}$$

$$_s\phi_p = \frac{s}{S}\,{}^{M}{}_f\phi_p \sin \omega t \tag{521}$$

The total field:

$$\phi_{total} = {}_f\phi_p + j_s\phi_p \tag{522}$$

The voltage impressed upon the motor must be proportional to the corresponding fluxes, and consists of two components in time quadrature; one, $_fE_p$, for $_f\phi_p$, and the other, $_sE_p$, for $_s\phi_p$.

$$\dot{E}_o = {}_f\dot{E}_p + j_s\dot{E}_p; \text{ or } E_0 = \sqrt{{}_fE_p^2 + {}_sE_p^2} \tag{523}$$

The "repulsion" motor has the characteristics of a straight series motor. To a limited extent it has been used for commercial work on loads requiring series motors. It is widely used as a device to give starting torque to single-phase induction motors. A typical starting and operating torque curve is shown in Fig. 340 for a motor that starts as a "repulsion" motor and automatically changes to a squirrel-cage induction motor when the speed approaches synchronism.

**(f) Single-phase Constant-speed Motor (*Wagner Type BA*).**—The circuit diagram is shown in Fig. 341. The stator has a main inducing winding 4, while the rotor is provided with a commuted winding 5, short-circuited by means of the permanently interconnected brushes 6, 7, along an axis displaced from the stator winding. A centrifugally operated device, 9, short-circuits a number of points on the commuted winding, such as 8, when synchronous speed is approached. A device is also provided that automatically lifts the brushes, 6, 7, off the commutator as soon as the short-circuiting device has interconnected most of the points of the rotor winding. The characteristics of the motor are thereby changed from series to shunt and it operates at nearly constant and nearly synchronous speed for all loads. In practice, the centrifugal device is located within the rotor at the opposite end from the commutator. Lifting the brushes reduces the wear on the commutator and practically eliminates the noise under normal operation. The motor is usually started and

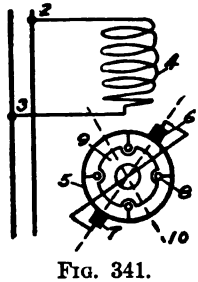


FIG. 341.

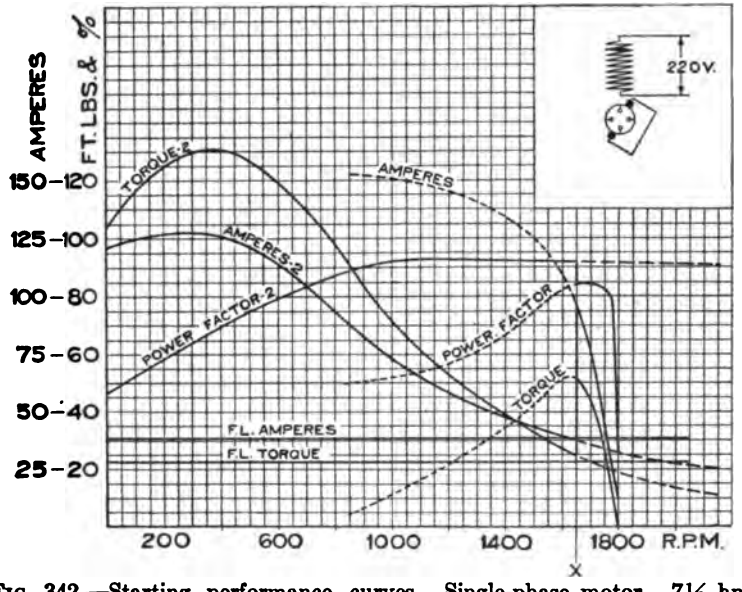



FIG. 342.—Starting performance curves. Single-phase motor. 7½ hp., 60 cycles, 4 pole, 220 volts. Type BA. (*Wagner Electric & Mfg. Co.*)

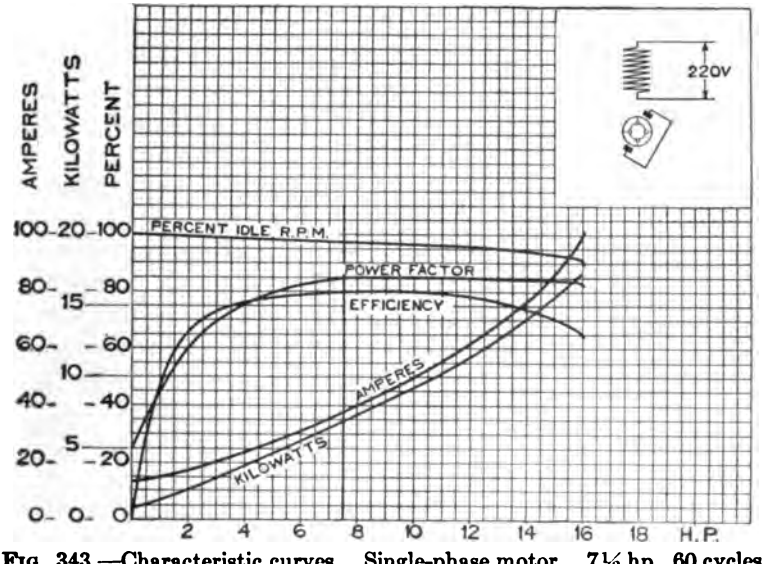



FIG. 343.—Characteristic curves. Single-phase motor. 7½ hp., 60 cycles, 4 pole, 220 volts. Type BA. (*Wagner Electric & Mfg. Co.*)

stopped by means of a single double-pole switch, except in some cases. An ordinary direct-current resistance starter is sometimes used in connection with the larger motors.

The starting performance curves are shown in Fig. 342 for the machine directly connected to the mains. From the start until the commutator is short-circuited the motor has series characteristics, the torque varying with the speed as shown by the curve *Torque*-2. The corresponding current and power factor values are shown by curves marked *Amperes*-2 and *Power Factor*-2 respectively. After the commutator has been fully short-circuited, the performance of the machine is indicated by the curves marked *Torque*, *Amperes* and *Power Factor*. The centrifugal device is set so as to operate at the speed marked $x$ in the figure, 1,650 r.p.m. When the short-circuiting of the commutator takes place the torque, from a value slightly in excess of full-load torque (Torque-2 curve) suddenly rises to over 50 ft.-lb. (torque curve) and then rapidly drops to whatever value the load requires. This change takes place in the short time interval required by the motor to rise in speed from 1,650 to 1,750 r.p.m. Likewise the current rises momentarily from 38 amp. to about 94 amp. and then rapidly falls to the value corresponding to the load on the motor.

The operating characteristics are shown in Fig. 343.

(*g*) **Single-phase Squirrel-cage Motor** (*Wagner Type BK*).—The disadvantages due to the lack of starting torque and low power factor at all loads inherent in the single-phase induction motor have been eliminated in the design of the single-phase squirrel-cage motor having both commuted and squirrel-cage windings with a double set of brushes spaced 90 electrical degrees on the commutator. Photographs of an assembled and dissembled motor are shown in Fig. 344. The circuit diagram is shown in Fig. 345 (*a*) and a section of the iron core showing the slots for the two windings and the intervening magnetic layer in Fig. 345 (*b*).

"The motor[1] consists of a rotor carrying a commuted winding, 6, located in a number of partly open slots, 18, placed near the outer periphery of the rotor laminations, 16, as shown in Fig. 345; and also a squirrel-cage winding the bars, 11, of which are located in holes 17, provided in the rotor punchings and interconnected at each end by

[1] FYNN, "Single-phase, Squirrel-cage Motors," *Trans.* A. I. E, E., 1915, Vol. XXXIV, page 2483.

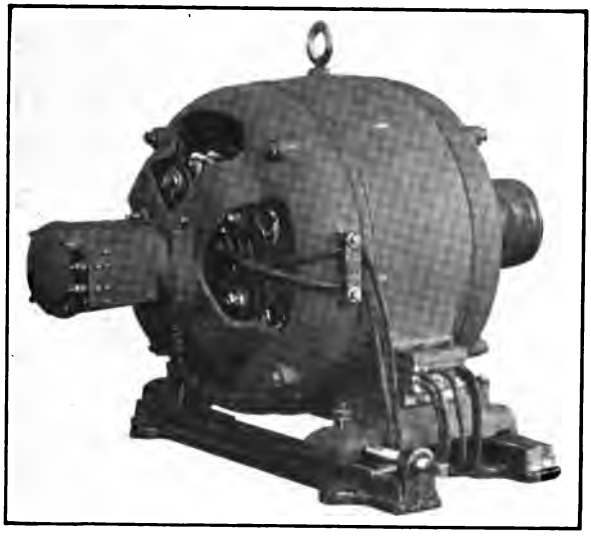


FIG. 344*a*.

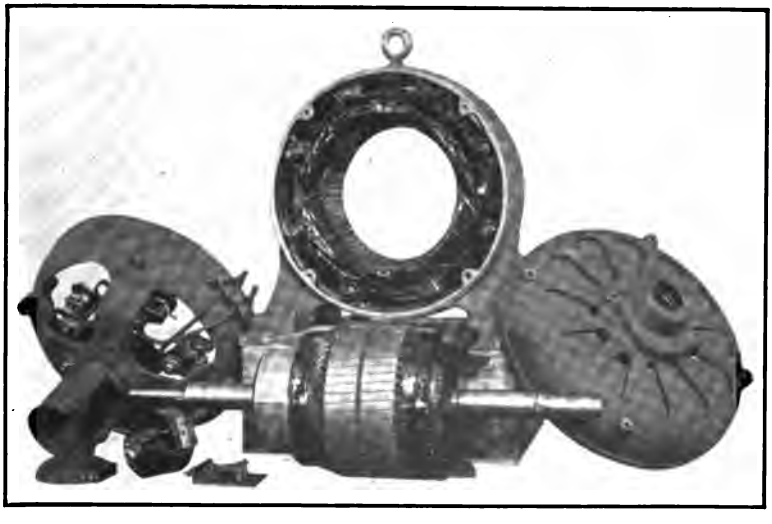


FIG. 344*b*.—Single-phase, squirrel-cage motor with commuted winding. Type BK. (*Wagner Electric & Mfg. Co.*)

means of conducting end rings, 12. The holes, 17, accommodating the squirrel-cage winding are so placed as to be separated by a certain amount of magnetic material, 20, from the slots, 18, carrying the

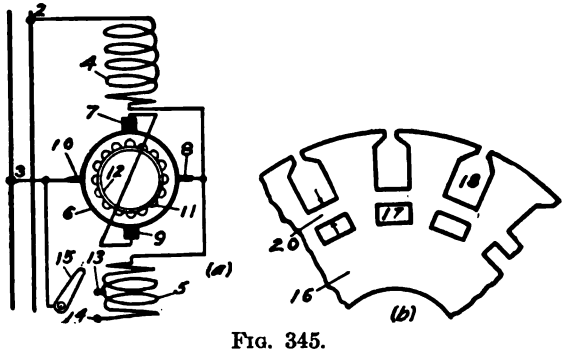



FIG. 345.

commuted winding." "The stator carries a main inducing winding, 4, and a coaxial compensating winding, 5, usually provided with a tap,

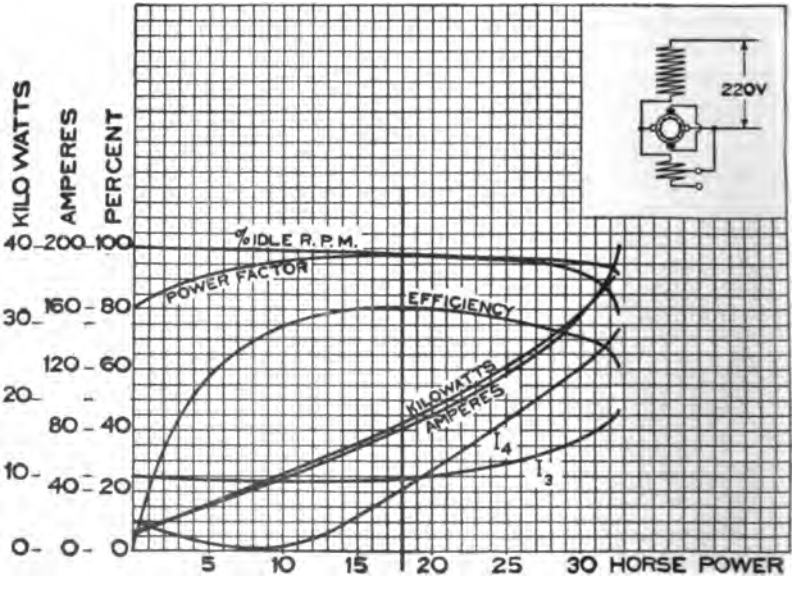



FIG. 346.—Characteristic curves. Single-phase motor. 18 hp., 60 cycles, 6 pole, 220 volts, 1200 r.p.m. Type BK. (*Wagner Electric & Mfg. Co.*)

13. The main or working brushes, 7, 9, are permanently short-circuited and placed in line with the axis or the stator windings. The

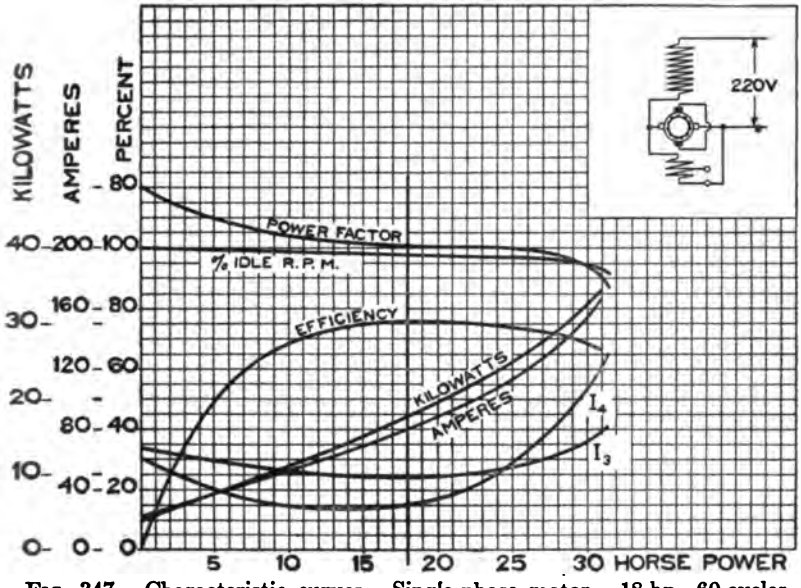



FIG. 347.—Characteristic curves. Single-phase motor. 18 hp., 60 cycles, 6 pole, 220 volts, 1200 r.p.m. Type BK. (*Wagner Electric & Mfg. Co.*)

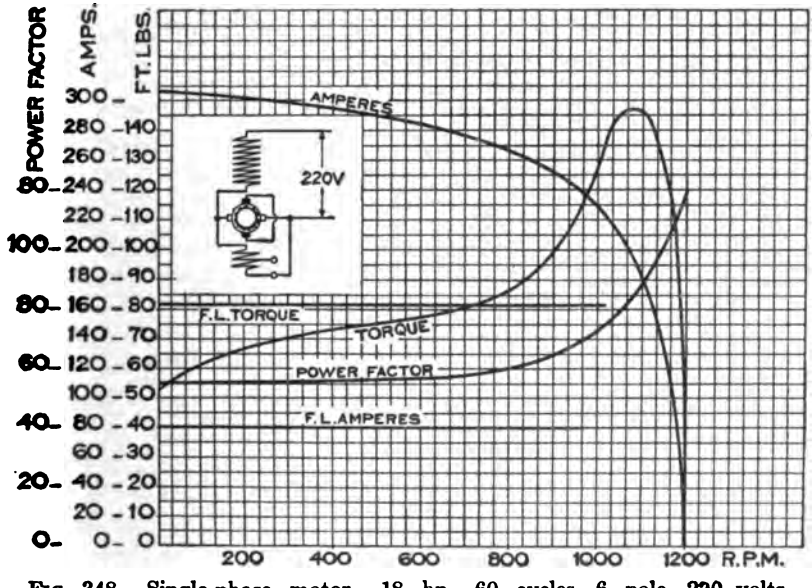



FIG. 348.—Single-phase motor. 18 hp., 60 cycles, 6 pole, 220 volts, 1200 r.p.m. Type BK. (*Wagner Electric & Mfg. Co.*)

auxiliary or exciting brushes, 8, 10, are displaced by 90 electrical degrees from the short-circuited brushes." At starting, the main stator winding, 4, is connected across the mains, 2, 3, in series with the commuted rotor winding, 6, by way of the exciting brushes, 8, 10. The brushes 7, 9, are short-circuited, and the circuit of the winding, 5, is open, this being indicated in Fig. 345*a* by showing the switch 15 in its "off" position. After the motor has reached its normal speed switch 15 is placed either on point 13 or point 14. If connected to the tap in the compensating winding, 5, the machine will operate with unity power factor under most loads. If switch 15 connects brush 10 to the end of 14 of the compensating winding 5, then the machine will operate with leading power factor at no load and with unity power factor at full load."

The corresponding characteristic operating curves are shown in Figs. 346, 347 and 348. The curves for the two connections differ mainly in the power factor; for maximum compensation the current leads at starting and at light loads. The efficiency is slightly higher for the normal compensation.

(*h*) **Other Types.**—Over 40 distinct types[1] of alternating-current commutator motors are known and quite a number have been manufactured and found to give satisfactory service. Compared to the polyphase, squirrel-cage induction motor all are complicated in design and as a consequence the first cost and maintenance charges are higher.

[1] FYNN, "Characteristics of Alternating-current Motors," *Trans.* A. I. E. E., 1915, Vol. XXXIV, page 1349.

### PROBLEMS

**1.** Draw the vector diagram for a series excited, inductively compensated alternating current commutator motor. Circuit diagram as in Fig. 330.

**2.** Draw the vector diagram for an inductively compensated and excited and series fed alternating current commutator motor. Circuit diagram as in Fig. 331.

For Experiments see page 547.

## CHAPTER XVIII

## INDUCTION OR ASYNCHRONOUS GENERATOR. GENERAL TRANSFORMER

(*a*) **The Induction Generator.**—In the discussion of the induction motor, Chap. XIII, it was shown that the speed of the rotating field of an ordinary squirrel-cage motor is slightly greater than the speed of the rotor. The slip of the rotor makes the magnetic lines of force cut the conductors, thereby inducing voltages which cause currents to flow in the rotor circuits. The reaction between the rotating field and rotor currents transfers energy from the stator to the rotor. The magnetic flux is therefore the connecting link by which electric energy is transferred from the stator circuit and transformed into mechanical energy in the rotor.

Let the rotor of the induction motor be belted to a direct-current motor or any prime mover whose speed can be varied; and let the stator be connected to the mains of the supply circuit. If the speed is increased until the rotor runs in synchronism with the rotating field, no voltage is induced and no current flows in the rotor circuits. The energy necessary to overcome the friction and windage comes from the prime mover and the energy dissipated by the losses in the stator from the supply mains. If the speed is still further increased, the rotor revolves faster than the field, or above synchronism, and voltage is again induced and the rotor currents flow in the opposite direction from what was the case when operating below synchronous speed. If the rotor currents are reversed with respect to the field, the resulting reaction is also in the opposite direction, or energy flows from the rotor to the stator. Hence, when operating above synchronous speed the mechanical energy supplied to the rotor is transmitted magnetically across the air gap and delivered as electric energy in the stator circuit. At speeds above synchronism, therefore, the induction motor is a generator and transfers and transforms the mechanical energy from the prime mover into electric energy in the stator or primary circuit.

*Primary Frequency.*—When operating as an induction motor the rotor slip depends on the load, but variations in load and slip in nowise affect the primary frequency. In precisely the same way when the machine is operated above synchronism, as a generator, the amount of power transmitted depends on the excess over synchronous speed, but the relative rotor speed does not affect the frequency in the stator circuit.

Therefore, if operated in parallel with ordinary alternators, the speed of the rotor varies with the load and the machine does not run in synchronism with the other machines. The frequency of the current in the stator of the induction generator is the same as for the other alternators, and the division of the load depends upon the governors of the prime movers.

*Excitation.*—The induction generator is not self-exciting; the reactive energy for the rotating magnetic field must be supplied from an outside source to the stator winding When operating in parallel with other alternators delivering power to constant-potential mains the exciting current automatically adjusts itself to the requirements for variations in the load. The power component is supplied through the rotor by the prime mover of the induction generator, but the quadrature component or reactive power required for the magnetic field comes from the other machines through the mains. Hence, whether operating as an induction motor or as an induction generator, the reactive power in the stator circuit must be supplied from an outside source. Operating in connection with over-excited synchronous motors is a desirable arrangement for induction generators as well as for induction motors. It is evident that the required reactive power could be supplied by stationary condensers The reactive power would oscillate between the condenser and the field of the induction generator with double the frequency of the voltage, appearing alternately in the magnetic field of the generator and the diclectric field of the condenser. The cost of the condensers would in many cases be prohibitive for using this arrangement in commercial plants, since the required reactive power is approximately 25 per cent. of the generator capacity.

*Circle Diagram.*—Since the induction generator is simply a reversed induction motor, having the same electric and magnetic circuits, the current locus for the generator is merely an extension of the locus for the induction motor. Draw the circle

diagram for the machine as an induction motor, Fig. 220, and complete the circle as shown in Fig. 349. The upper half of the diagram is the current locus of the induction motor, or for speeds below synchronism, while the lower half gives the corresponding current locus for speeds above synchronism, or, for the machine operating as an induction generator.

$MO = {}_mI$, the magnetizing current.
$P'M = I_1$, the stator current.
$P'O = I_{21}$, the rotor current.

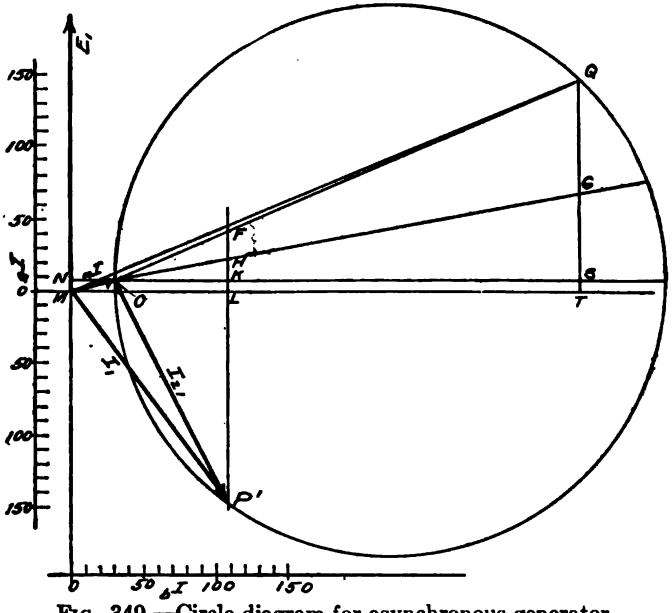



FIG. 349.—Circle diagram for asynchronous generator.

If the diagram represents values per phase, to obtain the corresponding quantities for the motor or generator multiply by $n$, the number of phases.

Under the same assumptions and approximations as for the induction motor diagram, Fig. 220, the corresponding values in watts may be obtained for the induction generator from the diagram in Fig. 349.

$$nE_1(P'F) = \text{mechanical input} \tag{530}$$

$$nE_1(MN) = \text{core loss, windage and friction} \tag{531}$$

$nE_1(HK)$ = primary copper loss (532)

$nE_1(HF)$ = secondary copper loss (533)

$nE_1(P'F - FL)$ = electrical output = $nE_1(P'L)$ (534)

Cos $P'ME_1$ = power factor of stator circuit (535)

$\frac{P'F - FL}{P'F}$ = efficiency = $\frac{P'L}{P'F}$ (536)

$ML = jb_1E_1$ = quadrature component of $I_1$ (537)

$nE_1(ML)$ = reactive power supplied by the mains (538)

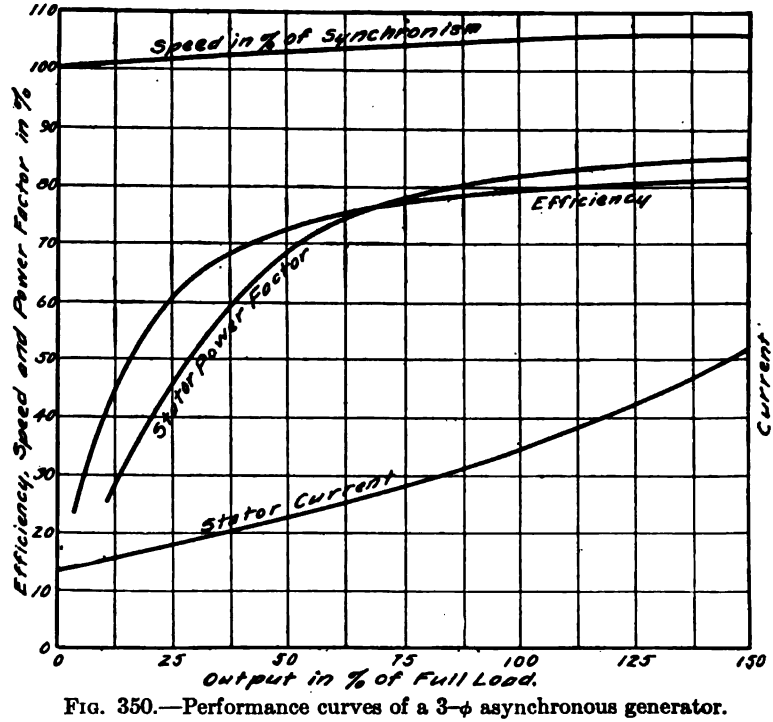



FIG. 350.—Performance curves of a 3-$\phi$ asynchronous generator.

From the circle diagram the performance curves, Fig. 350, may be calculated in the same manner as explained for the induction motor.

(*b*) **The General Transformer.**—In the stationary transformer the voltage is raised or lowered but the frequency remains the same in the two circuits. The electric energy received in the

primary is transmitted by the magnetic circuit to the secondary and transformed to electric energy at a higher or lower voltage but without change of frequency. In the induction motor the electric energy received in the stator is transmitted by the magnetic field across the air gap and transformed into the mechanical form and delivered through the rotor to the load. From the slip rings of a wound-rotor induction motor electric energy may be obtained in the same manner as from the secondary of the stationary transformer. Connecting the slip rings to an outside electric circuit, Fig. 351, the machine is both a motor and a transformer and both mechanical and electric power may be obtained simultaneously. The frequency of the voltage and current in the secondary circuit from the slip rings need not be the same as in the stator, but depends upon the relative speed of the rotating field and rotor. If the rotor is

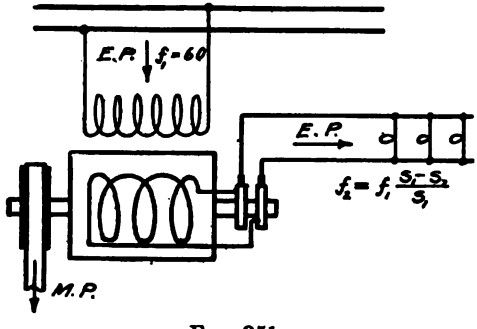



FIG. 351.

stationary the frequency of the secondary current from the slip rings is the same as in the primary and the machine is a stationary transformer with a large leakage flux.

Let the speed of the rotor be controlled independently of the rotating magnetic field, by belting to a variable-speed motor, which can operate in either direction. If the rotor revolves in the same direction as the stator field, the frequency of the current from the slip rings is proportional to the difference in the speeds. If the rotor is forced to rotate in the opposite direction, that is, run backward, the frequency of the secondary current is proportional to the sum of the speeds. Hence, if the backward rotation is considered negative, the frequency of the secondary currents is in all cases proportional to the algebraic difference in the speeds of the rotating field and the rotor.

$f_1$ = frequency of the primary or stator circuit.
$s_1$ = speed of the primary or rotating field.
$f_2$ = frequency of the secondary or rotor slip-ring circuit.
$s_2$ = speed of the rotor.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1}. \tag{539}$$

The voltage in the secondary depends upon the relative number of turns in the stator and rotor circuits and upon the relative speeds. Neglecting losses in the machine:

$$E_2 = E_1 \frac{n_2(s_1 - s_2)}{n_1 s_1} \tag{540}$$

Let ${}_s x_2$ = reactance of the secondary circuit at standstill, and $x_2$, the corresponding reactance at speed $s_2$.

$$x_2 = \frac{(s_1 - s_2)}{s_1} {}_s x_2 \tag{541}$$

$$\dot{I}_2 = \frac{\dot{E}_2}{Z_2} = \frac{\dot{E}_1 \frac{n_2}{n_1} \frac{(s_1 - s_2)}{s_1}}{\left[r_2 + j {}_s x_2 \frac{(s_1 - s_2)}{s_1}\right]} \tag{542}$$

The power relations of the several circuits, or the direction of the energy flow depends upon the relative speed of the rotating field and rotor. The reactive power, both for the stator and the rotor circuits comes from the primary mains at all rotor speeds.

The reaction between the rotating field and the rotor currents necessarily consists of two equal and opposite forces. With the forces equal, the work done by the field or by the rotor is directly proportional to their speeds. Neglecting losses in the machine, the power and speed relations may be grouped into four divisions.

1. For rotor speeds above synchronism.

$$s_2 > s_1.$$

The machine is an asynchronous generator and the mechanical power supplied through the rotor pulley is transformed into electric energy in both the primary and secondary circuits. The relative frequency in the two circuits is given in equation (543).

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{543}$$

2. For speeds between standstill and synchronism.

$$0 < s_2 < s_1.$$

The total power is supplied from the primary circuit. The rotor delivers mechanical power. The frequency of the current in the rotor circuit is proportional to the slip.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{544}$$

3. At standstill.

$$s_2 = 0.$$

With both windings stationary the machine operates as a stationary transformer. On account of the large air gap the magnetizing current is comparatively large, but all the power is supplied by the primary and appears in the electric form in the rotor secondary.

$$f_2 = f_1 \tag{545}$$

4. The rotor running backward.

$$s_2 < 0.$$

The machine is a combination generator and transformer. Electric energy is received from the primary and mechanical energy through the rotor pulley. From both sources the energy flow is through the rotor and slip rings to the secondary circuit. The frequency of the secondary circuit is greater than in the primary and equal to the sum (algebraic difference) of the speeds. $s_2$ is negative.

$$f_2 = f_1 \frac{s_1 - s_2}{s_1} \tag{546}$$

In cases (2) and (3) all the losses in the machine are supplied from the primary circuit. In (1), near synchronous speed and until the mechanical power supplied through the pulley is equal to or greater than the losses, the difference comes only from the primary circuit. In (4) the losses are supplied from both circuits.

With a single winding on the rotor a single-phase current is delivered through the slip rings. From a rotor-wound polyphase the corresponding polyphase currents are delivered from the slip rings. The machine can be used as a phase transformer since the secondary winding is independent of the primary.

The extreme flexibility of a machine consisting of a primary and a rotating secondary is apparent. The general transformer can operate as a motor, generator, transformer, phase converter, or frequency changer, and may even perform most of these transformations simultaneously.

### PROBLEMS

**1.** The machine in problem 3, Chap. XIII, is run as an induction generator. Find the variations of torque, current, power factor and efficiency by means of the circle diagram from 100 to 125 per cent. speed and plot curves.

**2.** Show that in the transition in changing an induction machine from a motor to a generator by increasing the speed; for certain speeds power is supplied both mechanically and electrically. Calculate by "McAllister's Method" the losses at no load when they are supplied electrically and when they are supplied mechanically.

For Experiments see page 548.

## CHAPTER XIX

## INSULATION. THE DIELECTRIC CIRCUIT

**A. Insulation. Leakage Current.**—The chief function of the dielectric is to provide insulation between conductors so as to confine as much of the electric current as possible to narrow paths, and thus establish definite electric circuits. In this respect a good dielectric is simply a very poor conductor, and hence the possibility of producing well-defined electric circuits depends directly upon the relative conductivity of the materials used in the conductor and the surrounding dielectric. In most dielectrics the resistance is very high, or the conductivity is extremely low as compared to metals. Thus the resistance of glass, rubber, porcelain and similar good insulators is of the order $10^{21}$ times that of copper, an enormously large ratio. However, in general, experimental data tend to prove that although the resistance may be very large, whenever a difference of potential exists a current flows through the dielectric and thus forms an electric circuit. This is the *true leakage current.*

In constant-potential systems the dielectric forms an infinite series of parallel circuits, each with a very high resistance. In large systems the cross-section of the dielectric through which the leakage current flows is very large, and hence while at any part the resistance is very high, still over the total section a considerable leakage current may flow. The dimensions of length and cross-section of the dielectric are generally so irregular that it becomes impracticable to determine the average resistance by means of space measurements and the specific resistance of the material. As an illustration consider the comparatively simple case of the insulation between the several turns in a transformer winding. Any attempt to accurately measure the length and cross-section of the dielectric for determining the leakage conductance would be futile since the leakage current tends to flow between any two points, at a difference of potential, and not merely between adjacent layers. Hence, while the law for the leakage current is simple, the practical determination of

the average length and cross-section of the irregularly shaped dielectric, by space measurements, makes a direct application impossible. The total equivalent conductance of any machine or system is usually determined by measuring the leakage current for any applied voltage and then applying Ohm's law.

$$_{l}I = {}_{g}gE \tag{551}$$

The measurements are made with direct currents. In alternating currents the leakage current is in phase with the voltage, and the power lost is therefore:

$$_{l}P = {}_{l}IE = {}_{g}gE^{2} = {}_{r}r\,{}_{l}I^{2} \tag{552}$$

In all cases the leakage current represents a transformation of electric energy into heat, light or chemical reactions. Since the process is not reversible, the energy is not returned to the circuit when the voltage decreases; hence the loss represents a drain of energy from the electric circuit in the same manner as the resistance losses in the metallic conductor.

Leakage through the dielectric is always undesirable. The energy loss may be small and in most cases the heat generated gives no troublesome rise in temperature, but very often the leakage current causes changes in the chemical composition of the dielectric that rapidly destroy its insulating properties.

Three factors of special importance in affecting both the leakage conductance and the rate of deterioration, should be mentioned, namely:

1. Moisture.
2. Temperature.
3. Voltage gradient.

Even very small amounts of moisture affect the leakage conductance to a marked degree. The leakage current is greatly increased and chemical reactions follow that produce deterioration at a more or less rapid rate. In many cases, electrical apparatus can not be protected from moisture and the selection of the most serviceable insulation for the given conditions must be made. In other cases, like the oil insulation of transformers, extreme care is exercised in removing all the water, and in keeping the apparatus well protected against moisture.

In a recent paper describing an extended investigation on press-board insulation, the effect of moisture is summarized: "The weakening effect in press-board, and very likely other

water absorbent insulations, may increase as great as the 5th and 6th power of the per cent. absorbed moisture. When the free moisture is above 3 per cent., the weakening effects due to its presence are quite pronounced."[1]

The temperature limit for various kinds of electrical apparatus is in almost all cases based directly on the characteristics of the dielectric used. The temperature range for commercial operation is seldom above 100°C. and in most cases considerably less, because at higher temperatures the rate of chemical change becomes too great, causing a rapid deterioration of the insulating properties of the dielectrics. It is, therefore, the rate of chemical change in the dielectrics that limits the permissible temperature and the rating of most electrical machinery, as is readily seen from the Standardization Rules of the A. I. E. E. "The weakening effects in insulation, as shown by the dielectric losses, power factors, and currents, may increase as great as the 5th or 6th power of the temperature."[1]

The voltage gradient at any point in the dielectric is of greater importance than the total voltage difference between two conductors, in determining the stresses and resultant strains and possible ruptures in the dielectric. Dielectrics are *electrically elastic* and have a more or less definite *elastic limit*, above which the insulation breaks down or ruptures. In the irregularly shaped dielectric between conductors, the stresses are not uniform, and the elastic limit may be reached in the layers near the conductors at a much lower total voltage than at points farther away. Thus the rupture may begin at the surface of the conductor long before the average stress exceeds the elastic limit of the particular dielectric, as illustrated by the corona, the use of condenser terminals, etc. It is therefore necessary to so design electrical apparatus that at no point the voltage gradient exceeds the elastic limit of the dielectric, or to provide safety appliances that will protect the apparatus against excessive voltage gradients.

In solid dielectrics the rupture, produced by a stress in excess of the elastic limit, is a permanent deformation or break, while in liquids and gases the insulating properties of the dielectric are immediately restored as soon as the electric stress is sufficiently reduced. In solids the puncture causes a short-circuit, generally followed by a "burn-out," due to the excessive

[1] MINTON, A. I. E. E. *Proc.*, July 1915, 1165.

leakage current flowing through the ruptured dielectric. In gases and liquids the insulation is automatically restored after the short-circuit arc is broken.

Briefly, the existence of a leakage current premises imperfect insulation. Under ideal conditions with perfect insulation, no leakage current could exist, and all of the electric circuit would be confined to the conductors.

**B. The Dielectric Circuit.**—When the conductors are at a difference of potential, an electric stress is exerted upon the materials in the intervening space. This stress produces what is equivalent to a strain in the electrically elastic dielectrics. The product of stress and strain represents energy. At potentials less than the rupturing voltage the strain is proportional to the stress and hence the energy stored in the dielectric is proportional to the square of the impressed voltage.

$$\text{Dielectrically stored energy} = \frac{CE^2}{2} \text{ (see Chap. I)} \qquad (553)$$

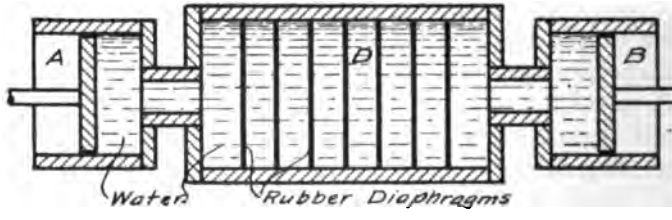



FIG. 352.

For voltages within the electrical elastic limit of the dielectric the process is reversible. Hence the energy flows from the electric circuit into the dielectric while the voltage increases, and returns to the electric circuit while the voltage decreases. This energy flow to and from the dielectric is separate and apart from the leakage current and follows independent laws. The leakage may interfere with the dielectric circuit, as both phenomena occur in the same space, but the basic processes are entirely different. A mechanical analogue will illustrate both the independence and the interference of the two sets of phenomena. Let three cylinders be arranged as shown in Fig. 352 with the pistons at the center position in cylinders *A* and *B*. Cylinder *D* is divided into a number of compartments by elastic rubber diaphragms and the space between the pistons is filled

with water. Move the two pistons in *A* and *B* to the right as shown in Fig. 353. A stress and consequent strain will appear in the rubber diaphragms, and energy is transferred from the pistons to the elastic rubber. If the rubber is perfectly elastic and frictionless, the energy stored in the diaphragm returns to the pistons upon removing the pressure applied by the pistons. Moving the pistons to the left, as in Fig. 354, energy is stored in the elastic diaphragms, and again returns to the piston during

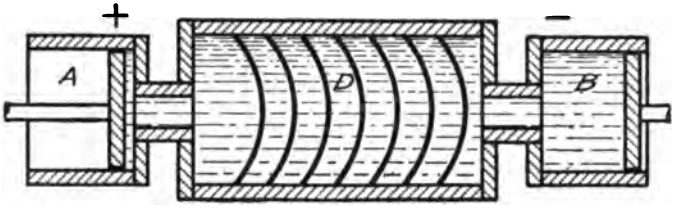



FIG. 353.

the return half cycle. This process is analogous to the energy flow in the dielectric circuit. If there are minute holes in the diaphragms allowing a little water to leak between the several compartments, this is analogous to the leakage current. With some water leaking through the diaphragms, the energy stored in the rubber in Figs. 353 and 354 would soon escape and the strain be removed. The energy thus transferred from the elastic rubber to the leaking water would not be returned to

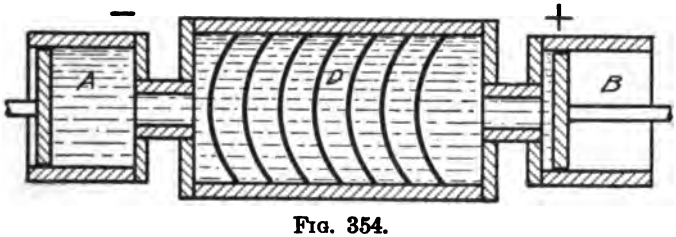



FIG. 354.

the pistons upon the removal of the applied force. The transfer and storing of energy in the mechanical model is analogous to the energy changes in the dielectric circuit and the leakage current. The one is due to the elastic properties of the dielectric and the other to imperfections in the insulation. With perfect insulation (no leakage) the dielectric circuit is free from interference; and conversely in conductors having little or no electric elasticity and no energy storage there is no dielectric circuit.

The strain in the dielectric is represented, both in magnitude and direction by dielectric lines of force, in precisely the same way as the intensity and direction of the magnetic field is represented by magnetic lines of force.

The elastance of the dielectric circuit corresponds to the reluctance of the magnetic circuit or the resistance of the electric circuit. Condensance (capacity) is the reciprocal of the elastance and hence corresponds to the permeance of the magnetic circuit or the conductance of the electric circuit. The unit of condensance is the farad (see Chap. I) and of the elastance the daraf.

$$\text{Elastance} = \frac{1}{\text{condensance}};\ S(\text{darafs}) = \frac{1}{C(\text{farads})} \qquad (554)$$

The specific condensance or permittivity, $\kappa$, is the ratio of the condensance of the given dielectric to that of air. The elastance of any given circuit also depends upon the dimensions of the dielectric and follows the same laws as the resistance of an electric circuit; that is, it varies directly as the length, $l$, and inversely as the cross-section, $A$.

$$S = \frac{4\pi l}{\kappa A};\ \text{or}\ C = \frac{\kappa A}{4\pi l}\ \text{abstatfarads (c.g.s. electrostatic units)} \qquad (555)$$

The factor $4\pi$ is introduced by the definition of the unit dielectric line of force (one line per cm.$^2$ on the surface of a sphere of unit radius).

To change from the c.g.s. electrostatic (abstat) to the c.g.s. electromagnetic absolute (ab) system of units, the constant $v^2$ must be introduced in the equations.

$$S = \frac{4\pi v^2 l}{\kappa A}\ \text{abdarafs; or}$$

$$C = \frac{\kappa A}{4\pi v^2 l}\ \text{abfarads (c.g.s. electromagnetic units)} \qquad (556)$$

The constant, $v = 3 \times 10^{10}$ cm. per sec. is the velocity of propagation of electric field in space, which is the same as the velocity of light.

The absolute electromagnetic units are reduced to darafs and farads by the factor $10^9$.

$$S = \frac{4\pi v^2 l}{\kappa A 10^9} = 113.1\frac{l}{\kappa A}10^{11}\ \text{darafs} \qquad (557)$$

$$C = \frac{\kappa A 10^9}{4\pi v^2 l} = 0.8842\frac{\kappa A}{l}10^{-13}\ \text{farads} \qquad (558)$$

$$= 0.8842\frac{\kappa A}{l}10^{-7}\ \text{microfarads.}$$

The relation between the voltage, flux and elastance or condensance in the dielectric circuit is the same as for the voltage, current and resistance or conductance of the electric circuit and hence may be stated in the same form as Ohm's law.

$$\text{Dielectric flux} = \frac{\text{voltage}}{\text{elastance}}; \qquad \psi = \frac{e}{S} \tag{559}$$

$$\text{Dielectric flux} = \text{condensance} \times \text{voltage}; \quad \psi = Ce \tag{560}$$

From 557 and 559:

$$\psi = \frac{\kappa A e 10^9}{4\pi v^2 l} \text{ lines of dielectric force}$$

$$= 0.8842\frac{\kappa A e}{l}10^{-13} \text{ lines of dielectric force} \tag{561}$$

and

$$e = \frac{4\pi v^2 l \psi}{\kappa A 10^9} \text{ volts} = 113.1\frac{l\psi}{\kappa A}10^{11} \text{ volts} \tag{562}$$

The potential or voltage gradient, $G$, in a dielectric circuit of constant cross-section and of homogenous material, is the ratio of the voltage and the length.

$$G = \frac{e}{l} \text{ volts per cm.} \tag{563}$$

The dielectric stress or field intensity in c.g.s. electromagnetic units:

$$K = \frac{G10^9}{4\pi v^2} \tag{564}$$

The flux density may be stated either as the permittivity times the field intensity or as the total flux divided by the cross-sectional area.

$$D = \kappa K = \frac{\psi}{A} = \frac{\kappa e 10^9}{4\pi v^2 l} \text{ for } e \text{ in volts} \tag{565}$$

The difficulty in applying the above equations to any dielectric circuit comes chiefly from the irregular shapes of the dielectric in electrical appliances. The dimensions are seldom simple geometrical forms. Often it is impracticable to determine either the cross-section or the length. Another factor is the small range in the permittivity of insulating materials, thus making it impossible to confine the dielectric flux to a definite path as may be done by both the electric and magnetic circuits.

TABLE XII

| Material | Permittivity | Material | Permittivity |
|---|---|---|---|
| Glass (easily fusible) | 2.0 to 5.0 | Air and other gases | 1.0 |
| Glass (difficult to fuse) | 5.0 to 10.0 | Alcohol, amyl | 15.0 |
| Gutta-percha | 3.0 to 5.0 | Alcohol, ethyl | 24.3 to 27.4 |
| Ice | 3.0 | Alcohol, methyl | 32.7 |
| Marble | 6.0 | Benzine | 1.9 |
| Mica | 5.0 to 7.0 | Benzol | 2.2 to 2.4 |
| Paper with turpentine | 2.4 | Olive oil | 3.0 to 3.2 |
| Paper or jute impregnated | 4.3 | Paraffin oil | 1.9 |
| Paraffin | 2.3 | Petroleum | 2.0 |
| Porcelain | 5.3 | Turpentine | 2.2 |
| Rubber | 2.4 | Bakelite | 6.6 to 16.0 |
| Rubber vulcanized | 2.5 to 3.5 | Micarta | 4.1 |
| Shellac | 2.7 to 4.1 | | |
| Silk | 1.6 | | |
| Sulphur | 4.0 | | |

Three important differences in the constants of the magnetic and dielectric circuits should be noted.

1. The permeability of soft steel or iron may reach several thousand while the permittivity of most dielectrics is less than ten. Hence while very great changes can be made in the strength of the magnetic field by using iron-clad circuits, only comparatively small changes can be secured in the dielectric field by using different materials. Table XII.

2. The permeability of iron and other magnetic materials varies widely for different magnetic densities, while the permittivity of most dielectrics is constant for all potentials up to the rupturing voltage. The dielectric field has no saturation point; and the strength of a magnetic field is not limited by a breakdown of the magnetic material.

3. The magnetic flux density may be increased indefinitely by increasing the m.m.f., for although the permeability, even in iron-clad circuits, is small above the saturation point, it is always positive. The practical limit for the magnetic density, in iron-clad circuits is the saturation point of the steel or iron used.

The dielectric field intensity increases directly with the impressed voltage until the insulation is pierced or breaks down. The rupturing voltage, therefore, limits the dielectric flux density.

(*a*) *Condensers in Series and in Parallel.*—Since the relation between the dielectric flux, voltage and elastance or condensance may be expressed by Ohm's law (559, 560) it follows that the total elastance or condensance may be found for series and

parallel arrangements in the same manner as the total resistance or conductance is found for similar electric circuits.

Thus in Fig. 355, the total elastance is the sum of the several elastances in series.

For series circuits:

$$S_t = S_1 + S_2 + S_3 + S_4 + \ldots + S_n \qquad (566)$$

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots + \frac{1}{C_n} \qquad (567)$$

Likewise for parallel circuits, as in Fig. 356, the sum of the several condensances is the total condensance.

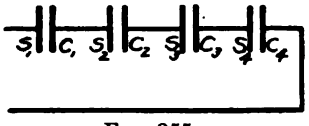


FIG. 355.



FIG. 356.

For parallel circuits:

$$C_t = C_1 + C_2 + C_3 + C_4 + \ldots + C_n \qquad (568)$$

$$\frac{1}{S_t} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \frac{1}{S_4} + \ldots + \frac{1}{S_n} \qquad (569)$$

The distribution of the voltage in series circuits and the current in parallel circuits is therefore the same as for the corresponding electric circuits.

(*b*) *Charging Current.*—The current, ${}_cI$, flowing in the electric circuit, supplying the energy to the dielectric circuit (corresponding to the magnetizing current for the magnetic circuit) is called the *charging*, condensance or capacity current. As energy is stored in and returned from the dielectric, the term *charging* current is appropriate; *but it should be kept clearly in mind that the energy supplied by the charging current is held within the dielectric and not located on the surface of the conductors.* As stated in Chap. IV, the charging current is proportional to the rate of change in the voltage; and hence for a sine voltage wave the current is of the same form and leads by 90°.

$$E = \frac{-j_c I}{2\pi f C} = -j_c x_c I \qquad (570)$$

Since the dielectric circuit is in parallel with the electric circuit

in the conductors, it is customary to express the relation between the voltage and the charging current by a susceptance instead of a reactance.

$$_cI = j_cbE \tag{571}$$

If there are no losses in the dielectric circuit the susceptance represents the reciprocal of the reactance, but with energy losses (series resistance, dielectric hysteresis, etc.) the equivalent resistance also enters the equations.

The reactive power, represented by the product of the voltage and the charging current, surges to and fro with double the frequency of the impressed voltage.

$$_rP = {}_cbE^2 = {}_cx_cI^2 \tag{572}$$

(*c*) *Energy Losses.*—In a constant dielectric field the energy loss in the dielectric is represented by the leakage. When an alternating voltage is impressed the energy loss is increased. This additional loss is in the dielectric circuit proper and is probably partly due to a molecular friction, similar to magnetic hysteresis, and partly to a viscous give in the elastic medium. From a series of tests in 1901, on commercial condensers of the paraffin-tinfoil type, Steinmetz[1] draws the following conclusions:

1. "With a sine wave of impressed e.m.f. the current in the condenser is directly proportional to the impressed e.m.f. and to the frequency; that is, the capacity of the condenser on an alternating current is constant, within the range of the tests (220 to 980 volts; 57 to 133 cycles)."

2. "The loss of the condenser is, within the range of the tests, proportional to the square of the impressed e.m.f.; that is, the power factor of the condenser at constant frequency is constant and independent of the impressed voltage."

3. "The power factor of the condenser does not seem to vary with the frequency; that is, the efficiency of the condenser appears to be independent of the frequency; or, in other words, the loss of energy per cycle is proportional to the square of the electrostatic field strength, but independent of the frequency."

4. "The power factor in well-made condensers is extremely low, averaging about 0.005 or the efficiency is about 99.5 per cent."

In a recent investigation[2] by means of the cyclograph more satisfactory measurements of the power losses were possible. For 60 cycles and a wide range of impressed voltage, "it is shown

[1] STEINMETZ, *Electrical World*, vol. XXXVII, 1065.

[2] MINTON, A. I. E. E. *Proc.*, July, 1915, 1165.

that the watts do not vary as the square of the voltage, but may vary from the 1.32 to the 2.52 power of the voltage." While the losses have been represented by a variety of empirical equations, no simple form seems to apply to all cases. Expressed in the form of a complex equation, with the conductance and susceptance constants determined for each case, the dielectric circuit admittance is:

$$Y_d = g_d + j_c b_d \tag{573}$$

In combination with the leakage the admittance of the circuits in the dielectric is:

$$Y_d = {}_lg + g_d + j_c b_d \tag{574}$$

The current flowing in the conductor supplying both the energy losses and the reactive power in the dielectric:

$${}_c\dot{I} = Y_d\dot{E} = ({}_lg + g_d + j_c b_d)\dot{E} \tag{575}$$

(*d*) *Residual Charge.*—Due to the heterogeneous nature of insulation materials and compositions, combined with a slow leakage, all the energy stored dielectrically is not released immediately upon short-circuiting the conductors. This is particularly the case with the use of direct currents when the voltage is applied for long periods in one direction. The energy stored in the dielectric at some distance from the conductor must be transmitted through the intervening layers. This transmission is not all made instantaneously and is often complicated by leakage of various amounts in the several layers, and hence an appreciable length of time is required both to fully energize and to discharge the dielectric. On this account a single discharge of a cable or other apparatus having considerable condensance may not be sufficient. The two sides should be short-circuited for some time or several "groundings" may be necessary to remove the stored energy, and thus prevent any dangerous shock to the workmen.

(*e*) *Dielectric Induction* (*Electrostatic Influence*).—The e.m.f. in an alternating circuit induces in all nearby conductors, by dielectric induction, voltage opposite in direction but proportional in magnitude. The induced voltage reacts upon the impressed e.m.f. in somewhat the same manner as the two currents in mutual magnetic induction. The induced voltages cause currents to flow in the conductors thereby changing the distribution of the

energy content in the system. As the impressed voltage is continually changing in value, the induced voltage is likewise either increasing or decreasing. This necessitates a flow of current in all nearby conductors with attendant conversion of some electric energy into heat by the $rI^2$ losses. The source for this heat energy is in the primary circuit. Therefore, the phenomenon of dielectric induction (electrostatic influence) increases both the condensance and the energy loss through the dielectric. That part of the charging current due to mutual dielectric induction may also be represented by components in phase and in quadrature with the impressed voltage, and the equivalent conductance and susceptance may be included in the notation used in equation (572).

(*f*) *Disruptive Strength. Piercing Pressure.*—When the voltage between two conductors is gradually increased the stress on the intervening dielectric likewise increases until the pressure is suddenly equalized by a discharge or formation of a short-circuit through the dielectric. If sufficient energy is supplied to the conductors the current continues to flow in the form of an arc at a comparatively low voltage, depending upon the resistance of the breakdown path. Instead of stating the disruptive strength of an insulating material in terms of the density of dielectric lines of force, it is customary to give the voltage at which a given thickness of the specified material will be punctured. This is called the *disruptive strength* or *piercing pressure*, and its value depends upon many factors. These may be grouped in two divisions: (*g*) factors affecting the specific dielectric strength of any given material; (*h*) factors affecting the voltage gradient at various points in the dielectric circuits.

TABLE XIII.—APPROXIMATE PIERCING PRESSURE FOR 1 MM. THICKNESS

| | |
|---|---|
| Boiled linseed oil | 8,000 volts |
| Dry wood fiber | 13,000 volts |
| Insulating varnish | 50,000 volts |
| Melted paraffin | 8,000 volts |
| Mica | 58,000 volts |
| Micanite | 33,000 volts |
| Oiled linen | 12,500 volts |
| Paraffined paper | 30,000 volts |
| Porcelain | 13,000 volts |
| Transformer oil | 9,000 volts |
| Turpentine | 6,500 volts |
| Vulcanized rubber | 10,000 volts |

(*g*) *Specific Dielectric Strength.*—The more important factors are: (1) material; (2) moisture; (3) temperature; (4) thickness; (5) mechanical stresses; (6) length of time the voltage is applied.

1. The specific piercing pressure varies widely for different materials, and bears no direct relation to the insulation resistance. Solids like rubber, glass, mica, porcelain or fiber have much higher disruptive strengths than liquids and gases. But in a solid a puncture is not readily repaired, while in liquids and gases the path of the disruptive spark is quickly filled and the dielectric automatically regains its normal strength. For definite chemical compounds, the piercing pressure for a given thickness

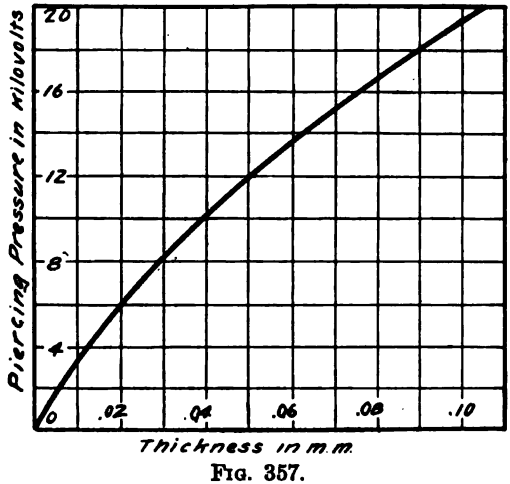



FIG. 357.

is fairly constant, but many insulating compositions are mere physical mixtures and a lack of uniformity necessarily produces a great variation in the disruptive strength.

2. The presence of moisture, especially in solids and liquids, decreases the disruptive strength. As most materials are more or less hygroscopic, the moisture content must be carefully determined.

3. The temperature within ordinary limits has little effect upon the disruptive strength, provided no chemical changes are produced in the material. For commercial operation the permissible rise in temperature is definitely limited so as to prevent chemical changes in the material.

4. The disruptive strength is not directly proportional to the thickness of the dielectric. Usually the thickness increases faster than the piercing pressure. This is illustrated for mica in Fig. 357. For very thin layers the variation is generally the reverse.

5. In the presence of mechanical stresses the piercing pressure is decreased.

6. The length of time during which the pressure is applied has considerable effect upon the piercing pressure. There seems to be something similar to fatigue or a slightly viscous give in the material, requiring a few seconds or minutes to reach equilibrium. In testing insulation on commercial machines it is customary to apply the pressure for one minute when they are warm, as this provides the necessary time duration and also conforms with working temperature conditions.

In some materials like oiled press boards it requires several minutes for the fatigue to reach a permanent value at any given voltage. For practical application of the factors affecting the dielectric strength a number of empirical formulas have been devised. In commercial work the data given in handbooks and the Standardization Rules of the A. I. E. E. are the main guides in determining the permanent limits for electrical designs.

(*h*) *The Voltage Gradient.*—The voltage gradients in the space surrounding conductors at a difference of potential depend directly on the relative density of the dielectric lines of force. The factors that determine the distribution of the dielectric flux may be discussed under four divisions.

1. One homogeneous dielectric.

2. Two or more insulating materials forming parallel dielectric circuits.

3. Two or more insulating materials forming series dielectric circuits.

4. Combinations of series and parallel circuits.

1. *One Dielectric.*—In the space surrounding two conductors, at a difference of potential, the distribution of the lines of force depends directly upon the geometric form or the dimensions of the dielectric. In dielectric circuits of simple form the flux density and voltage gradient may be readily calculated.

*First Example. Lead-covered Cable.*—Given a single conductor, lead-covered cable with an insulating dielectric of one homogeneous material. When a difference of voltage exists between

the conductor and the lead sheath, the direction of the stress, Fig. 358, and hence the flux is radial. Since all the lines connect the conductor and the lead sheath, the density is greatest at the surface of the conductor and least at the lead sheath. Concentric cylinders form equipotential surfaces. From equation (562), Fig. 359, the voltage required to pass the flux, $\psi$, through the elemental cylinder 1 cm. long, of $(dx)$ thickness and at $x$ distance from the center is:

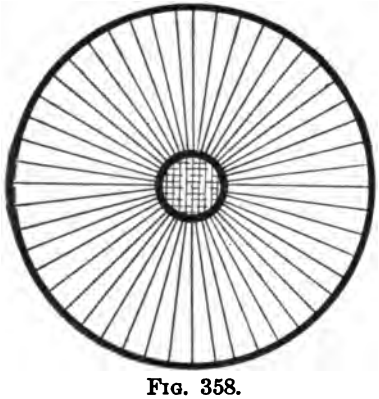


FIG. 358.

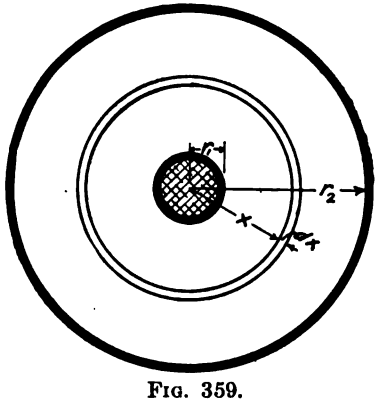



FIG. 359.

$$de = \frac{4\pi v^2 \psi\, dx}{\kappa 2\pi x\, 10^9} = \frac{18\psi 10^{11}}{\kappa x}\, dx \text{ volts} \tag{576}$$

Let $r_1$ = radius of conductor; and
$r_2$ = radius (inside) of lead sheath.

The voltage between the conductor and sheath is, therefore:

$$e = \frac{2\, v^2 \psi}{\kappa 10^9} \int_{r_1}^{r_2} \frac{dx}{x} = \frac{2\, v^2 \psi}{\kappa 10^9} \log_\epsilon \frac{r_2}{r_1} \text{ volts} \tag{577}$$

$$\psi = \frac{\kappa e 10^9}{2v^2 \log_\epsilon \frac{r_2}{r_1}} = Ce \text{ lines per cm. length of cable} \tag{578}$$

$$G = \frac{de}{dx} = \frac{2v^2\psi}{\kappa x\, 10^9} = \frac{e}{x \log_\epsilon \frac{r_2}{r_1}} \text{ volts} = \frac{0.4343e}{x \log_{10} \frac{r_2}{r_1}} \tag{579}$$

For constant voltage applied to any given cable the curve for equation (579) is that of an hyperbola referred to its asymptotes. In Fig. 360 $r_1 = 5$, $r_2 = 30$ and the ordinates represent

either the voltage gradient or the dielectric flux density, for values of $x$ between 5 and 30.

The maximum voltage gradient is at the surface of the conductor:

$$^{M}G = \frac{e}{r_1 \log_e \frac{r_2}{r_1}} = \frac{0.4343e}{r_1 \log_{10} \frac{r_2}{r_1}} \qquad (580)$$

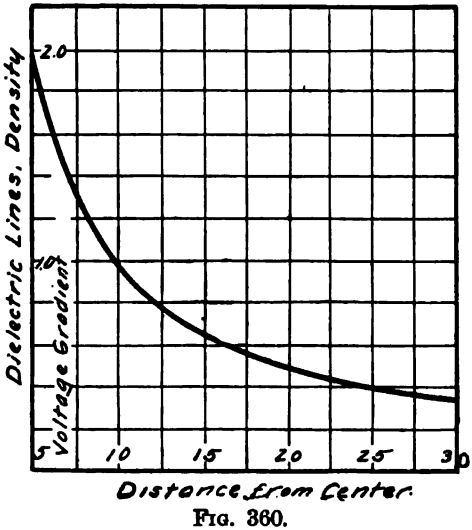



FIG. 360.

*Second Example. Two Parallel Plates.*—If the plates are of considerable size, the distribution of the flux, in the area between the plates, is uniform except at or near the edges. Near

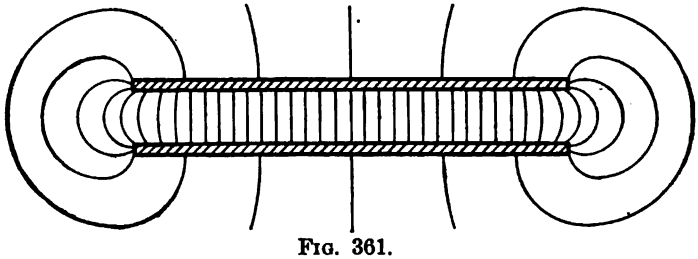


FIG. 361.

the edges the density is greater, and at points outside, less than in the space between the plates, Fig. 361.

Let $x$ = the distance between the plates.

$A$ = the area over which the flux is uniformly distributed.

$$\text{The elastance, } S = \frac{4\pi v^2 x}{\kappa A 10^9} = (\text{a constant})\,\frac{x}{A}, \text{ darafs} \qquad (581)$$

$$\text{The condensance, } C = \frac{\kappa A 10^9}{4\pi v^2 x} = (\text{a constant})\,\frac{A}{x}, \text{ farads} \qquad (582)$$

$$\text{Dielectric flux, } \psi = \frac{e}{S} = Ce = \frac{\kappa A e 10^9}{4\pi v^2 x} = (\text{a constant})\,\frac{Ae}{x} \text{ lines} \qquad (583)$$

$$\text{Flux density, } D = \frac{\psi}{A} = \frac{\kappa e 10^9}{4\pi v^2 x} = (\text{a constant})\,\frac{e}{x}, \text{ lines per } \overline{\text{cm.}}^2 \qquad (584)$$

$$\text{Voltage, } e = \frac{4\pi v^2 x \psi}{\kappa A 10^9} = (\text{a constant})\,\frac{x\psi}{A} \text{ volts} \qquad (585)$$

$$\text{Voltage gradient, } G = \frac{de}{dx} = \frac{4\pi v^2 \psi}{\kappa A 10^9} = (\text{a constant})\,\frac{\psi}{A}, \text{ volts per cm.} \qquad (586)$$

*Third Example. Two Parallel Round Conductors in Air.*— The most important practical example is the transmission line and the discussion deals with the problem under the restrictions of high-tension transmission line conditions. It is therefore assumed that:

1. The diameters of the two conductors are equal.
2. The distance between the conductors is large as compared to the diameter of each wire.

Since the two conductors have constant diameters and are parallel in position, equipotential surfaces must be cylinders and hence the distribution of the lines of force is the same in any normal plane. All phases of the problem may therefore be shown on a cross-section perpendicular to the conductors. Consider a neutral plane midway between the conductors and perpendicular to the plane joining their centers, as indicated by the line $NN'$ in Fig. 362. The two wires are equally positive and negative referred to the line $NN'$. When taken independently the fields for each are radial straight lines. The resultant field is the superposition of the separate fields, and the flux lines assume curved paths as shown in Fig. 363. These curved paths are arcs of circles passing through the points $A$ and $B$ and hence have their centers along the line $NN'$. This may be shown as in Fig. 364. Take any point $P$ in a normal

plane. Let $AP = s$ and $BP = u$. The dielectric stress or field intensity at the point $P$ due to the negative voltage of $A$ is

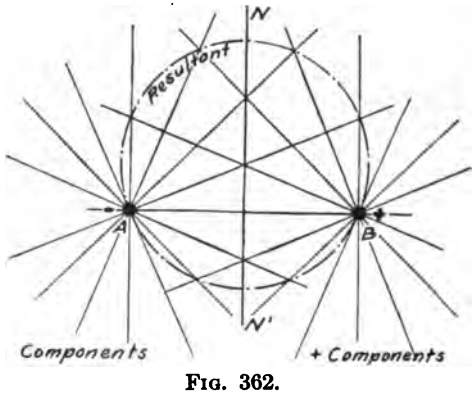



FIG. 362.

along $PA$ and inversely proportional to the distance $s$. Likewise the stress due to the positive voltage of $B$ is in the direction

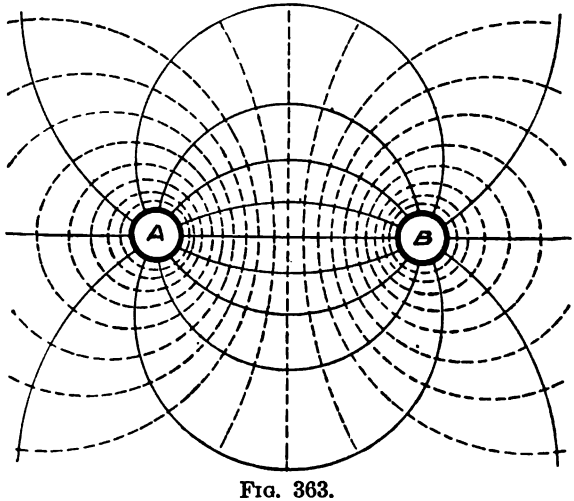



FIG. 363.

$BP$ and inversely proportional to the distance $u$. Lay off the lines $PN$ and $PH$ in Fig. 364, so that

$$PN : PH :: \frac{1}{u} : \frac{1}{s} \qquad (587)$$

Complete the parallelogram $PHKN$. Then the line $PK$ rep-

resents, in direction and relative magnitude, the dielectric field intensity at the point $P$. For the two triangles, $APB$ and $PHK$, two sides are proportional and the included angles equal, hence the triangles are similar. Therefore, the angles $ABP$ and $HPK$ are equal. Hence the line $PK$ must be tangent to a circle passing through the three points $A$, $P$ and $B$. Since $P$ is any point in the normal plane, the direction of the dielectric lines of force between the points $A$ and $B$ is along arcs of circles passing through $A$ and $B$.

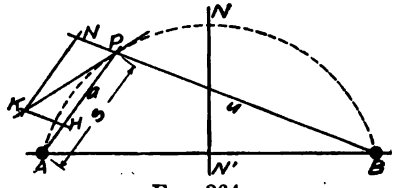



FIG. 364.

The equipotential surfaces are perpendicular at every point to the direction of the flux lines and therefore must be cylinders with their axes in the plane passing through both conductors, as indicated by the equipotential circles, dotted lines in Fig. 363.

From the diagram it is evident that the maximum flux density and voltage gradient must be in the plane passing through the centers of the two conductors, or along the line $XX'$ in Figs. 363, 365. Since the maximum values are of special importance in

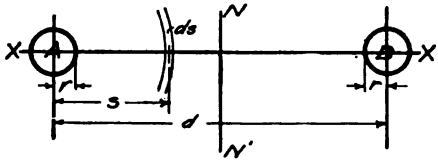



FIG. 365.

commercial problems, and in order to simplify the expressions, the discussion will be limited to the plane represented by the line $XX'$.

Let $d$ = the distance between the centers of the conductors.

$r$ = the radius of each conductor.

$s$ = distance of any point $a$ from the center of conductor $A$.

$\lambda$ = proportionality factor for the system of units used.

$\psi$ = total flux.
$\kappa$ = (permittivity of air) = 1.0.
$e_n$ = voltage to neutral.

At the point $a$ the dielectric field intensity due to the voltage difference from $A$ to $NN'$ is proportional to $\frac{1}{s}$; and for the voltage difference from $NN'$ to $B$ it is similarly proportional to $\frac{1}{d-s}$, hence:

$$K_a = \psi\left(\frac{1}{s} + \frac{1}{d-s}\right) \tag{588}$$

The voltage absorbed in any elemental zone, $(ds)$ is:

$$de_n = \lambda K_a ds$$
$$= \lambda\psi\left(\frac{1}{s} + \frac{1}{d-s}\right)(ds) \tag{589}$$

The total voltage from $A$ to neutral, $NN'$, is therefore:

$$e_n = \lambda\psi\int_r^{\frac{d}{2}}\left(\frac{1}{s} + \frac{1}{d-s}\right)(ds) \tag{590}$$

$$= \lambda\psi \log_\epsilon\left(\frac{d-r}{r}\right) \tag{591}$$

Therefore,

$$\psi = \frac{e_n}{\lambda \log_\epsilon\left(\frac{d-r}{r}\right)} \tag{592}$$

Voltage gradient:

$$G = \frac{de_n}{ds} = \lambda\psi\left(\frac{1}{s} + \frac{1}{d-s}\right)$$
$$= \frac{e_n}{\log_\epsilon\left(\frac{d-r}{r}\right)}\left(\frac{1}{s} + \frac{1}{d-s}\right)$$
$$= \frac{e_n d}{(sd - s^2)\log_\epsilon\left(\frac{d-r}{r}\right)} \text{ volts per cm.} \tag{593}$$

Maximum voltage gradient:

$$^{M}G = \frac{e_n d}{(rd - r^2)\log_\epsilon\left(\frac{d-r}{r}\right)} \text{ volts per cm.} \tag{594}$$

Under the assumption that $r$ is small as compared to $d$, which is the case in transmission-line problems, the maximum voltage gradient becomes:

$$^{M}G = \frac{e_n}{r \log_e\left(\frac{d}{r}\right)} = \frac{0.4343 e_n}{r \log_{10}\left(\frac{d}{r}\right)} \tag{595}$$

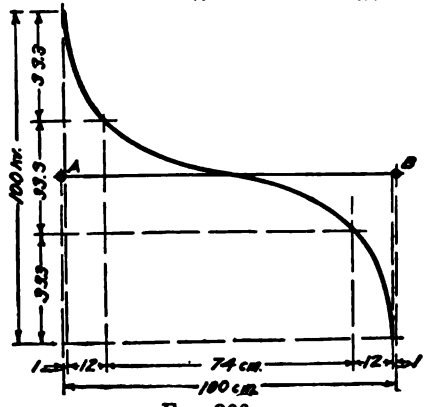



FIG. 366*a*.

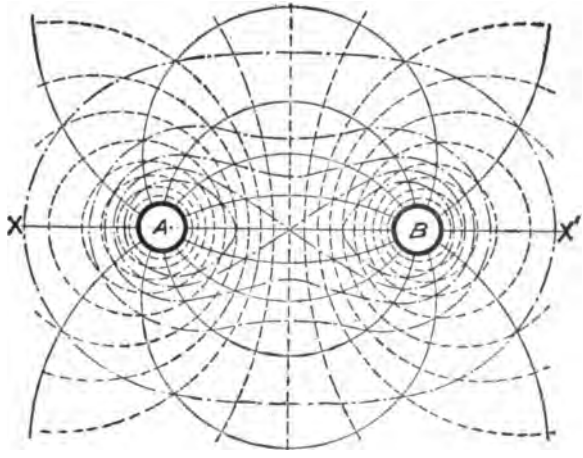



FIG. 366*b*.

For $e$, the total voltage between conductors, the maximum gradient:

$$^{M}G = \frac{e}{2r \log_e\left(\frac{d}{r}\right)} = \frac{0.4343 e}{2r \log_{10}\left(\frac{d}{r}\right)} \tag{596}$$

In Fig. 366*a* the voltages are plotted as ordinates with the distances from the wires as abscissæ in the plane through the wires. The broken lines in Fig. 366*b* show the equigradient lines in the field around the conductors.

*Fourth Example. Condenser Bushings for High-tension Apparatus.*—By means of concentric cylindrical layers differing in length as well as in diameter it is possible to obtain practically the same voltage gradient in all parts of the insulating layers. The total cylindrical insulation is divided into several concentric cylinders around the transformer lead with the layers separated by metal sheaths (tinfoil). The cylinder near the central conductor is longest and the outside layer in contact with the transformer casing shortest as indicated by a cross-sectional view in Fig. 368. The heavy line at the outer surface shows the metal holder or ring by which the condenser bushing is attached to the transformer case.



FIG. 367.—448,000 volt flashover test on one of the early condenser bushings. (*Westinghouse Elec. & Mfg. Co.*)

Let the radius of the conducting lead be $r$, and the radii of the successive tinfoil cylinders be $r_1, r_2, r_3 \ldots$. Let $e_1, e_2, e_3, e_4 \ldots$ be the corresponding voltages between $r - r_1$, $r_1 - r_2$, $r_2 - r_3$, $r_3 - r_4 \ldots$ as indicated in Fig. 368.

The potential gradients at the conductor surface and at the outside surfaces of the successive tinfoil cylinders (that is, on the inside surface of the insulating cylinders) are:

$$g_c = \frac{e_1}{r \log_e \frac{r_1}{r}}; \; g_1 = \frac{e_2}{r_1 \log_e \frac{r_2}{r_1}}; \; g_2 = \frac{e_3}{r_2 \log_e \frac{r_3}{r_2}}; \; \ldots \quad (598)$$

With the same material in the several insulation cylinders, the flux density and hence the dielectric gradient should be the same for the most desirable distribution of the flux, hence:

$$g_c = g_1 = g_2 = g_3 = \ldots \quad (599)$$

The numerical value of the gradient depends upon the material used and the assumed factor of safety; usually about 25 per cent. less than the rupturing voltage. The gradient through each insulating cylinder is greatest at the inner and least at the outer surface, as was shown under the first example. The permissible difference must be assumed for any design in much the same manner as the selection of a suitable factor of safety. This assumed ratio determines the approximate thickness of each insulating cylinder since the density at the outer and inner surfaces necessarily must be inversely as the radii. Hence the values of $e_1, e_2, e_3, e_4, \ldots$ may be found from equation (598).

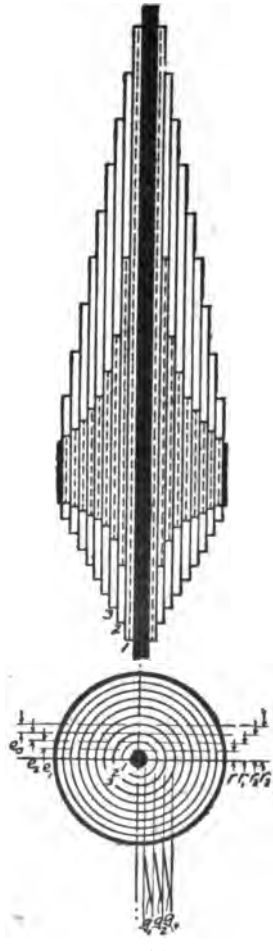


FIG. 368.

Let it be assumed that all the flux between the transformer lead and casing passes through the insulating cylinders and hence that there is no leakage through the surrounding air. The error introduced by this assumption will be noted in the next paragraph. The flux, $\psi$, passing through the successive insulating cylinders is constant for any given total voltage, $e$, and hence for $e_1, e_2, e_3, e_4, \ldots$ since

$$e = e_1 + e_2 + e_3 + e_4 + \ldots \quad (600)$$

$\psi = Ce$, and therefore,

$$C_1 e_1 = C_2 e_2 = C_3 e_3 = C_4 e_4 = \ldots \quad (601)$$

or, the condensances of the several cylinders are inversely proportional to the corresponding voltages. From equation (578), the condensance of a cylindrical insulator between two conductors as the above tinfoil layers, is;



FIG. 369.—150,000 volt indoor condenser bushing. (*Westinghouse Elec. & Mfg. Co.*)

$$C = \frac{\kappa l 10^9}{2v^2 \log_\epsilon \frac{r_2}{r_1}} \text{ m.f.s.} \qquad (602)$$

$l$ = the length in centimeters.
$r_2$ = the outside radius of insulating cylinder.
$r_1$ = the inside radius of insulating cylinder.

Hence;

$$\frac{l_1 c_1}{\log_\epsilon \frac{r_1}{r}} = \frac{l_2 c_2}{\log_\epsilon \frac{r_2}{r_1}} = \frac{l_3 c_3}{\log_\epsilon \frac{r_3}{r_2}} = \quad . \quad . \quad . \quad (603)$$

By assuming a value for the length of either the inner or the outer layer, the length of all the other cylinders may be calculated. The shaded portion in Fig. 368 gives the shape of the bushing as determined from the above equations. It is, however, more convenient to manufacture a form tapering uniformly as shown in Fig. 367. This modifies the voltage distribution, increasing the voltage gradient near the casing and the lead and proportionately decreasing it in the middle layers.

To prevent hygroscopic action it has been found advantageous to give the part of the bushing above the oil a smooth cylindrical form as shown in Fig. 369.

2. *Two or More Dielectrics in Parallel.*—Let the space between two metallic plates be divided into three equal parts, filled with rubber, air and glass as indicated in Fig. 369. With a difference of voltage existing between the plates, the density of the dielectric flux in the three insulating materials will be directly proportional to their permittivities, or approximately as 3:1:5. By varying the impressed voltage, the flux density also changes, but the relative flux densities are still in the same ratio. The voltage distribution in the space may be represented by a straight line, Fig. 370; and the voltage gradient $G = \frac{de}{dx}$ is a constant. In order to illustrate the

elemental principles it was assumed that there is no leakage, *i.e.*, the dielectric flux is confined to a definite path, as is the case with electric circuits. Under actual conditions there are no sharp divisions between good and poor conductors of dielectric flux. A good dielectric like glass has a permittivity only about five times that of air or of a vacuum. Hence, in the above illustration there would be no material available for separating the rubber and the air, or the glass and the air. Likewise, the flux can not be confined to the direct path between the two plates but also passes through the air on the sides. In the discussion of the condenser bushings in the last paragraph, it was assumed that all the flux between the casing and the transformer lead passed through

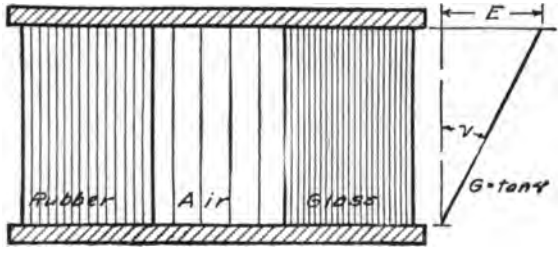



FIG. 370.

the insulating cylinders. However, since the permittivity of the solid material is probably at best only four or five times as great as that of the surrounding air, it is evident that a considerable share of the flux leaks around the solid dielectric. The flux density for any path is in all cases proportional to the permittivity of the material, and inversely proportional to the length. The flux distribution may be represented as in Fig. 371*a*; and the equivalent dielectric circuits by Fig. 371*b*.

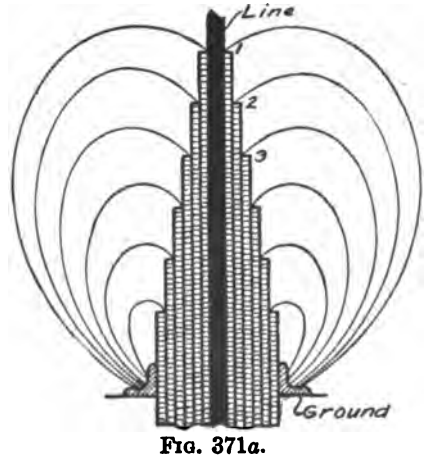



FIG. 371*a*.

3. *Two or More Dielectrics in Series.*—Let three equal thicknesses of rubber, air and glass fill the space between two parallel metallic plates, as indicated in Fig. 372; and consider only that portion having uniform distribution

of the field flux. The dielectric flux passes through the three materials in series and hence the flux densities in the glass, air and rubber are the same. The voltages required to force the flux through equal distances of the three materials are inversely proportional to the permittivities This may be indicated graphically by plotting the relation between volts and length of flux path as in Fig. 372. For each material the curve is a straight line, but the slope differs in the three cases. The voltage gradients are therefore in the ratio of $\frac{1}{3}:\frac{1}{1}:\frac{1}{5}$ or as 5:15:3. Hence while the air occupies only one-third of the space it takes $\frac{15}{23}$ of the total voltage. The voltage gradient in the air is therefore almost twice as great as would be the case if the rubber and glass were removed. If the voltage gradient is near the breakdown strength of air alone between the plates, the insertion of the rubber and glass would so increase the gradient in the air that corona and sparks would appear. For example, let the voltage between the plates be 20 kv. times the distance in cm. apart. With air alone as the dielectric the voltage gradient would be 20 kv. which is below the rupturing voltage. After inserting the rubber and glass plates the voltage gradient in the air is 39.1 kv., far above its dielectric strength, and the air space is aglow with corona and spark discharges.

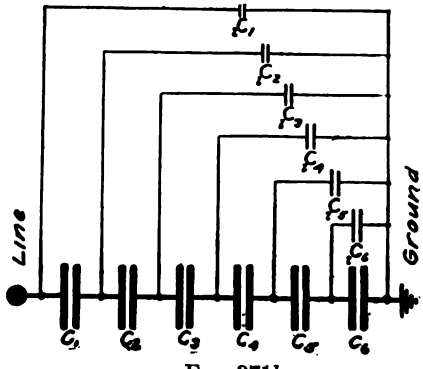



FIG. 371b.

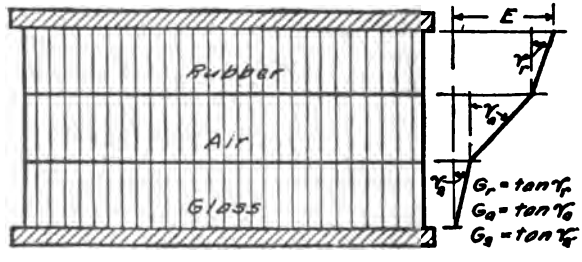



FIG. 372.

The space dimension factor affecting the distribution of the lines of force may to some extent be counterbalanced by selecting

and arranging insulating materials in the proper order. Thus in a cable having only one insulating material the flux density and hence the voltage gradient is greatest at the surface of the conductor and least at the lead sheath. The voltage to neutral and thickness of insulation are as shown in Fig. 373.[1] By arranging four insulating layers of permittivities 6.1, 4.7, 4.2, and 4.0 and of relative thickness as shown in Fig. 374, the voltage to neutral is almost a straight line (the tangent of which represents the voltage gradient at any given point) and the total thickness is only two-thirds of the single dielectric without increasing the maximum of the voltage gradient.

4. *Combination of Series and Parallel Circuits.*—From the discussion of the simple series and parallel dielectric circuits it is seen that the laws are similar to those of the electric circuits.

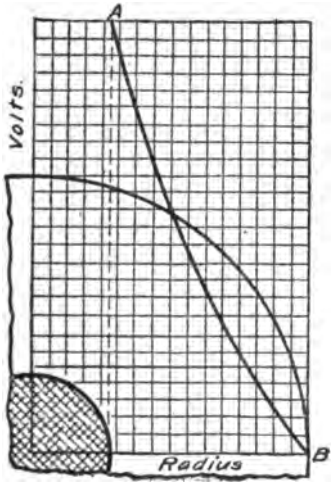



FIG. 373.

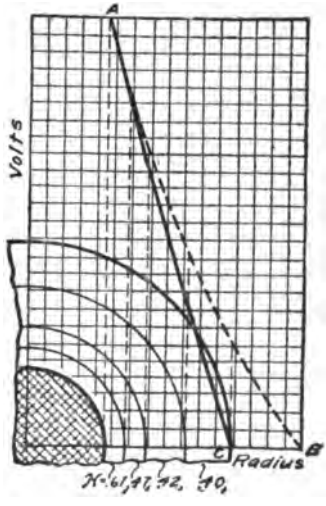



FIG. 374.

In principle, therefore, the combination of series and parallel circuits offers no difficulties, and the laws may be easily formulated. The calculation of the numerical values for electrical appliances, however, is in most cases very difficult because of the irregular shape of the dielectric. Electrical conductors are usually of simple geometric forms. Wires, bars, ribbons, cables, etc., whose dimensions are readily determined, form the bulk of resistance problems in electric circuits. In the dielectric circuit the same simple law applies: namely, the elastance varies directly as the length and the specific constant of the material

[1] JONA, "Insulating Material in High-tension Cables," St. Louis International Congress.

used and inversely as the cross-section, but the shape of the dielectric makes it very difficult to determine the dimensions. In most cases it is manifestly impossible to measure either the length or the cross-section of the dielectric circuit, and as a consequence the exact distribution of the dielectric flux can not be calculated.

*Suspension Insulator.*—As an example of a simple case consider the dielectric circuits in a suspension insulator. Let the several units be of the same size and type, and hence the condensances of the several insulators are equal as indicated in Fig. 375. The dielectric flux is, however, not entirely confined to the path through the string of insulators since the condensance between the metal parts and the ground is comparable with the condensance across the insulator. Hence, the actual dielectric circuit between the transmission line and the supporting arm (ground) of the tower consists of a combination of series and parallel condensances, $c_1$ and $c_2$ as indicated in Fig. 375. The total condensance of the string of $n$ insulators is therefore not equal to $\frac{c_2}{n}$ but depends upon the number, arrangement and relative values of $c_1$ and $c_2$. Let $c_2 = kc_1$.

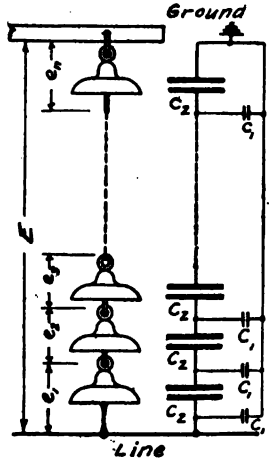


FIG. 375.

The total condensance of the suspension insulator may be found as follows:

For one insulator:

$$C_1 = c_1 + c_2 = c_1(1 + k) \tag{604}$$

For two insulators:

$$C_2 = c_1 + \frac{1}{\frac{1}{c_1 + c_2} + \frac{1}{c_2}} = c_1 + c_2 - \frac{c_2^2}{2c_2 + c_1} \tag{605}$$

For three insulators:

$$C_3 = c_1 + \frac{1}{\frac{1}{C_2} + \frac{1}{c_2}} = c_1 + c_2 - \frac{c_2^2}{c_2 + C_2}$$

$$= c_1 + c_2 - \frac{c_2^2}{2c_2 + c_1 - \frac{c_2^2}{2c_2 + c_1}} \tag{606}$$

For a string of $n$ insulators:

$$C_n = c_1 + c_2 - \frac{c_2^2}{c_2 + C_{n-1}}$$

$$= c_1 + c_2 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{2c_2 + c_1 - \cfrac{c_2^2}{\cdots\cdots \over 2c_2 + c_1}}}} \qquad (607)$$

The fraction is continued to $n - 1$ of the $2c_2 + c_1$ terms.

The voltage distribution is therefore not uniform but greatest across the insulator nearest the line and least across the insulator next to the crossarm. Let $E$ = the total voltage across a suspension insulator of $n$ units, and $e_1$ the voltage across the first unit nearest the line, then:

$$e_1 = \frac{E}{k}\left(\frac{C_n}{c_1} - 1\right) \qquad (608)$$

Voltage across the second insulator,

$$e_2 = e_1 - \frac{E - e_1}{k} \qquad (609)$$

Voltage across the third insulator,

$$e_3 = e_2 - \frac{E - (e_1 + e_2)}{k} \qquad (610)$$

Voltage across the $n$th insulator,

$$e_n = e_{n-1} - \frac{E - (e_1 + e_2 + e_3 + \ldots + e_{n-1})}{k} \qquad (611)$$

or

$$E = \frac{e_1 k}{\frac{C_n}{c_1} - 1} \qquad (612)$$

Hence if ${}_a e_1$ is the arc-over voltage for one unit, the arc-over strength of the string of $n$ units is:

$${}_a E = \frac{{}_a e_1 k}{\frac{C_n}{c_1} - 1} \qquad (613)$$

The chief factor that makes it difficult to calculate dielectric flux distribution is the small range in specific condensance

or permittivity of insulating materials including the vacuum. The electric circuits may be confined to narrow definite paths because of the enormous difference in the resistance of good conductors, like copper, and poor conductors, like rubber, air or a vacuum. On account of this difference in range of specific resistance and of specific elastance the electric circuit can be confined to the wires in a transmission line with very small leakage; while the dielectric circuit embraces all the space surrounding the conductors. Combinations of several insulating materials may, to a certain extent, modify the distribution of the dielectric flux, but can not confine it entirely to any definite path. Fortunately, the relative distribution of the dielectric flux is of little importance unless the voltage gradient at any point approaches or exceeds the dielectric strength of the material. For all low-voltage apparatus no calculations are necessary. For high-voltage systems the dielectric circuit and the flux distribution are of great importance and apparatus for continuous operation must be so designed that at no point shall the maximum voltage gradient exceed the dielectric strength of the insulation.

**Problem**

**1.** Given a transmission line having suspension insulators with nine disks in each string. $E = 150{,}000$ volts between mains. Star connected with neutral grounded. Let $k = 9.5$.

Find the voltage across each disk.

Plot curve with the number of disks on the axis of abscissæ and the voltage across the disks as ordinates.

**2.** A three-phase transmission line of 4/0 hard drawn copper wires has 60,000 volts between the mains. Spacing of wires: six feet, equilateral triangle. Find the maximum voltage gradient at surface of conductors.

For Experiments see page 549.

## CHAPTER XX

## ELECTRIC AND DIELECTRIC CIRCUITS THROUGH AIR. CORONA

Under ordinary conditions and for low voltages, air is very nearly a perfect insulator and its dielectric properties are almost the same as for a vacuum. In other words, the conductivity of air is practically zero, the permittivity unity, and the energy loss extremely small. For higher voltages the air insulation may break down, either partially or completely, causing important changes in the electric and dielectric circuits.

For illustration, consider two parallel, smooth, bare wires, spaced as in a commercial high-tension transmission line, with dry, clean air as the dielectric. Let the room be dark, or if the wires are strung in the open, let the observations be taken at night. If the voltage be gradually increased until the gradient near the wires reaches 30 kv. per cm. ($^{\kappa}E$) a faint purplish light may be seen at or near the surface of the wires, and a hissing sound may be heard. By still further increasing the voltage both the luminous glow and the hissing sound increase in volume and intensity. The phenomenon, of which the light and sound are indications, is called the *corona*. A full understanding of the laws of the corona is of great importance in designing long-distance transmission lines, and all other high-voltage apparatus.

The lowest voltage gradient at which corona appears, a fairly definite value, is called the *critical visual corona voltage*. If the wires are far apart in comparison to their diameters, as is the case in transmission lines, the voltage may be increased far beyond the critical visual corona value before a disruptive spark passes between the wires. The voltage at which the spark breaks through the intervening air is called the *disruptive* or *spark-over voltage*, and this is also a fairly definite value, which may be calculated from the dimensions of the system. As soon as spark-over occurs the corona disappears. If the impressed voltage is only a little above the disruptive strength of the air, the spark-overs will be intermittent. After each spark passes, the corona

reappears and persists until the next spark disrupts the insulating air. The corona may be considered as a partial, and the spark-over as the complete breakdown of the insulation; and although differing widely in their characteristics, they are fundamentally two stages of the same group of phenomena. Since corona depends directly upon changes in the insulation of the air, the phenomena relate primarily to the electric circuit (leakage) between the wires. Changes in the insulation, however, with the resulting variations in the leakage current modify the condensance and therefore the corona indirectly also affects the dielectric circuit.

(*a*) **Theory.**—Extended discussions of various physical theories, some of which are largely "metaphysical fairy tales," by which groups of physical phenomena may be coördinated or "explained" are outside the scope of this book. The laws of energy transmission and transformation form the natural foundation upon which the engineer may safely build. Terms like *magnetic and dielectric fields or lines of force, current, potential,* etc., are merely pictorial illustrations which enable the student to use the *visual memory method.* In discussing dielectric phenomena in the preceding chapters the mechanical analogue of stress and strain in an electrically elastic medium has been used for the same purpose. It is essential that the student should fully realize that dielectric phenomena consist of *energy changes in time and space,* and follow just as simple laws as do the better known phenomena of the electric and magnetic circuits. The stress and strain analogue may be used to advantage when studying the corona and spark-over phenomena because the attention is kept focused on the energy changes causing the phenomena, which are of fundamental importance.

Following the epoch-making experiments on cathode rays by J. J. Thomson (1897), a large number of physicists, chemists and engineers have conducted extensive researches on the electric conductivity of gases in general, and of air in particular. Thomson showed that the particles moving in the cathode stream were much smaller than the ordinary atom or molecule; in fact only about $\frac{1}{1800}$ of the mass of a hydrogen atom. Moreover, it was found that these *corpuscles* or *electrons* invariably carried a charge of negative electricity of about $4.7 \times 10^{-10}$ c.g.s. electrostatic units, which is approximately the quantity carried in electrolysis by a hydrogen ion. That these quantities are not

merely averages but that the individual units are of equal magnitude has been shown experimentally by Millikan[1] in an elegant and convincing manner.

From experimental observations it is known that the conductivity of air and other gases is greatly modified by the presence of radioactive substances, by the action of the cathode rays and ultra-violet light, by electric stress causing corona, and by other factors. According to the electron theory, the changes in the conductivity of the air are due to similar variations in the number of free electrons, and the conduction of the electric energy through the air is accomplished by the space movements of electrons and ions. Under normal conditions the electron is combined with, or connected to, an ordinary molecule. The rapid motion and frequent collisions of the molecules in a gas often knock off an electron carrying a negative charge and leaving a positively charged ion of practically the same mass as the original molecule. Various combinations are possible between the free electrons, the positive ions and the neutral molecules. An electron combining with a positive ion forms a neutral molecule; positive ions may combine with neutral molecules forming larger aggregate masses, etc. It is evident that so many combinations, and combinations of combinations are possible, that almost any range of experimental data may be accounted for, or explained by the electron theory.

Since it is always advantageous to observe, as directly as possible, the energy changes that cause any physical phenomenon, a minimum of theoretical scaffolding is desirable. For this reason the simple stress and strain principle has an advantage over the more elaborate electron theory. The quantitative relations between the several factors that determine the appearance and extent of the corona and spark-over on transmission lines or other electrical apparatus, have been derived from experimental data and are now available in the form of empirical equations. The researches of Ryan,[2] Mershon,[2] and Steinmetz[2] and especially the extensive investigations of Peek[2] and Whitehead[2] are of great importance since they deal with the problem from the engineer's point of view and give summaries of the

[1] R. A. Millikan, *Phil. Mag.*, Vol. 19, p. 209, Feb., 1910; *Science*, Sept. 30, 1910.

[2] *Trans.* A. I. E. E., XXVIII to XXXIV.

results in forms that may readily be applied to engineering problems.

(*b*) **Visual Corona.**—The more important factors that enter into the equations for determining the visual corona voltage on parallel wires are:

1. The diameter of the wire.
2. The spacing of the wires.
3. The distance from the wire.
4. The condition of the conductor surface.
5. The air density.
6. The shape of the voltage wave.

Other factors like the conductor material, amount of current flowing in the wire, the frequency within ordinary commercial range and humidity of the air have little if any effect on the starting voltage of visual corona.

*Factors* 1, 2 *and* 3.—The first three factors may be discussed together. Let the conductors be wires with smooth surfaces, like those used in transmission lines; the air temperature and pressure normal (25°C., 76 cm.); and the voltage wave of a simple sine form.

From equation (595) the maximum voltage gradient between parallel wires is at the surface of the conductor and

$$_{n}G = \frac{e_n}{r \log_e \frac{d}{r}}$$

It has, however, been found experimentally that the corona appears on wires of different sizes for a constant gradient at a distance of $0.301\sqrt{r}$ from the surface of the conductor, and not for a constant gradient at the surface itself. The value of the voltage gradient at the distance $0.301\sqrt{r}$ at which visual corona is produced is 29.8 kv.-cm. or approximately 30 kv.-cm. For sine waves this is the maximum value and hence the corresponding effective values are 21.1 and 21.2 kv.-cm.

Let $_{v}e$ = visual corona voltage.

$_{o}G$ = visual corona gradient (at a distance $0.301\sqrt{r}$ from the surface of the conductor), a constant.

$x = r + 0.301\sqrt{r}$.

$d$ = distance between centers.

$_{v}G$ = visual corona gradient at the surface of the wire.

$$_{\delta}G = \frac{{}_{v}e}{x \log_{\epsilon} \frac{d}{r}} = \frac{{}_{v}e}{(r + 0.301\sqrt{r}) \log_{\epsilon} \frac{d}{r}} = \frac{0.4343\,{}_{v}e}{(r + 0.301\sqrt{r}) \log_{10} \frac{d}{r}} \quad (616)$$

$$_{\delta}G = \frac{{}_{v}e}{r \log_{\epsilon} \frac{d}{r}} = {}_{\delta}G\left(1 + \frac{0.301}{\sqrt{r}}\right) \text{ kv.-cm.} \quad (617)$$

Hence,

$$_{v}e = {}_{\delta}G\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (618)$$

For sine voltage waves:

$$_{v}^{m}E = 29.8\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (619)$$

$$_{v}E = 21.1\left(1 + \frac{0.301}{\sqrt{r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (620)$$

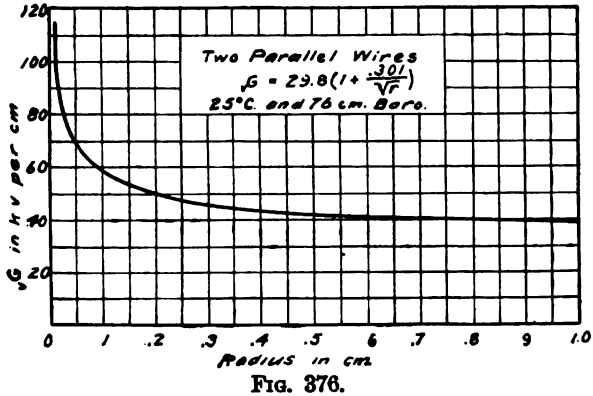



FIG. 376.

The relation between the surface gradient $_{\delta}G$ and the radius of the wire as given by equation (617) is shown graphically in Fig. 376. This curve agrees very closely with smooth wires at 25°C. temperature and 76 cm. pressure. If it be assumed that a definite amount of work is required to rupture the electrically elastic air, it follows that a stress in excess of the elastic limit must exist over a certain distance before the breakdown occurs in order that the product of stress and strain may supply the required energy. Hence the smaller the wire, the greater is the distance, and therefore the higher the voltage gradient at the surface of the wire.

*Factor* 4.—For conductors with rough surfaces or for cables the voltage gradient for visual corona is less than for smooth wires. A correction may be introduced by using an *irregularity factor* $m$ as in equation (621). The value of the irregularity factor varies greatly, and in general must be determined by direct measurements. Dirt, oil and water on the surface of the conductor also affect the gradient for visual corona and the necessary corrections may be included in the irregularity factor $m$.

$$_{v}G = {}_{o}Gm\left(1 + \frac{0.301}{\sqrt{r}}\right) \text{ kv.-cm.} \tag{621}$$

*Factor* 5.—Changes in air density, whether produced by changes in temperature or barometric pressure, affect the voltage gradient of visual corona. Normal conditions are given by 25°C. temperature and 76 cm. pressure. The *density factor* for any temperature, $t$, in centigrade degrees, and any barometric pressure, $b$, in centimeters mercury is given by equation (622).

$$\delta = \frac{3.92b}{273 + t} \tag{622}$$

Including both the density and irregularity factors, the expressions for the surface gradient and voltage to neutral required to produce visual corona are:

$$_{v}G = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) \text{ kv.-cm.} \tag{623}$$

$$_{v}e = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{e}\frac{d}{r} \text{ kv. to neutral} \tag{624}$$

For sine voltage waves:

$$_{v}^{m}E = 29.8m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{e}\frac{d}{r} \text{ kv. to neutral} \tag{625}$$

$$_{v}E = 21.1m\delta\left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{e}\frac{d}{r} \text{ kv. to neutral} \tag{626}$$

For smooth wires $m = 1$ and for normal temperature (25°C.) and pressure (76 cm.) $\delta = 1$.

*Factor* 6.—For peaked voltage waves the effective voltage is less and for flat-topped waves greater than in equation (626). The variation depends upon the distortion and no general equation has been derived.

*Concentric Cylinders.*—For smooth concentric cylinders the voltage gradient required to give visual corona near the inside

cylinder is slightly larger than for parallel wires as shown in equation (627).[1]

$$G = 31.0\delta\left(1 + \frac{0.308}{\sqrt{\delta r}}\right) \text{ kv.-cm.} \qquad (627)$$

*Spheres.*—For spheres of equal diameters and spaced like the parallel wires, far enough apart to give corona at a lower voltage than required for the disruptive spark, the voltage gradient at the surface is:

$$G = 27.2\delta\left(1 + \frac{0.54}{\sqrt{\delta r}}\right) \text{ kv.-cm.} \qquad (628)$$

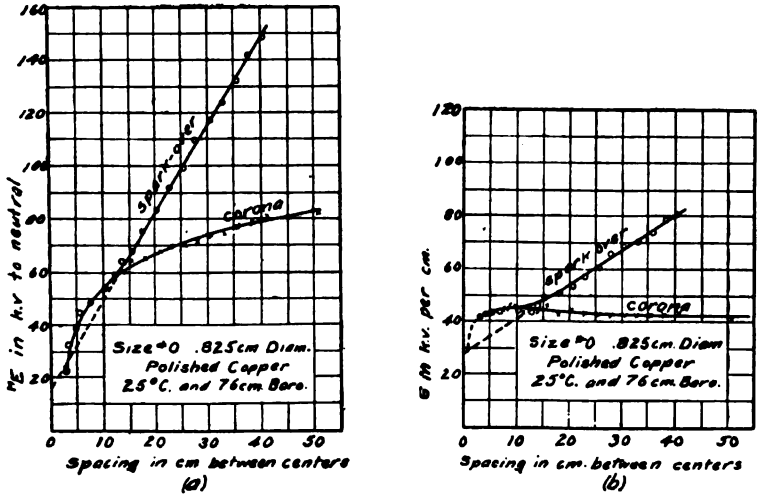



FIG. 377.

(*c*) **Spark-over.**—In visual corona the air breaks down near the wires in the space filled with the luminous glow. If the voltage is increased, the corona increases in volume and intensity. By increasing the voltage still further until the stress becomes sufficient to pierce all of the intervening dielectric, a disruptive spark passes between the wires. At small spacing the spark-over may occur before the corona. A certain critical spacing, depending upon the size of the wire, must be used in order to have a decided difference in voltage for visual corona and spark-over. This may be seen from the experimental curves shown in Figs. 377 and 378.[1] The lower curve shows the voltage required

[1] PEEK, *Trans.* A. I. E. E., XXXI.

to produce corona at the different spacings while the upper curve shows the corresponding voltages required to produce spark-over. Below the point where the two curves intersect the spark-over occurs before the voltage gradient is sufficient to produce corona. The corona and spark-over curves (extended) intersect at a voltage gradient of approximately 30 kv.-cm.

The ratio between the spacing, $d$, and the radius of the wires for the point where the corona and spark-over curves intersect, is found to be $\frac{d}{r} = 30$. Hence if it is assumed that the spark-

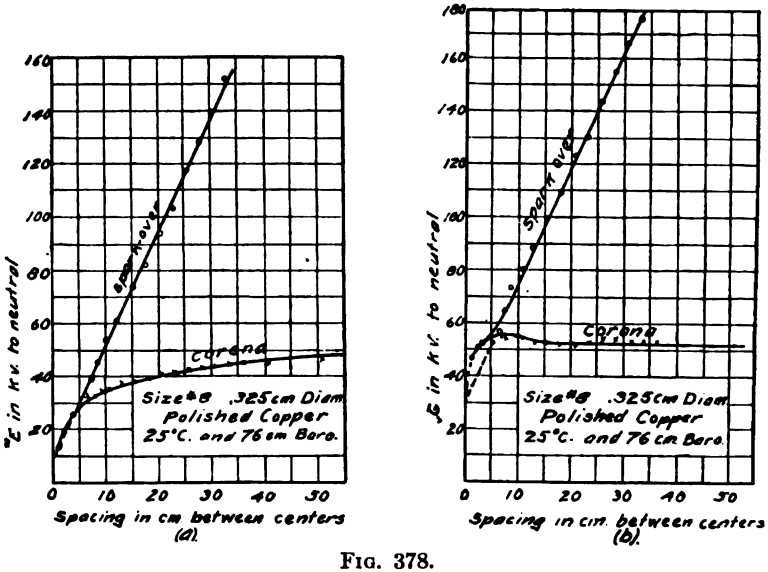



FIG. 378.

over gradient is a straight line, it is determined by these two points. Therefore, the spark-over gradient, ${}_sG$, under normal conditions, is:

$$ {}_sG = {}_oG\left(1 + \frac{0.301}{\sqrt{r}} \cdot \frac{d}{r} \cdot \frac{1}{30}\right) \text{ kv.-cm.} $$

$$ = 30\left(1 + \frac{0.01d}{r\sqrt{r}}\right) \text{ kv.-cm.} \qquad (630) $$

At the point of intersection of the two curves the ratio $\frac{d}{r} = 30$, and the spark-over gradient equals the visual corona gradient in equation (617).

Corrections for changes in air density may be made by means of the density factor, $\delta$.

$$G = 30\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) \text{ kv.-cm.} \qquad (631)$$

The corresponding spark-over voltage,

$$e = Gr \log_e \frac{d}{r} = 30\,\delta\left(1 + \frac{0.01\,d}{r\sqrt{\delta r}}\right) r \log_e \frac{d}{r} \text{ kv. to neutral} \qquad (632)$$

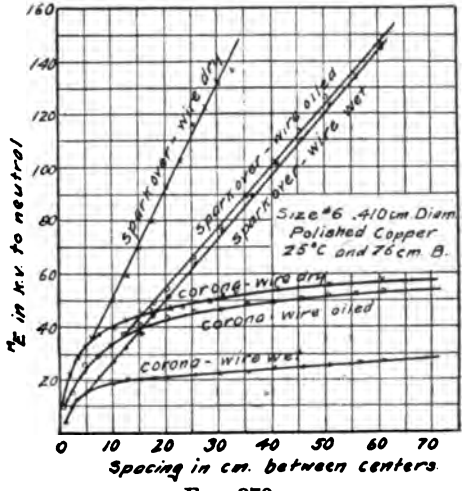



FIG. 379.

For sine voltage waves,

$$E = 30\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) r \log_e \frac{d}{r} \text{ kv. to neutral} \qquad (633)$$

$$E = 21.2\,\delta\left(1 + \frac{0.01d}{r\sqrt{\delta r}}\right) r \log_e \frac{d}{r} \text{ kv. to neutral} \qquad (634)$$

The effect of water or oil on the surface of the wires upon the spark-over voltage is shown in Fig. 379.[1]

A simple apparatus for experimental work on the corona and spark-over between parallel wires is shown in Fig. 380. Each wire is supported by two insulators having shields made from the sheath of a lead-covered cable. The wires must be kept taut and carefully spaced. The voltage gradient is less on the

[1] PEEK, *Trans.* A. I. E. E., XXXI.

shields than on the intervening wires, and visual corona will form along the whole length of wire between the shields.




FIG. 380.—Apparatus for measuring voltage for visual corona. The rims of the insulators are covered by lead sheath shields.

*The Sphere Spark Gap.*—The general solution of the voltage gradient between spheres of any radius, $R$, and of any spacing, $D$, leads to complicated expressions that can not be readily applied to practical problems. The voltage gradient is at a maximum for the points of intersection of the surfaces with the line joining the centers of the two spheres. For spheres of equal radii, $R$ cm., and spaced a distance, $D$ cm., between the surfaces, and with a difference of $e$ kilovolts potential, the maximum voltage gradient is:

$$G = \frac{e}{D}(f) \text{ kv.-cm.} \qquad (635)$$

The factor $(f)$ is a function of $R$ and $D$ and for ungrounded spheres may be expressed as in equation (636).[1]

$$(f) = \tfrac{1}{2}\left[\frac{D}{R} + 1 + \sqrt{\left(\frac{D}{R} + 1\right)^2 + 8}\right] \qquad (636)$$

The values of $(f)$ for different ratios of $D$ and $R$ are given in Table XIV.

TABLE XIV[2]

| $\frac{D}{R}$ | $(f)$ | $\frac{D}{R}$ | $(f)$ |
|---|---|---|---|
| 0.0 | 1.0000 | 0.8 | 1.2881 |
| 0.1 | 1.0337 | 0.9 | 1.3268 |
| 0.2 | 1.0681 | 1.0 | 1.3660 |
| 0.3 | 1.1032 | 1.5 | 1.5687 |
| 0.4 | 1.1390 | 2.0 | 1.7808 |
| 0.5 | 1.1754 | 3.0 | 2.2247 |
| 0.6 | 1.2124 | 4.0 | 2.6861 |
| 0.7 | 1.2500 | 5.0 | 3.1583 |

With one sphere grounded different values must be used since the dielectric field is not symmetrical. The distance from the ground also affects the flux distribution.

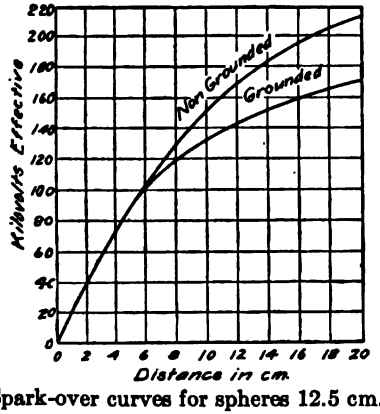



FIG. 381.—Spark-over curves for spheres 12.5 cm. in diameter.

In commercial work the sphere spark gap is important as a convenient instrument for measuring high voltages. For spheres of diameters greater than the spacing, spark-over occurs at a lower voltage than that required to form corona. For spacings

[1] DEAN, *General Electric Review*, XVI, 148.
[2] PEEK, *General Electric Review*, XVI, 287.

within the limits $0.5\sqrt{R}$ and $2R$ the voltage gradient is practically constant.

$$_{o}G = 27.2\left(1 + \frac{0.54}{\sqrt{\delta R}}\right) \text{ kv.-cm.} \qquad (637)$$

The spark-over voltage for different spacings may be found directly from the curves in Figs. 381 and 382. It is essential, when using the sphere spark gap for measuring high voltages, that a high resistance be inserted in the leads to prevent an excessive current flowing across the arc and to eliminate abnormal voltages. For this purpose glass tubes filled with water are well adapted. The dimensions of the tubes and the quality of the water should be adjusted so as to limit the current in the spark-over arc from ¼ to 1 amp.

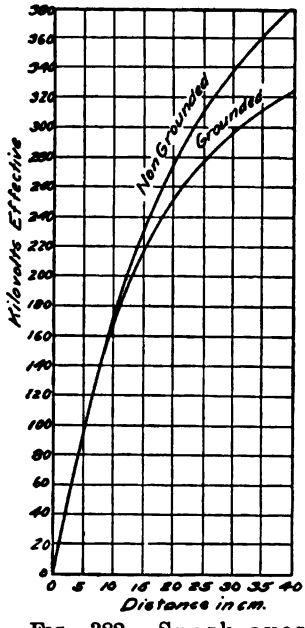



FIG. 382.—Spark-over curves for spheres 25 cm. in diameter.

(*d*) **Corona Energy Losses.**—If the voltage between two parallel wires is raised beyond the point required for the visual corona the energy losses rapidly increase. Under normal atmospheric conditions and for smooth wires and constant frequency the energy loss varies as the square of the voltage in excess of the critical disruptive voltage.

Let $_{a}W$ = energy loss in the air.
$E$ = voltage to neutral.
$_{d}E$ = critical disruptive voltage.

Then,

$$_{a}W \propto (E - {}_{d}E)^2 \qquad (638)$$

The critical disruptive voltage is obtained when the gradient at the surface of the wire equals $_{o}G$.

The values for $_{d}E$ for different spacings and sizes of wire are obtained from the relation between voltage gradient, spacing and size of wire already derived. $_{d}E = {}_{o}Gm\delta r \log_{\epsilon}\frac{d}{r}$. For trans-

mission lines ${}_dG = 21.1$ (effective values). For cables between 0.583 cm. and 1.18 cm. diameter and various spacings ${}_dG = 18.1$ kv.-cm.

For higher frequencies the losses increase but not in direct proportion. There seems to be a loss proportional to the frequency superimposed upon a constant loss, and the relation between the energy loss and the frequency may be expressed as in (639).

$$ {}_aW \propto (f + 25)(E - {}_dE)^2 \tag{639} $$

Correction for variations in air density and surface roughness may be made by introducing the density factor, $\delta$, and irregularity factor, $m$, into the equation for the critical disruptive voltage, ${}_dE$.

$$ {}_dE = \delta m_d G r \log_e \frac{d}{r} \tag{640} $$

The equation for the energy loss above the critical voltage for visual corona as derived from experimental data in kilowatts per kilometer, is:

$$ {}_aW = 241 \frac{\sqrt{r}}{\delta\sqrt{d}} (f + 25)(E - {}_dE)^2 10^{-5} \text{ kw./km.} \tag{641} $$

(*e*) **The Alternating-current Corona Cycle.**—Let a sufficiently high voltage, of sine wave shape and of any commercial frequency, be impressed on two parallel, smooth wires, properly spaced, so as to give corona without spark-over. The appearance of the corona is a continuous uniform glow or luminous brush surrounding the wires. However, if observations be taken through a stroboscope, it is found that the corona pulsates in synchronism with the impressed voltage. By turning the stroboscope through 360 electrical degrees it may be seen that the corona disappears twice each cycle (that is, during the portions of the cycle when the instantaneous voltage is less than the required value to produce visual corona); and that the appearance of the corona for the positive and negative half waves is decidedly different, Fig. 383.[1] The luminous brush extending a distance from the wires is generated by the positive half wave, while the negative half produces a bright glow in close contact with the wire. With the neutral grounded, the luminous brush, or positive corona, appears alternately on the two wires, but with ordinary frequencies the

[1] For a photographic study of the positive and negative corona see Peek, *Trans.* A. I. E. E., XXXI, 1089.

pulsations overlap so rapidly that when viewed without the aid of a stroboscope the corona appears to be continuous.

(*f*) **The Dielectric Circuit.**—From the preceding discussion it is seen that the corona primarily affects the electric circuit (leakage) between the conductors. The corona results from a partial



(1) Without stroboscope—72,000 volts.



− (2) With stroboscope—72,000 volts. +



\+ (3) Stroboscope rotated 180°. −

FIG. 383.

breakdown of the insulation with a resultant leakage current and corresponding energy loss. In the space filled by the corona the resistivity is less than for air under normal conditions. As a consequence of this change in the resistance of the layer near the conductors, the condensance of the dielectric circuit is increased.

Because the insertion of a layer of comparatively low resistance, as compared to the normal air, around the wires, is in effect an increase in the conductor diameter, and this increases the condensance of the dielectric circuit. The specific resistance of the corona, while less than for normal air is still much greater than for metallic conductors. The corona, therefore, introduces a fairly large resistance into the dielectric circuit in series with the condensance, through which the reactive energy flows when passing to and from the electrically elastic air outside the corona zone. For alternating currents the corona and the resistance of the electric circuit (leakage) through the air, and likewise the condensance and series (corona) resistance of the dielectric circuit, pulsate in synchronism with the impressed voltage, with alternate values for the successive half waves.

## PROBLEMS

**1.** Calculate the required voltage to neutral to give visual corona on two smooth parallel wires of No. 2 A.W.G. copper, for 5, 10, 15, 20, 25 and 30 cm. spacing between centers. Bar. = 74 cm. Temp. = 20°C. Plot curves with spacing as abscissæ and visual corona voltage as ordinates.

**2.** Find the spark-over voltage for No. 2 copper wires spaced as in problem 1. Plot spark-over voltage curve as ordinates and distance between centers as abscissæ.

**3.** Calculate the voltage gradients at the surface of the wire corresponding to the spacings and size of wire given in problem 1 with 50,000 volts a.c., 1-phase between wires. Plot the voltage gradients as ordinates and distance between centers as abscissæ.

**4.** Same as problems 1, 2 and 3 but with No. 12 wire.

**5.** An alternating-current voltage of 24,000 volts, 60-cycle, is impressed on two No. 00 A.W.G. copper conductors placed at spacings of 2, 4, 6, 8, 12, 16, 20, 24, 36 and 48 in. Find the maximum surface gradients for the several spacings. Plot curves with voltage gradients as ordinates and spacings as abscissæ.

**6.** In problem 5 let the wire be No. 16 A.W.G. Find the voltage gradients and plot curve as in problem 5.

**7.** What is the corona loss at 150,000 volts on 10 miles of single-phase line consisting of No. 0000 wire with 10-foot spacing? What size of wire is required to eliminate the corona with the same voltage? Elevation 3000 ft.

For Experiments see page 550.

## CHAPTER XXI

## SHORT TRANSMISSION LINES

In direct-current systems the voltage drop on the transmission line is equal to the product of the resistance and the current. The voltage at the generator is the arithmetical sum of the voltage at the receiver and the line drop, and hence the voltage at the load is always less than at the generator. For alternating currents the transmission line problem is much more complex. The magnetic field surrounding each wire and the dielectric field between the wires as well as the ohmic resistance of the wires are distributed along the transmission line, thus forming an interwoven network of series and parallel circuits over the whole length of the line. Through this complex circuit, power is transmitted to a load that may vary both in magnitude and power factor. The voltage and current values along the transmission line depend not only upon the magnitude of the component parts but also upon their time-phase and space-phase relations; and the problem of voltage regulation becomes radically different from the simple conditions existing in direct-current systems. The voltage at the load may be less, equal to, or even greater than the voltage at the generator. The voltage along the line may be greater than at either end, and the conditions may be so adjusted as to automatically keep constant voltage at three points on the system for wide variations of load. The basic factors in the problem are the resistance, inductance and condensance of the transmission line, the so-called *line constants;* and it will be necessary to first derive expressions by which these may be found quantitatively for any given line.

(*a*) **Line Resistance.**—In commercial calculations the ohmic resistance is generally used, and only in special cases are corrections for obtaining the *effective resistance,* introduced.

$R$ = resistance.
$l$ = length of wire.
$r$ = radius of wire.
$\rho_o$ = resistivity at 0° temperature.

$\rho_t$ = resistivity at $t°$ temperature.
$\alpha$ = temperature coefficient, usually taken at 0.0042 between 0° and 50°C. for an initial temperature of 0°.
$\rho_t = \rho_o(1 + \alpha t)$.

The resistance varies directly as the length and inversely as the cross-section of the conductor.

$$R_t = \frac{\rho_t l}{\pi r^2} \tag{612}$$

(*b*) **Line Inductance.**—As the inductance in a circuit is a measure of the magnetic field produced by unit current it depends upon the size and spacing of the conductors and the permeability of the conductors and the surrounding medium. Let the conductor be a cylindrical wire in a single-phase circuit, Fig. 384.

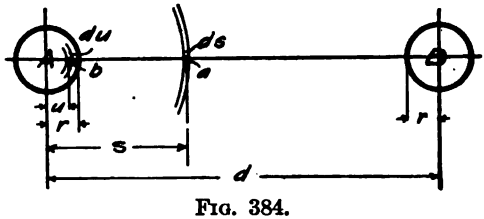



FIG. 384.

$\mathcal{H}_a$ = strength of the magnetic field, at any point $a$ outside the wire.
$\mathcal{H}_b$ = strength of the magnetic field, at any point $b$ inside the wire.
$s$ = the distance from $a$ to the center of the wire in cm.
$u$ = the distance from $b$ to the center of the wire in cm.
$\mathfrak{B}_a$ = flux density at $a$.
$\mathfrak{B}_b$ = flux density at $b$.
$r$ = radius of the wire in cm.
$d$ = distance between centers of the conductors in cm.
$\mu'$ = permeability outside the wire.
$\mu$ = permeability inside the wire.

Inside the conductor with uniform current density the m.m.f. for any cylindrical shell a centimeter in length and of $u$ radius equals $4\pi I \frac{u^2}{r^2}$ and the reluctance is proportional to $\frac{2\pi u}{du}$.

$$\mathcal{H}_b = \frac{\text{m.m.f.}}{\text{length of path}} = \frac{2Iu}{r^2} \tag{613}$$

$$\mathfrak{B}_b = \mu\mathcal{H}_b = \frac{2\mu Iu}{r^2} \tag{614}$$

The number of lines in a cylindrical shell $du$ of radius $u$ and 1 cm. long is $d\phi_b$.

$$d\phi_b = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{2\mu I u du}{r^2} \tag{615}$$

As these lines interlink only with that part of the current lying inside of the corresponding cylindrical shell it is desirable to find the equivalent flux, which multiplied by the total current equals the integral of the products of the separate lines into their respective parts of the current.

Let $\phi_{o-r}$ be the equivalent flux surrounding the whole current.

$$\phi_{o-r} = \int_0^r \frac{\pi u^2}{\pi r^2}\, d\phi_b = \int_0^r \frac{2\mu I u^3 du}{r^4} = \frac{\mu I}{2} \tag{616}$$

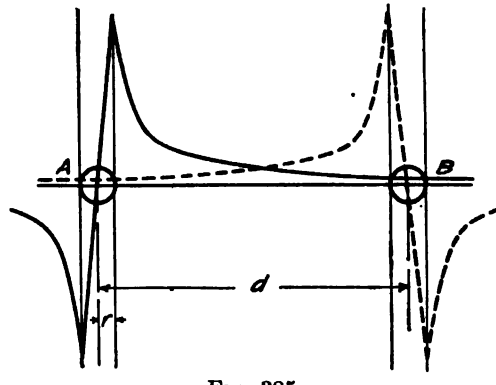



FIG. 385.

Hence the flux inside the wire is equivalent to $\frac{\mu I}{2}$ lines interlinking the total current $I$ in the wire, and the equivalent inductance $= \frac{\mu}{2}$.

Outside the conductor, Fig. 384, at a distance $s$ from the center of the wire, and for 1 cm. length of wire,

$$\mathfrak{IC}_a = \frac{4\pi I}{2\pi s} = \frac{2I}{s} \tag{617}$$

$$\mathfrak{B}_a = \frac{2\mu' I}{s} \tag{618}$$

$$d\phi_a = \mathfrak{B}_a ds = \frac{2\mu' I ds}{s} \tag{619}$$

$$\phi_{r-d} = \int_r^{d-r} \frac{2\mu' I ds}{s} = 2\mu' I \log_e \left(\frac{d-r}{r}\right) \qquad (620)$$

Hence total flux per centimeter length of conductor:

$$\phi = \phi_{r-d} + \phi_{o-r} = 2I\,\mu' \log_e \left(\frac{d-r}{r}\right) + \frac{\mu I}{2} \qquad (621)$$

$$L = \frac{\phi}{I} = 2\mu' \log_e \left(\frac{d-r}{r}\right) + \frac{\mu}{2} \qquad (622)$$

The flux distribution for each wire in a single-phase line is shown diagrammatically in Fig. 385 and the total flux for both wires in Fig. 386.

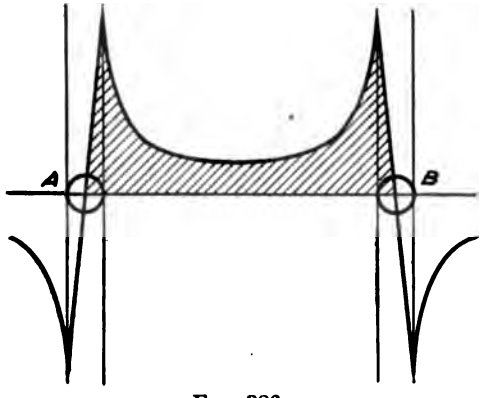



FIG. 386.

For copper, aluminum and all non-magnetic materials the permeability is unity; hence for most transmission lines,

$$\mu = \mu' = 1$$

Therefore, in copper or aluminum aerial single-phase transmission lines,

$L = 2 \log_e \left(\frac{d-r}{r}\right) + 0.5$ in electromagnetic c.g.s. units.

$$= [2 \log_e \left(\frac{d-r}{r}\right) + 0.5]10^{-9} \text{ henrys per cm. of conductor} \qquad (623)$$

Transforming from the Naperian to the common system of logarithms,

$$L = [4.61 \log\left(\frac{d-r}{r}\right) + 0.5]10^{-9} \text{ henrys per cm. of conductor.}$$

$$= 3.048\,[4.61 \log\left(\frac{d-r}{r}\right) + 0.5]10^{-5} \text{ henrys per 1,000 ft. of conductor.}$$

$$= [74.1 \log\left(\frac{d-r}{r}\right) + 8.05]\,10^{-5} \text{ henrys per mile of conductor} \quad (624)$$

In electrical handbooks, the corresponding reactances for 25, 60 and 100 cycles are given in tabular form, for the more common spacings and sizes of wire. For iron wire $\mu$ is not unity and the inductance inside the wire varies with the permeability.

(*c*) **Line Condensance.**—When a difference of potential exists between two conductors a stress is exerted upon the dielectric between the conductors. This stress produces the equivalent of

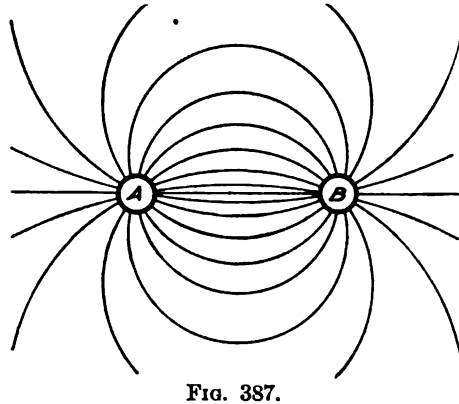



FIG. 387.

a strain and the product of the stress and strain measures the energy stored in the dielectric. Thus in Fig. 387 let *A* and *B* represent the two conductors (wires) of a single-phase transmission line. The voltage applied to the line exerts a stress and produces a strain in the dielectric (air) between the wires. With alternating currents the stress and the accompanying strain increase until the maximum point of the wave is reached and then decrease to zero, reverse, reaches a maximum in the other direction and again decrease to zero. Thus electric energy is stored in the dielectric and returned to the electric circuit twice during each voltage cycle. The strain is usually

represented by dielectric lines of force or dielectric flux, its relative magnitude by the flux density, and the total strain by the total number of lines of force. The distribution of the stress and strain in the dielectric circuit follows simple laws similar to those of the magnetic and electric circuits. In Fig. 388, let $A$ and $B$ represent the two wires in a single-phase line.

$d$ = distance between centers of the conductors.
$r$ = radius of each wire.
$s$ = distance of a point $a$ from center of conductor $A$.
$K_a$ = dielectric field intensity at a point $a$.
$D_a$ = dielectric flux density at a point $a$.
$\kappa$ = permittivity (specific inductive capacity).
$\gamma$ = proportionality factor.
$NN'$ = neutral plane.
$E_n$ = the voltage to neutral.

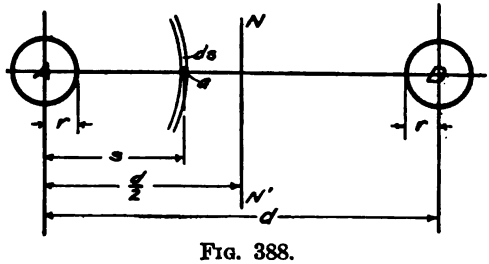



FIG. 388.

The dielectric stresses are symmetrical with respect to the neutral plane, and the voltage from the neutral plane is positive to one wire and negative to the other.

As the total stress in the dielectric is the same for any path between the two conductors, the discussion may be confined to the condition along the straight line connecting the centers. The flux density along this line is proportional to the total dielectric flux between the wires for different values of $d$ and $r$, and the proportionality factor is determined by the system of units employed. Since the dielectric stress varies inversely as the distance from the center of each wire and directly as the voltage, the dielectric field intensity at the point $a$ is expressed by equation (625).

$$K_a = \gamma \frac{\Psi}{\kappa}\left[\frac{1}{s} + \frac{1}{d-s}\right] \qquad (625)$$

The voltage absorbed in any elemental zone, is equal to the product of the field intensity and the distance.

$$de = K_s ds \tag{627}$$

The total voltage from conductor $A$ to neutral is therefore the integral of equation (627) between the limits of $r$ and $\frac{d}{2}$.

$$E_n = \int_r^{\frac{d}{2}} K_s ds = \gamma \frac{\Psi}{\kappa} \int_r^{\frac{d}{2}} \left[\frac{1}{s} + \frac{1}{d-s}\right] ds = \gamma \frac{\Psi}{\kappa} \log_e \left(\frac{d-r}{r}\right) \tag{628}$$

$$C = \frac{\Psi}{E_n} = \frac{\kappa}{\gamma \log_e \left(\frac{d-r}{r}\right)} \tag{629}$$

For air $\kappa = 1$, and for the electrostatic c.g.s. system of units $\gamma = 2$.

$$C = \frac{1}{2 \log_e \left(\frac{d-r}{r}\right)} \text{ per cm. of wire in c.g.s. electrostatic units} \tag{630}$$

To change from the electrostatic to the electromagnetic system of units, equation (630) must be divided by the square of the velocity of propagation of an electric field in space (equal to the velocity of light).

$$C = \frac{1}{2v^2 \log_e \left(\frac{d-r}{r}\right)} \text{ per cm. of wire in c.g.s. electromagnetic units.} \tag{631}$$

For circuits in which $r$ is small as compared to $d$, the equation may be simplified:

$$C = \frac{1}{2v^2 \log \frac{d}{r}} \tag{632}$$

$$v = 3 \times 10^{10} \text{ cm.-sec.}$$

Reduced to farads,

$$C = \frac{10^9}{2v^2 \log_e \frac{d}{r}} \tag{633}$$

For microfarads and common system of logarithms,

$$C = \frac{0.241}{10^6 \log \frac{d}{r}} \text{ microfarads per cm. of wire to neutral.}$$

$$= \frac{7{,}360}{10^6 \log \frac{d}{r}} \text{ microfarads per 1,000 ft. of wire to neutral.}$$

$$= \frac{3.883}{100 \log \frac{d}{r}} \text{ microfarads per mile of wire to neutral} \quad (634)$$

In electrical handbooks the *charging currents* for a wide range of wire sizes and spacings and for standard frequencies are given in tabular form. It is generally more convenient to use the charging current in commercial problems than derive the microfarads by equation (634).

$$_c\dot{I} = 2\pi f C\dot{E} = {}_cb\dot{E} \quad (635)$$

(*d*) **Velocity of Propagation of an Electric Field.**—The speed at which the electric field travels through space may be expressed in terms of the inductance and condensance of the transmission line.

From equation (620):

$L' = 2 \log_e \left(\frac{d-r}{r}\right)$, the inductance outside the conductor in air per cm. of length of line.

From equation (631):

$C = \frac{1}{2v^2 \log_e \left(\frac{d-r}{r}\right)}$, the condensance outside the conductor in air per cm. of length of line.

Therefore,

$$CL' = \frac{1}{v^2}; \text{ or, } v = \frac{1}{\sqrt{CL'}} \quad (636)$$

Due to retardation produced by the losses the actual speed is somewhat less. Since $v$, the velocity of propagation of an electro-magnetic field (same as the speed of light) is known, it is possible to calculate either the inductance or the condensance when the other is given. This method is sometimes used in commercial problems when it is difficult to measure both quantities.

(*e*) **Line Leakage.**—In addition to the resistance, inductance and condensance which, as already shown, may be calculated with a fair degree of accuracy, or obtained from tables in electrical handbooks, another factor, more or less constant enters into

the transmission-line problem. The dielectric between the conductors is not a perfect insulator and hence a current in phase with the voltage leaks through the dielectric throughout the whole length of the line. As a result power is dissipated along the transmission line in proportion to the product of the leakage current and the line voltage. The amount of this leakage and consequent power loss are not so readily predetermined as the resistance and reactance factors. Even in aerial high-tension transmission lines the leakage depends on many factors that are difficult to define, or whose quantitative values are either not known, or continually vary within wide limits. The temperature, humidity and barometric pressure of the air, the amount of dust in the air and the velocity of the wind, the condition of the insulators and line, the wave shape and maximum voltage, corona and other factors enter into the problem and it is evident that exact calculations are impracticable. Fortunately the leakage loss in commercial lines in good condition is relatively small and may, in most cases, be neglected.

(*f*) **Transmission-line Constants.**—The transmission-line consists of a series circuit having resistance and inductance interwoven with parallel circuits having conductance and susceptance between the conductors. The properties of the transmission line may be stated in terms of four constants, called the *transmission-line constants*.

(1) $r$ = effective resistance.
$rI$ = voltage consumed in phase with the current.
$rI^2$ = power consumed by the ohmic resistance, mutual inductance, magnetic hysteresis. Series circuit.
(2) $x$ = effective reactance.
$jxI$ = voltage consumed in quadrature with the current.
$jxI^2$ = reactive power from self and mutual inductance. Series circuit.
(3) $g$ = effective conductance.
$gE$ = current consumed in phase with the voltage.
$gE^2$ = power consumed by leakage, dielectric induction, dielectric hysteresis, etc. Parallel circuits between the line wires.
(4) $b$ = effective susceptance.
$jbE$ = current consumed in quadrature with the voltage.
$jbE^2$ = reactive power due to line condensance and dielectric induction. Parallel circuits between the line wires.

These constants are uniformly distributed over the whole length of the transmission line and hence the elemental unit of the network is the differential length, *dl*, and the corresponding line constants, *dr*, *dx*, *dg* and *db*. The complete solution of the problem for transmission lines with uniformly distributed resistance, inductance, condensance and leakage is given in Chap. XXVII. The derived equations are complicated and their application to commercial lines involves long and tedious calculations.

For short transmission lines certain assumptions may be made that greatly reduce the labor involved in making the calculations. The results obtained by short-cut methods are in most cases sufficiently close approximations to the actual values for ordinary commercial work.

(*g*) **Equations for Short Lines under Special Assumptions.**—*Case I.—When both line condensance and line leakage are omitted from the calculations.*

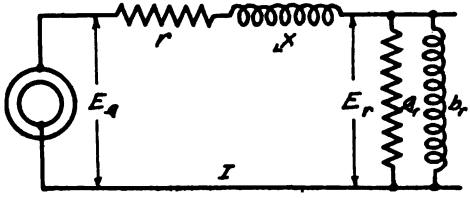



FIG. 389.

The corresponding circuit and vector diagrams are shown in Figs. 389 and 390. With the condensance and leakage omitted the problem becomes simply the line resistance and inductance in series with the receiver load.

$E_r$ = voltage at the receiver end.
$E_g$ = voltage at the generator end.
$r$ = line resistance.
$x$ = line inductive reactance.
$\cos \theta_r$ = power factor at receiver end.
$\cos \theta_g$ = power factor at generator end.
$g_r$ = conductance of receiver.
$b_r$ = susceptance of receiver.

From the vector diagram in Fig. 390 we have:

$$E_g^2 = (E_r \cos \theta_r + rI)^2 + (E_r \sin \theta_r + xI)^2 \qquad (638)$$

$$\cos \theta_g = \frac{E_r \cos \theta_r + rI}{E_g} \qquad (639)$$

**Mershon's Diagram.**—For any given commercial transmission line operating at constant frequency the impedance is constant and hence the voltage triangle, *ABD*, Fig. 390, is directly proportional to the current. The line regulation is dependent both upon the impedance drop and the phase relations of the voltages or upon the power factor. With a variable power factor it is desirable to have some convenient way in which to calculate the

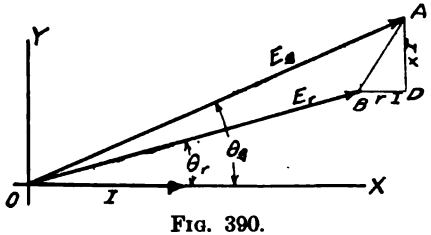



FIG. 390.

line regulation for commercial loads of any power factor. A number of graphical charts have been proposed and several are in commercial use. Mershon's diagram is convenient and frequently used. Let *OB* in Fig. 390 be rotated around *O* as a center as shown in Fig. 391. Let the system operate with a constant voltage at the receiver end and express the voltages in per cent. with *OB* the receiver voltage as 100 per cent. Draw

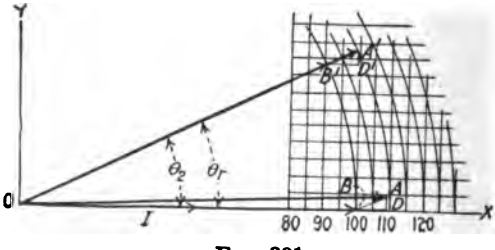



FIG. 391.

concentric circles at convenient differences as 5 per cent. in Fig. 391. A section of Fig. 391 drawn to a larger scale with circles 1 per cent. apart is shown in Fig. 392. Let the distance *OB* along the *X*-axis be the power-factor scale for the receiver load. To find the regulation proceed as follows:

Given: The size, length and spacing of the line conductors, the load and power factor at the receiver end and the receiver voltage.

(*a*) Find the resistance and reactance drops in per cent. of the receiver voltage from the given data.

(*b*) Starting on the *X*-axis at the given power factor follow the ordinate until it intersects the *OB* circle. From the point of

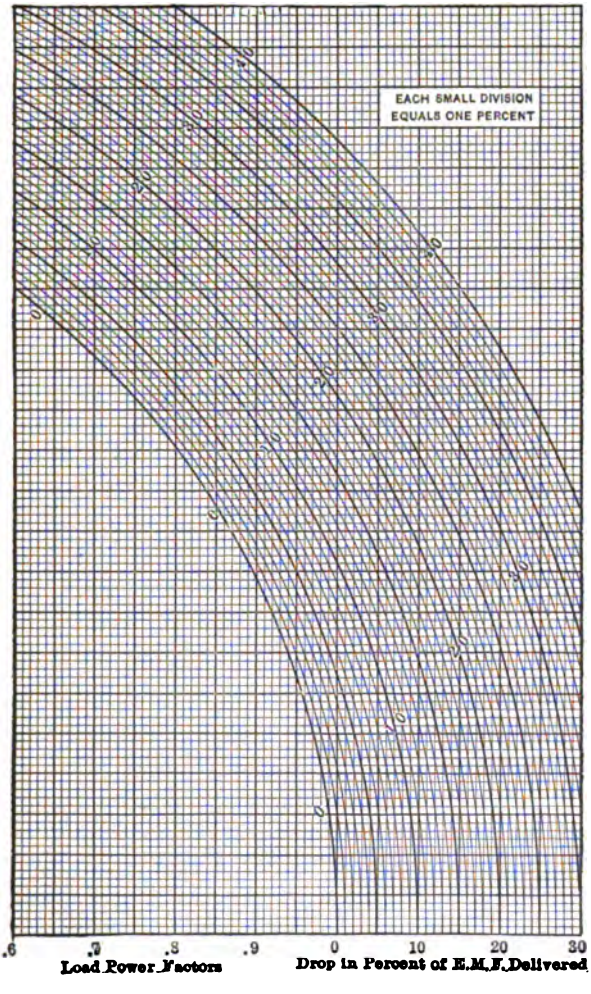



FIG. 392.

intersection lay off along the coördinate parallel to the *X*-axis the resistance drop in per cent. as found under (*a*). From this point lay off on the ordinate upward the reactance drop in per cent. Follow the arc of the circle passing through the last point

down to the $X$-axis and obtain the per cent. regulation for the given load and power factor.

An example will make the process plain.

Given: Single-phase transmission line, 2 miles long, of No. 0 copper wire, spaced 24 in. Receiver voltage 2,400, receiver load 220 kw. at 90 per cent. power factor; $f = 25$ cycles.

($a$) Find line resistance and reactance drops.

$$\text{Line current} = \frac{220 \times 1{,}000 \times 100}{2{,}400 \times 85} = 107.8 \text{ amp.}$$

Line resistance (handbook tables, per mile of No. 0 = 0.534) = 2.14 ohms.

Line reactance (handbook tables, per mile of No. 0 = 0.265) = 1.06 ohms.

$$\text{Resistance drop} = \frac{107.8 \times 2.14}{2{,}400} = 9.6 \text{ per cent.}$$

$$\text{Reactance drop} = \frac{107.8 \times 1.06}{2{,}400} = 4.8 \text{ per cent.}$$

($b$) From the 90 per cent. point on the $X$-axis, Fig. 391, follow the ordinate until it intersects the $OB$ circle, at $B'$. Lay off the resistance drop of 9.6 per cent. parallel to the $X$-axis $B'D'$. From $D'$ lay off the reactance drop, 4.8 per cent. parallel to the $Y$-axis to $A'$. Follow the circle passing through $A'$ to the $X$-axis and the regulation is found to be 11 per cent. of the receiver voltage. Expressed in terms of the generator voltage the regulation $= \frac{11}{100 + 11} = 9.9$ per cent.

In order to reverse the process and find the size of wire for a given load, power factor and line drop one must solve by *trial and error*. As commercial wires differ in diameter by definite steps the nearest size is readily found.

*Case II.*—Let the *additional assumption* be made *that the power factor at the generator is the same as at the receiver end of the line*, a condition that approximately obtains in short lines. This simplifies the equations as may be seen from Fig. 393. If $\cos \theta_g - \cos \theta_r$ is small, the projection of vector $E_g$ along $E_r$ produced, is almost equal to $E_g$, or $FH$ may be neglected in the calculations.

Hence (approximately)

$$E_g - E_r = BF = I(r \cos \theta_r + {}_{\iota}x \sin \theta_r) \qquad (640)$$

This gives the line drop per wire to neutral. Between lines the voltage drops for single-phase and three-phase lines are:

$$e_{1\phi} = 2I(r \cos \theta_r + {}_{\iota}x \sin \theta_r) \text{ for single-phase} \quad (641)$$

$$e_{3\phi} = \sqrt{3}I(r \cos \theta_r + {}_{\iota}x \sin \theta_r) \text{ for three-phase} \quad (642)$$

In order to facilitate the line-drop calculations graphical methods are often used. The chart in Fig. 394 is convenient for rapidly calculating line drop under the conditions assumed. When using the chart proceed as follows:

Find the given size of wire on the positive $X$-axis. Follow the ordinate to the proper power-factor line; from there move along the abscissa to scale I or scale II on the right-hand edge. This gives the factor $r \cos \theta_r$. Similarly in order to find $x \sin \theta_r$, find the given size of wire along the $Y$-axis. Follow the line to the left until it intersects the desired spacing curve. From the

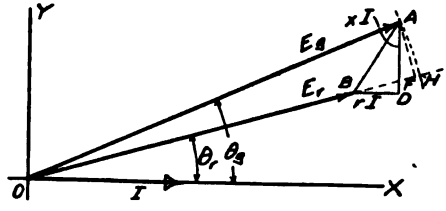



FIG. 393.

point of intersection follow the ordinate to the $X$-axis and obtain the reactance for 100 cycles. Then follow the arc of the circle to the intersection of the desired frequency line. From this intersection move horizontally to the right and intersect with the desired power-factor line. The ordinate passing through this intersection gives, on the upper scale, the desired value for $x \sin \theta_r$. An example will illustrate the use of the chart:

| | |
|---|---|
| Three-phase line, distance | 25 miles. |
| Spacing of wires | 30 in. |
| Full load to be delivered | 4,000 kw. |
| Voltage at receiver end | 30,000 volts. |
| Frequency | 60 cycles. |
| Power factor of load | 85 per cent. |
| Size of wire, copper | No. 0 A.W.G. |

In the right-hand quadrant follow the line for No. 0 wire and find the intersection with the 85 per cent. power-factor line. $r \cos \theta_r = 0.440$. In the left-hand quadrant find intersection of No. 0 wire and the 30 in. spacing curve, move along the ordinate to the $X$-axis, follow the circular arc to the 60-cycle line, then

move horizontally to the 85 per cent. power-factor line, and from the intersection vertically to the upper scale. $x \sin \theta_r = 0.335$. Total drop per mile of wire per ampere = 0.440 + 0.335 = 0.775 volt.

$$I = \frac{4{,}000 \times 1{,}000}{30{,}000 \times \sqrt{3} \times 0.85} = 90.7 \text{ amp.}$$

$$\text{Total drop} = \sqrt{3} \times 90.7 \times 25 \times 0.775 = 3{,}440 \text{ volts.}$$

$$\text{Regulation} = \left(\frac{30{,}000 + 3{,}440}{30{,}000}\right) 100 - 100 = 11.4 \text{ per cent.}$$

*Case III.—Let the total line condensance be represented by a condenser across the middle of the line as indicated in Fig.* 395. *No leakage.*

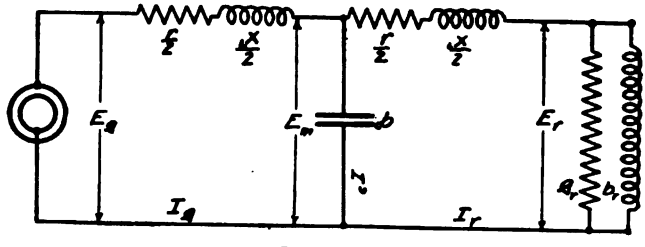



FIG. 395.

In addition to the notation used under Case *I*, let,

$jb$ = line susceptance.

$E_m$ = voltage at middle of line.

$jI$ = condensance or charging current.

$I_r$ = current at receiver end.

$I_g$ = current at generator end.

Beginning at the receiver end of the line we have:

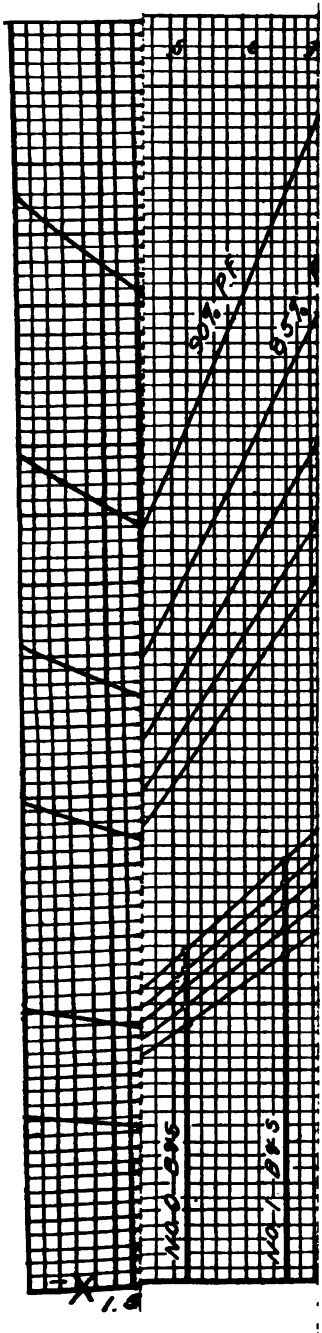
$$\dot{I}_r = \dot{E}_r(g_r - jb_r) \tag{643}$$

$$\dot{E}_m = \dot{E}_r + \left(\frac{r + jx}{2}\right)\dot{I}_r = \dot{E}_r\left[1 + \frac{(r + jx)(g_r - jb_r)}{2}\right] \tag{644}$$

$$\dot{I}_g = \dot{I}_r + jb\dot{E}_m = \dot{E}_r\left\{g_r - jb_r + jb\left[1 + \frac{(r + jx)(g_r - jb_r)}{2}\right]\right\}$$

$$= \dot{E}_r\left\{\left[g_r + \frac{jb}{2}(rb_r - xg_r)\right] + j\left[jb - b_r + \frac{jb}{2}(rg_r + xb_r)\right]\right\} \tag{645}$$

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$$\dot{E}_g = \dot{E}_m + \frac{(r+j_l x)}{2} I_g = \dot{E}_r\Big[1 + \frac{(r+j_l x)}{2}\frac{(g_r - jb_r)}{2} + \frac{(r+j_l x)}{2}\frac{(g_r - jb_r)}{2} + \frac{j_c b(r+j_l x)}{2} + \frac{j_c b(r+j_l x)^2(g_r - jb_r)}{4}\Big]$$

$$= \dot{E}_r\Big[1 + (r+j_l x)\Big(g_r - jb_r + \frac{j_c b}{2}\Big) + \frac{j_c b}{4}(r+j_l x)^2(g_r - jb_r)\Big] \qquad (646)$$

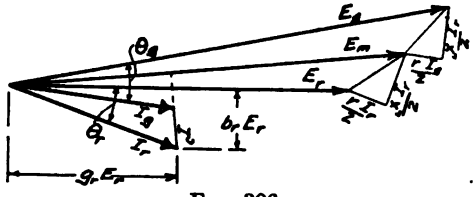



FIG. 396.

The current and voltage relations are shown graphically by the vector diagram in Fig. 396.

*Case IV.—Let the total condensance of the line be represented by three condensers, one-sixth at each end, and two-thirds at the middle. No leakage.* The circuit diagram under the above assumptions is shown in Fig. 397. In addition to the notation used in Case II, we have, as indicated in the diagram:

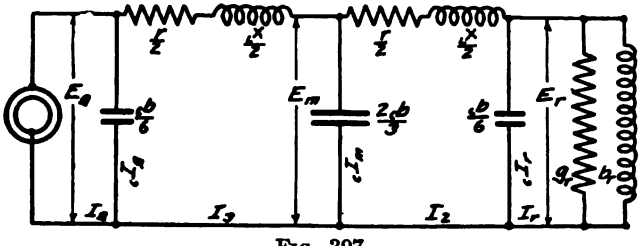



FIG. 397.

$_cI_r$ = condensance or charging current at receiver end.
$_cI_m$ = condensance or charging current at middle of line.
$_cI_g$ = condensance or charging current at generator end.

Starting at the receiver end and proceeding toward the generator, we have:

$$\dot{I}_r = \dot{E}_r(g_r - jb_r) \qquad (647)$$

$$\dot{I}_2 = \dot{I}_r + {}_c\dot{I}_r = \dot{I}_r + j_c\frac{b\dot{E}_r}{6} = \dot{E}_r\Big((g_r - jb_r + \frac{j_c b}{6}\Big) \qquad (648)$$

$$\dot{E}_m = \dot{E}_r + \left(\frac{r + j_l x}{2}\right)\dot{I}_2 = \dot{E}_r\left[1 + \left(\frac{r + j_l x}{2}\right)\left(g_r - jb_r + \frac{j_c b}{6}\right)\right] \tag{649}$$

$$\begin{aligned}\dot{I}_3 &= \dot{I}_2 + {}_c\dot{I}_m = \dot{I}_2 + \left(\frac{j2_c b}{3}\right)\dot{E}_m \\ &= \dot{E}_r\left[g_r - jb_r + j\tfrac{5}{6}{}_c b + \frac{j_c b}{3}(r + j_l x)(g_r - jb_r + \frac{j_c b}{6})\right]\end{aligned} \tag{650}$$

$$\begin{aligned}\dot{E}_g &= \dot{E}_m + \left(\frac{r + j_l x}{2}\right)\dot{I}_3 = \dot{E}_r\left[1 + (r + j_l x)(g_r - jb_r + \frac{j_c b}{2})\right. \\ &\qquad \left. + \frac{j_c b}{6}(g_r - jb_r + \frac{j_c b}{6})(r + j_l x)^2\right]\end{aligned} \tag{651}$$

$$\begin{aligned}\dot{I}_g &= \dot{I}_3 + {}_c\dot{I}_g = \dot{I}_3 + \frac{j_c b}{6}\dot{E}_g \\ &= \dot{E}_r\left[(g_r - jb_r + j_c b) + \frac{j_c b}{6}(r + j_l x)(3g_r - 3jb_r + \tfrac{5}{6}j_c b)\right. \\ &\qquad \left. - \frac{{}_c b^2}{36}(r + j_l x)^2(g_r - jb_r + \frac{j_c b}{6})\right]\end{aligned} \tag{652}$$

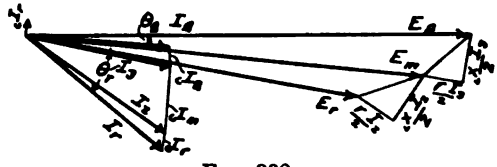


FIG. 398.

The current and voltage relations at the receiver, generator and at the middle of the line are shown graphically by the vector diagram in Fig. 398.

*Case V.*—For moderately long transmission lines a closer approximation to the actual condition of uniformly distributed inductance, condensance and resistance is necessary. The complete solution gives complicated equations that are not readily applied but which must be used for very long lines, as shown in Chap. XXVII. These equations may be expanded into rapidly converging series, and the first few terms give results sufficiently accurate for commercial purposes for lines up to 150 miles long. These approximate equations are derived in Chap. XXVII, page 493.

In transmission problems in polyphase circuits it is most convenient to calculate the line characteristics of one line to neutral.

The solution of any given problem is also made on the basis of one line to neutral. Thus in a three-phase circuit by taking one-third of the load and the voltage to neutral the solution is the same as for a single wire to neutral in a single-phase system.

(*h*) **Line Transformers.**—In the above discussion the transformers at both ends of the transmission line are not included. Usually it is desirable to calculate the voltage relations on the low side at both the generator and receiver ends. This introduces both the step-up and the step-down transformers between the generators and the receiver distribution system. The resistance, inductance and condensance of the transformers may be considered as part of the line constants, and a complete solution obtained from one set of equations. This is often done in commercial problems. The resistance and reactance of the transformers at both ends of the line are added to the constants of the line proper and all calculations made in terms of the high-tension circuit. For preliminary calculations, or in the absence of definite information, it is customary to assume the transformer resistance drop to be 1 per cent. and the inductive reactance drop 6 per cent. of the impressed voltage. The actual values may vary from 0.5 to 2.0 per cent. for the resistance and from 3.5 to 9.0 per cent. for the reactance. The lower limit for the resistance and the higher values for the inductance refer to transformers on long distance, high-voltage transmission lines. At the receiver end the equivalent values for the step-down transformers may be found as follows:

Let $P_r$ = power delivered to receiver.
$_xd_{rt}$ = per cent. reactance drop assumed for the transformer.
$_rd_{rt}$ = per cent. resistance drop assumed for the transformer.
$I_r$ = line current at receiver.
$E_r$ = receiver voltage between mains.
$_nE_r = \frac{E_r}{\sqrt{3}}$ = receiver voltage to neutral.

$$_xd_{rt} = \frac{100 I_r x_{rt}}{_nE_r} \tag{653}$$

$$_rd_{rt} = \frac{100 I_r r_{rt}}{_nE_r} \tag{654}$$

$$I_r = \frac{P_r}{\sqrt{3} E_r \cos \theta_r} \tag{655}$$

$$x_{rt} = {}_xd_{rt}\frac{E_r^2 \cos\theta_r}{100P_r} \text{ ohms} \tag{656}$$

$$r_{rt} = {}_rd_{rt}\frac{E_r^2 \cos\theta_r}{100P_r} \text{ ohms} \tag{657}$$

Similar equations give the constants for the transformers at the generating station. The transformers are considered as connected from line to neutral in making the above calculations, whether the actual connections are in delta or star.

(*i*) **Line Regulation.**—From the previous discussion in this chapter, it is evident that the transmission-line drop depends upon the line constants and also upon both the magnitude and power factor of the load. This interaction of the reactive component of the receiver circuit with the impedance of the line forms the basis for voltage regulation on the whole transmission system and merits careful consideration. For long transmission lines it is necessary to consider the distribution of the inductance and condensance on the line, as discussed in Chap. XXVII, in determining the effects of variations in the power factor of the receiver circuit. For short lines this is not necessary, and the equations and computations may be greatly simplified by assuming that the transmission line consists merely of a resistance and an inductance in series with the receiver. A closer approximation for somewhat longer lines may be made by considering a condenser equivalent to the line condensance connected across the line at the receiver end. This condenser may be included in, and considered as part of, the receiver circuit, thus requiring no changes in the equations derived under the condition that the line condensance is omitted. Let the discussion be for a single-phase circuit. The same equations may be applied to balanced three-phase circuits by using either *equivalent single-phase* values or solving *for one-third of the load with one line wire and voltage to neutral.*

$z_l = r_l + jx_l$ impedance of transmission line.
$y_r = g_r - jb_r$ admittance of receiver circuit.
$E_g$ = voltage at generator end of line.
$E_r$ = voltage at receiver end of line.
$I_l$ = current in transmission line.

For any given transmission line the resistance and reactance are constants while the load in the receiver circuit may vary both

in magnitude and power factor. In the discussion it will be assumed that the generator voltage, the line resistance and reactance are constants while the receiver conductance and susceptance are variable, as indicated in Fig. 399. It is necessary to derive the equations for the receiver voltage and line current in terms of the given constants and for any desired load.

$$\dot{I}_l = \dot{E}_r y_r \tag{658}$$

$$\dot{E}_g = \dot{E}_r + \dot{I}_l z_l = \dot{E}_r(1 + y_r z_l) \tag{659}$$

$$\dot{E}_r = \frac{\dot{E}_g}{1 + y_r z_l} = \frac{\dot{E}_g}{(1 + r_l g_r \pm x_l b_r) + j(x_l g_r \mp r_l b_r)} \tag{660}$$

$$I_l = \frac{\dot{E}_g y_r}{1 + y_r z_l} = \frac{\dot{E}_g(g_r - jb_r)}{(1 + r_l g_r \pm x_l b_r) + j(x_l g_r \mp r_l b_r)} \tag{661}$$

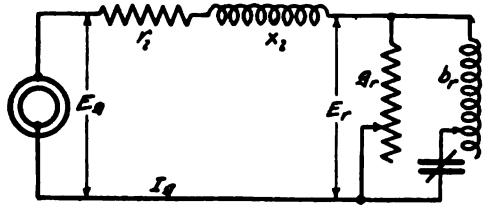



FIG. 399.

For absolute values:

$$E_r = \frac{E_g}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{662}$$

$$I_l = \frac{E_g \sqrt{g_r^2 + b_r^2}}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{663}$$

For convenience let $\alpha$ represent the ratio of the receiver and generator voltages.

$$\alpha = \frac{E_r}{E_g} = \frac{1}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}} \tag{664}$$

$$\text{The regulation} = \frac{E_g - E_r}{E_r} = \frac{1 - \alpha}{\alpha} \tag{665}$$

The power delivered to the receiver circuit:

$$P_r = E_r^2 g_r = E_g^2 \alpha^2 g_r \tag{666}$$

Efficiency of transmission

$$= \frac{E_r^2 g_r}{E_r^2 g_r + r_l I_l^2} = \frac{g_r}{g_r + r_l y_r^2} \tag{667}$$

With a constant voltage at the generator and a given transmission line, the power transmitted, current in the line, voltage at the receiver, efficiency of transmission and voltage regulation all depend upon both the magnitude and relative values of the conductance and the susceptance of the receiver circuit. In order to determine the effects produced by either variable it is convenient to discuss the problem under the following assumptions:

Case I. $g_r$ variable, $b_r = 0$.
Case II. $b_r$ variable, $g_r = 0$.
Case III. $b_r$ variable, $g_r$ constant.
Case IV. $g_r$ variable, $b_r$ constant.
Case V. Maximum efficiency of transmission.
Case VI. Maximum voltage at receiver circuit.

*Case I. $g_r$ variable, $b_r = 0$.*—In this case the receiver voltage, line current, ratio of voltages, power delivered and efficiency of transmission are given in equations (668) and (674).

$$\dot{E}_r = \frac{\dot{E}_g}{(1 + r_l g_r) + j x_l g_r} \tag{668}$$

$$\dot{I}_l = \frac{g_r \dot{E}_g}{(1 + r_l g_r) + j x_l g_r} \tag{669}$$

In absolute values:

$$E_r = \frac{E_g}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{670}$$

$$I_l = \frac{g_r E_g}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{671}$$

$$\alpha = \frac{1}{\sqrt{(1 + r_l g_r)^2 + (x_l g_r)^2}} \tag{672}$$

$$P_r = E_r^2 g_r = E_g^2 \alpha^2 g_r \tag{673}$$

Efficiency of transmission

$$= \frac{1}{1 + r_l g_r} \tag{674}$$

*Case II. $b_r$ variable, $g_r = 0$.*—The equations are readily derived in the same manner as under Case I. Since no power is delivered to the receiver circuit the assumed conditions are seldom realized in commercial systems.

*Case III. $b_r$ variable, $g_r$ constant.*—From equation (666) the power delivered to the receiver circuit is:

$$P_r = E_g^2 \alpha^2 g_r$$

For constant conductance, $g_r$, in the receiver, the power will be a maximum when $\alpha$ is a maximum, or when $\frac{d\alpha}{db_r}$ equals zero.

$$\text{Hence} \quad x_l(1 + r_l g_r + x_l b_r) - r_l(x_l g_r - r_l b_r) = 0$$

$$\text{or} \quad \frac{x_l}{r_l^2 + x_l^2} + b_r = b_l + b_r = 0 \tag{675}$$

Hence for maximum power the line susceptance must be equal in magnitude to the receiver susceptance but of opposite sign. That is, if the line has inductive reactance the receiver must have condensive reactance of an equal amount. Substituting this value for $b_r$ in equations (660) to (667) inclusive we have:

$$\dot{E}_r = \frac{\dot{E}_g}{1 + y_r z_l} = \frac{\dot{E}_g}{(r_l + jx_l)(g_r + g_l)} \tag{676}$$

$$E_r = \frac{E_g}{Z_l(g_r + g_l)} \tag{677}$$

$$\dot{I}_l = \frac{\dot{E}_g y_r}{1 + Z_l y_r} = \frac{\dot{E}_g(g_r + jb_l)}{(1 + r_l g_r - x_l b_l) + j(r_l b_l + x_l g_r)} \tag{678}$$

$$\dot{I}_l = \dot{E}_g \sqrt{\frac{g_r^2 + b_l^2}{(1 + r_l g_r - x_l b_l)^2 + (r_l b_l + x_l g_r)^2}} = \dot{E}_g \frac{\sqrt{g_r^2 + b_r^2}}{Z_l(g_r + g_l)} \tag{679}$$

$$\alpha = \frac{1}{(r_l + jx_l)(g_r + g_l)} \tag{680}$$

Maximum power,

$$P_r = E_g^2 \alpha^2 g_r \tag{681}$$

Power factor at receiver $= \cos \theta_r$; where

$$\theta_r = \tan^{-1} \frac{b_r}{g_r} = \tan^{-1} \frac{(-b_r)}{g_r} \tag{682}$$

Power factor at generator $= \cos \theta_g$; where

$$\theta_g = \tan^{-1} \left( \frac{x_r + x_l}{r_r + r_l} \right) \tag{683}$$

*Case IV. $g_r$ variable, $b_r$ constant.*—The power delivered to the receiver, $P_r = E_g^2 \alpha^2 g_r$, will be a maximum when $\alpha^2 g_r$ is maximum, or when

$$\frac{d(\alpha^2 g_r)}{dg_r} = 0$$

$$\frac{d(\alpha^2 g_r)}{dg_r} = \frac{d}{dg_r}\left[\frac{g_r}{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}\right] = 0$$

$$\therefore (1 + r_l g_r + x_l b_r) - 2g_r(r_l + r_l g_r + x_l b_r) = 0 \tag{684}$$

Hence the conductance for maximum power in the receiver circuit is:

$$g_r = \sqrt{g_l^2 + (b_r + b_l)^2} \tag{685}$$

Substituting this value in the equations for receiver voltage and power delivered to the receiver the following expressions are obtained:

$$\dot{E}_r = \frac{\dot{E}_g(g_l - jb_l)}{\sqrt{2g_r(g_r + g_l)}} \tag{686}$$

$$\dot{E}_r = \frac{\dot{E}_g Y_l}{\sqrt{2g_r(g_r + g_l)}} \tag{687}$$

$$P_r = \frac{E_g^2 Y_l^2}{2(g_r + g_l)} = \frac{E_g^2}{2\left[r_l + \sqrt{r_l^2 + \left(x_l + x_r \frac{z_l}{z_r}\right)^2}\right]} \tag{688}$$

$$\alpha = \frac{Y_l}{\sqrt{2g_r(g_r + g_l)}} \tag{689}$$

If in the above expressions the receiver susceptance be taken equal to minus the line susceptance, $b_r = -b_l$, as derived in Case III for the condition of maximum power as a function of the receiver susceptance we have:

$$b_r = -b_l;\ g_r = g_l;\ y_r = y_l;\ x_r = -x_l;\ r_r = r_l;\ z_r = z_l \tag{690}$$

$$Y_l = \sqrt{g_l^2 + b_l^2} = \sqrt{g_r^2 + b_r^2} = Y_r \tag{691}$$

$$Z_l = \sqrt{r_l^2 + x_l^2} = \sqrt{r_r^2 + x_r^2} = z_r \tag{692}$$

With the inductive reactance in the line balanced by an equal condensive reactance in the receiver, the maximum power delivered becomes the same as for direct currents. For, substituting the values given in (690) in equations (688) and (689):

$${}^{m}P_r = \frac{E_g^2}{4r_l} \tag{693}$$

$${}^{m}\alpha = \frac{Y_l}{2g_l} = \frac{Z_l}{2r_l} \tag{694}$$

*Case V. Maximum Efficiency of Transmission.*—Since the line loss varies as the square of the current and for any given load the current is a minimum when the power factor is unity, the maximum efficiency for any given load is obtained when the current is in phase with the voltage.

Hence,

$$\cos \theta_g = 1, \text{ or } x_r = -x_l \tag{695}$$

Substituting in equations (664) and (666):

$$\alpha = \frac{Z_r}{(r_r + r_l)} \tag{696}$$

$$P_r = \frac{E_g^2 r_r}{(r_r + r_l)^2} \tag{697}$$

The power at maximum efficiency will be a maximum when $g_r = g_l$ as explained under Case IV.

Hence, maximum $P_r$ at maximum efficiency $= \frac{E_g^2}{4r_l}$,

and

$$\alpha = \frac{Z_l}{2r_l} \tag{698}$$

*Case VI. Maximum Voltage at Receiver Circuit.*—The ratio of the receiver and generator voltages, $\alpha$, contains both the constants of the line and the receiver conductance and susceptance. For any given line and constant generator voltage the receiver voltage depends upon both $g_r$ and $b_r$.

$$\alpha = \frac{E_r}{E_g} = \frac{1}{\sqrt{(1 + r_l g_r + x_l b_r)^2 + (x_l g_r - r_l b_r)^2}}$$

Hence for maximum receiver voltage,

$$\frac{d\alpha}{dg_r} = 0 \quad \text{and} \quad \frac{d\alpha}{db_r} = 0 \tag{699}$$

These conditions are fulfilled when $g_r = 0$, and $b_r = -b_l$. Hence, for maximum receiver voltage a condition of resonance exists and the line resistance alone opposes the flow of the current. Substituting in equations (664) and (666):

$$\alpha = \frac{E_r}{E_g} = \frac{Z_l}{r_l} = \frac{Y_l}{g_l} \tag{700}$$

$$I_l = \frac{E_g}{r_l} \tag{701}$$

**PROBLEM**

**1.** Show that in a three-phase transmission line with equilateral spacing of conductors the condensance "to neutral" is twice the condensance between two conductors. Show that the inductance "per wire" is one-half the inductance of two wires.

## CHAPTER XXII

## PHASE CONTROL OR REGULATION BY POWER FACTOR

The interaction of the line reactance and the receiver susceptance provides a basis for voltage regulation in alternating-current systems that is not available in direct-current systems. The receiver voltage may be varied, within limits, independently of the power delivered by adjusting the receiver susceptance while the generator voltage is held constant.

This method of voltage control in a circuit delivering power over an inductive line by varying the receiver power factor is known as *phase control* or *regulation by power factor.* It is used to a large extent in transmission lines supplying power to electric railways through synchronous motors or rotary converters, and is the accepted standard in long-distance transmission systems. For short lines the required variation in the receiver susceptance is supplied by adjusting the field excitation of the synchronous motors or rotary converters. The required variation is provided automatically by a series winding on the field coils. At no load or light loads the fields are under-excited and therefore the current is lagging; at overload the compound field winding over-excites the synchronous machine which then takes a leading current from the line. The receiver susceptance is in this manner automatically adjusted from inductive at no load to condensive at overload and provides the means for voltage regulation. In some cases shunt-wound converters or synchronous motors are used and the field excitation is controlled by Tirrill regulators in much the same manner as was explained for voltage regulation of alternators in Chap. XIV.

In long-distance transmission lines synchronous condensers are installed at the receiver end, with their field excitation controlled by Tirrill regulators. These machines carry no load but automatically supply the leading or lagging reactive component of the current and thereby keep the voltage at the receiver constant, at some predetermined value, for all loads. The line condensance forms an important factor in the calculations as is shown in Chap. XXVII.

For short lines, only the inductive reactance need be considered. In many systems the inductive reactance in the line does not give a sufficient range in voltage regulation for the changes in load. The additional reactance required for phase control is obtained by inserting reactive coils in the line or by a magnetic bridge in the transformers; methods often used in transmission lines supplying power to electric railways. Hence, while the resistance in the system is kept at a minimum, the reactance is often greatly increased by inserting reactive coils in order to secure the advantage of automatic voltage regulation by phase control.

A diagrammatic representation of the complete circuit, omitting line and transformer condensance, in a system consisting of generators, step-up transformers, transmission lines, step-down transformers and synchronous motors or rotary converters is shown in Fig. 400

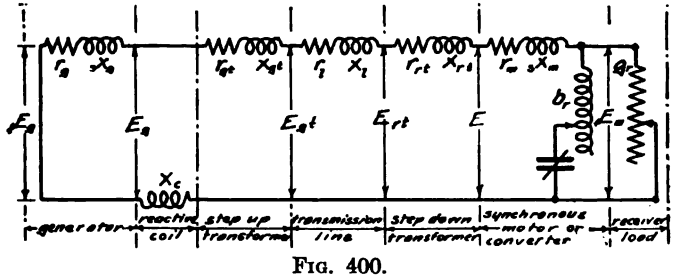



FIG. 400.

$g_r$ = receiver conductance, from the direct-current load.

$b_r$ = receiver susceptance, from the synchronous motor or converter, may be either inductive or condensive, and is obtained by under- or over-exciting the motor field.

$r_m$, $_sx_m$ = resistance and synchronous reactance of synchronous motor or converter.

$r_{rt}$, $x_{rt}$ = resistance and reactance of step-down transformers.

$r_l$, $x_l$ = resistance and reactance of transmission line.

$r_{gt}$, $x_{gt}$ = resistance and reactance of step-up transformers.

$x_o$ = reactance of inserted reactive coil.

$r_g$, $_sx_g$ = resistance and synchronous reactance of generator.

$_fE_g$ = nominal induced voltage in generator, proportional to the field excitation.

$E_g$ = voltage at generator terminals.

$E_r$ = voltage at terminals of synchronous motor.
${}_fE_m$ = nominal induced voltage in synchronous motor, proportional to the field excitation.

If the calculations are made for the nominal induced voltages the synchronous impedance of both the generator and the synchronous motor must be included. If the terminal voltages of the generator and synchronous motor are used in the equations only the impedances between the two points enter into the calculations. Since the resistance and reactance in the line, step-up and step-down transformers and reactive coil form one series circuit the notation may be abbreviated.

Let $R = r_{rt} + r_l + r_{gt}$ = total resistance between generator and motor terminals.
$X = x_{rt} + x_l + x_c + x_{gt}$ = total reactance between generator and motor terminals.
$Z = R + jX$ = total impedance.

Selecting $E_r$ as reference vector:

$$\dot{I} = g_r\dot{E}_r - jb_r\dot{E}_r = {}_g\dot{I} + j_b\dot{I} \tag{711}$$

$$\dot{E}_g = \dot{E}_r + Z\dot{I} = (\dot{E}_r + R_g\dot{I} - X_b\dot{I}) + j(X_g\dot{I} + R_b\dot{I}) \tag{712}$$

$$E^2{}_g = (E_r + R_gI - X_bI)^2 + (X_gI + R_bI)^2 \tag{713}$$

*This is the fundamental equation for phase control.*

From (713):

$$E_r = \sqrt{E_g{}^2 - (X_gI + R_bI)^2} - (R_gI - X_bI) \tag{714}$$

$${}_bI = \frac{E_rX}{Z^2} - \sqrt{\frac{E_g{}^2}{Z^2} - \left(\frac{E_rR}{Z^2} + {}_gI\right)^2} \tag{715}$$

$$= \frac{ErX - \sqrt{E^2Z^2 - (E_rR + {}_gIZ^2)^2}}{Z^2}$$

The maximum load, or power component of current at which unity power factor can be maintained with the supply voltage, $E_g$, constant, is found by equating the quantity under the radical to zero.

$${}_gI_{max} = \frac{E_gZ - E_rR}{Z^2} \tag{716}$$

Phase control is in most cases used for regulating the voltage so as to automatically hold a predetermined ratio between the generator and receiver voltages for all loads. This ratio may be equal to, less or greater than unity.

First, $E_r = E_g$; second, $E_r < E_g$; third, $E_r > E_g$.

Let $E_g$ and $E_r$ be constant and the load variable.

In the problem four current values are of special importance.

(*a*) ${}_bI_o$ = the reactive component at no load.
(*b*) ${}_gI_{max}$ = the power component at maximum load.
(*c*) ${}_gI_u$ = the power component at which the reactive component is zero.
(*d*) ${}_gI_f$ = the power component at full load.

At no load ${}_gI = 0$, and from equation (715):

$$ {}_bI = \frac{E_rX - \sqrt{E_g{}^2Z^2 - E_r{}^2R^2}}{Z^2} = \frac{E_rX - \sqrt{(E_g{}^2 - E_r{}^2)\,R^2 + E_g{}^2X^2}}{Z^2} \tag{717} $$

For $E_r = E_g$, ${}_bI = 0$; (718)

$E_r < E_g$, ${}_bI < 0$; hence current lagging (719)

$E_r > E_g$, ${}_bI > 0$; hence current leading (720)

When power is delivered to the receiver circuit the field excitation of the synchronous motors or converters must be increased if the voltages shall remain constant.

Hence with load, ${}_gI > 0$.

For $E_r = E_g$, ${}_bI > 0$, hence current leading (721)

$E_r < E_g$, ${}_bI \begin{matrix} < \\ = \\ > \end{matrix} 0$, hence current lagging, in phase or leading (722)

$E_r > E_g$, ${}_bI > 0$, hence current leading (723)

When $E_r$ is less than $E_g$ the current is lagging at no load and for light loads, in phase at some load depending upon the ratio selected and leading for larger loads. The average quadrature component for a variable load is therefore less when $E_r < E_g$ than for either $E_r = E_g$ or $E_r > E_g$. If the choice of the ratio is made on the basis of a minimum copper loss the shunt field of the synchronous motor or converter must be adjusted to give unity power factor at such load as to require a minimum quadrature component over the given range. With a continually varying load the actual minimum may be difficult to determine. Generally the reduction in efficiency due to copper losses over the possible minimum is of little importance. However, these

losses affect the temperature rise in the machines and thereby affect the permissible output. On this account it is customary to so adjust the shunt field as to give unity power factor at the generator at full load. The possible maximum load and the no-load reactive current depend on the amount of reactance in the line; the larger the required overload capacity the larger the reactance in the line and consequently the larger the no-load current. As the line reactance may be adjusted by inserting reactive coils, a compromise is usually made in a more or less arbitrary manner, between a desirable large overload capacity and a likewise desirable minimum no-load current. It is generally assumed that ${}_bI_o = k{}_gI_f$, in which the factor $k$ depends on the conditions of the system and may vary widely, probably from 20 per cent. to 50 per cent. Hence, in problems of phase control seven quantities are involved; $E_r$, $E_g$, $X$, ${}_gI_f$, ${}_gI_u$, ${}_bI_o$, $I_{max}$; of these, two are given by the load requirements ($E_r$, ${}_gI_f$), two are assumed (${}_gI_u$, ${}_bI_o$) and the remaining three are calculated. Assuming ${}_gI_u = {}_gI_f$, ${}_bI_o = k{}_gI_u$ and letting the reactive component produced by the compound field winding vary as a linear function of the load, ${}_bI = k({}_gI_u - {}_gI)$, then by substituting in equation (713) we have:

*At no load:*

$$ {}_gI = 0,\ {}_bI = k{}_gI_u \tag{724} $$

$$ E_g^2 = (E_r - kX{}_gI_u)^2 + (-kR{}_gI_u)^2 \tag{725} $$

*At full load:*

$$ {}_gI = {}_gI_u,\ {}_bI = 0 \tag{726} $$

$$ E_g^2 = (E_r + R{}_gI_u)^2 + (X{}_gI_u)^2 \tag{727} $$

From (725) and (727):

$$ X = \frac{\dfrac{kE_r}{{}_gI_u} \pm \sqrt{\dfrac{E_r^2}{{}_gI_u^2}(k^2+1) - \left[\dfrac{E_r}{{}_gI_u} - R(k^2-1)\right]^2}}{(k^2-1)} \tag{728} $$

$E_g$ may be found by substituting the value of $X$ from (724) in either (725) or (727); and ${}_gI_{max}$ by substituting the above derived values in equation (716). Similar equations may be derived if the maximum load current, ${}_gI_{max}$, is assumed instead of the no-load current, ${}_bI_o$, as above.

The double sign before the radical in equation (728) shows that two values for the reactance, $X_1$ and $X_2$, satisfy the given

conditions. Hence two values of the generator voltage are found to produce the required constant voltage at the receiver; or by dividing the reactance in two parts, $X_1$ and $X_2 - X_1$, three points on the system may be kept at constant voltage for wide variations in load. It is therefore possible to automatically keep the voltages constant at the receiver, $E_r$, at the generator terminals, $E_g$, and also the nominal induced voltage in the generator, ${}_fE_g$, which is proportional to the field excitation. Since the generator armature has a resistance, $r_g$, in addition to the reactance, ${}_sx_g$, the equations for the two values of $X$ that satisfy the required conditions of constant voltage at the receiver and generator terminals with constant field excitation of the generator are given by equations (729) and (730).

$$X_1 = \frac{\frac{kE_r}{{}_gI_u} - \sqrt{\frac{E^2{}_r}{{}_gI_u{}^2}(k^2+1) - \left[\frac{E_r}{{}_gI_u} - R(k^2-1)\right]^2}}{(k^2-1)} \tag{729}$$

$$X_2 = \frac{\frac{kE_r}{{}_gI_u} + \sqrt{\frac{E_r{}^2}{{}_gI_u{}^2}(k^2+1) - \left[\frac{E_r}{{}_gI_u} - (R+r_g)(k^2-1)\right]^2}}{(k^2-1)} \tag{730}$$

Referring to Fig. 461, for constant $E_r$, $E_g$, and ${}_fE_g$,

$$X_1 = X_{gt} + X_l + X_{rt} + X_e \tag{731}$$

$$X_2 = X_1 + {}_sX_g \tag{732}$$

The value of ${}_sX_g$ may be adjusted by means of a reactive coil in addition to the normal synchronous reactance of the generator.

## CHAPTER XXIII

## COMMERCIAL WAVE FORMS—HARMONICS

The preceding chapters have dealt almost exclusively with alternating current and voltage waves of the simple harmonic form. The fundamental relations of alternating-current circuits have been discussed under the assumption that the instantaneous values of the voltage and the current could be expressed by $e = {}^{\prime\prime}E \sin \omega t$ and $i = {}^{\prime\prime}I \sin \omega t$. Both in theoretical discussions and in practical operation the sine wave is taken as the standard and any deviation is termed a distortion. Although the sine wave is the ideal form sought by both the operating and designing engineer, many factors in both the generators and distribution system cause distortion in the current and voltage wave shapes. As mentioned in Chap. II and illustrated by Figs. 18, 20 and 21 the distortions may be so large that even the semblance of a fundamental sine wave is lost.

(*a*) **Fourier's Series.**—Before attempting an analysis of the several factors that produce distortions in the shape of the voltage and current waves it is desirable to note two fundamental characteristics:

(1) The waves are periodic, that is, the successive cycles are alike.

(2) The function is single valued. At any point in the system, at any instant, the voltage or current has only one value.

In 1822, long before alternating currents came into commercial use, Fourier published his researches and proved that any single-valued, periodic function can be completely expressed by a simple trigonometric series (741), now known as Fourier's series.

$$\begin{aligned} y &= A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \ . \ . \ . \ + A_n \sin nx \\ &\quad + B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \ . \ . \ . \ + B_n \cos nx \end{aligned} \tag{741}$$

The same series is often written in another form as shown in equation (742) and using the notation applicable to any distorted voltage wave.

$$e = {}^{\prime\prime}E_1 \sin(\omega t + \gamma_1) + {}^{\prime\prime}E_2 \sin(2\omega t + \gamma_2) + {}^{\prime\prime}E_3 \sin(3\omega t + \gamma_3) + {}^{\prime\prime}E_4 \sin(4\omega t + \gamma_4) + \ldots {}^{\prime\prime}E_n \sin(n\omega t + \gamma_n) \ldots \quad (742)$$

The first term in (742) is called the fundamental and the other terms are the harmonics. The transformation factors between (742) and (741) are:

$$y = e; x = \omega t$$

$${}^{\prime\prime}E_1 = \sqrt{A_1^2 + B_1^2}; {}^{\prime\prime}E_2 = \sqrt{A_2^2 + B_2^2}; {}^{\prime\prime}E_3 = \sqrt{A_3^2 + B_3^2}, \text{ etc.}$$

$$\gamma_1 = \tan^{-1}\frac{B_1}{A_1}; \gamma_2 = \tan^{-1}\frac{B_2}{A_2}; \gamma_3 = \tan^{-1}\frac{B_3}{A_3}; \text{ etc.}$$

Any voltage or current wave may in this manner be expressed by a series of sine waves of multiple frequency and of different maximum values and phase positions. While the series has an infinite number of terms only a few are required to express the elemental characteristics of waves in commercial power systems. In most cases only a few of the harmonics are present and three or four terms fully express the distorted voltage or current wave.

(*b*) **Distorted Voltage Waves.**—In commercial power systems the alternating voltage is produced by rotating machinery and hence the positive and negative halves of the wave are equal in magnitude and similar in shape. It is readily seen that waves with equal positive and negative halves can have no even harmonics. Hence for voltage waves in power circuits,

$$e = {}^{\prime\prime}E_1 \sin(\omega t + \gamma_1) + {}^{\prime\prime}E_3 \sin(3\omega t + \gamma_3) + {}^{\prime\prime}E_5 \sin(5\omega t + \gamma_5) + \ldots {}^{\prime\prime}E_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1}] \quad (743)$$

In most power circuits the voltage wave consists of a fundamental combined with the third and fifth harmonics, while sometimes the seventh and ninth harmonics are of importance. Except in special cases harmonics above the eleventh may be neglected. Thus, in Fig. 401, is shown a wave consisting of the fundamental and a third harmonic. The expression for the instantaneous voltage, equation (744), has two terms only. The coefficients ${}^{\prime\prime}E_5$, ${}^{\prime\prime}E_7$, ${}^{\prime\prime}E_9 \ldots {}^{\prime\prime}E_{2n-1}$ are equal to zero and hence all but the first two terms of the series vanish.

$$e = 100 \sin(\omega t + 2°) + 9.7 \sin(3\omega t - 27°) \quad (744)$$

In this case the maximum of the fundamental is greater than the actual voltage in the circuit.

In Fig. 402 and in equation (745) is shown a wave consisting of a fundamental and the third and fifth harmonics. The maximum

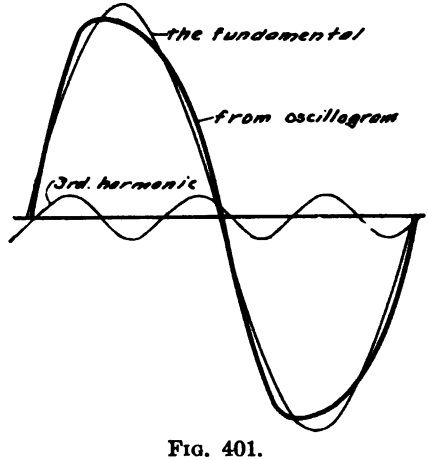



FIG. 401.

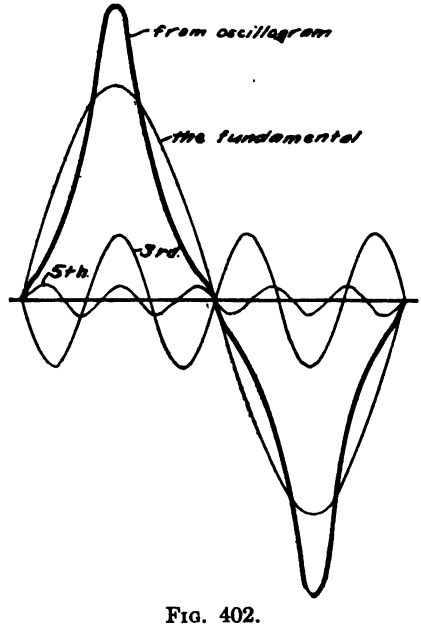



FIG. 402.

of the fundamental is considerably less than the maximum of the actual voltage wave.

$$e = 100 \sin(\omega t) + 28.2 \sin(3\omega t + 180°) + 5.4 \sin(5\omega t) \quad (745)$$

The three component waves reach a positive maximum at the same time, producing a peaked voltage wave whose maximum is equal to the sum of the three maxima or 133.6 per cent. of the fundamental. The waves shown in Figs. 401 and 402 were taken by an oscillograph from two machines in the laboratory. The fundamental and component waves were found by analysis as will be explained in a later paragraph. It is, however, evident that not only the relative magnitude of the fundamental and the harmonics but also their phase position affect the shape of the resultant wave. By keeping the same magnitude of the three component waves as in Fig. 402, but changing the phase positions of the harmonics by 180° the resulting complex wave is changed

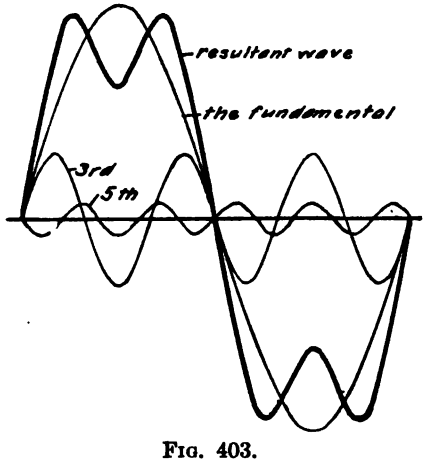



FIG. 403.

into the form shown in Fig. 403 and the corresponding equation is given in (746)

$$e = 100 \sin(\omega t) + 28.2 \sin(3\omega t) + 5.4 \sin(5\omega t + 180°) \quad (746)$$

The effect of phase position is more fully illustrated in Figs. 404 and 405. In Fig. 404 is shown a series of waves formed by combining the same fundamental with a 20 per cent. third harmonic but differing by 30° in phase angle. Similar effects due to changes of phase angle of 30° between a fundamental and a 20 per cent. fifth harmonic are shown in Fig. 405. The component curves and corresponding equations are given in each case. From these illustrations it is apparent that the combination of a fundamental with two or three harmonics may produce

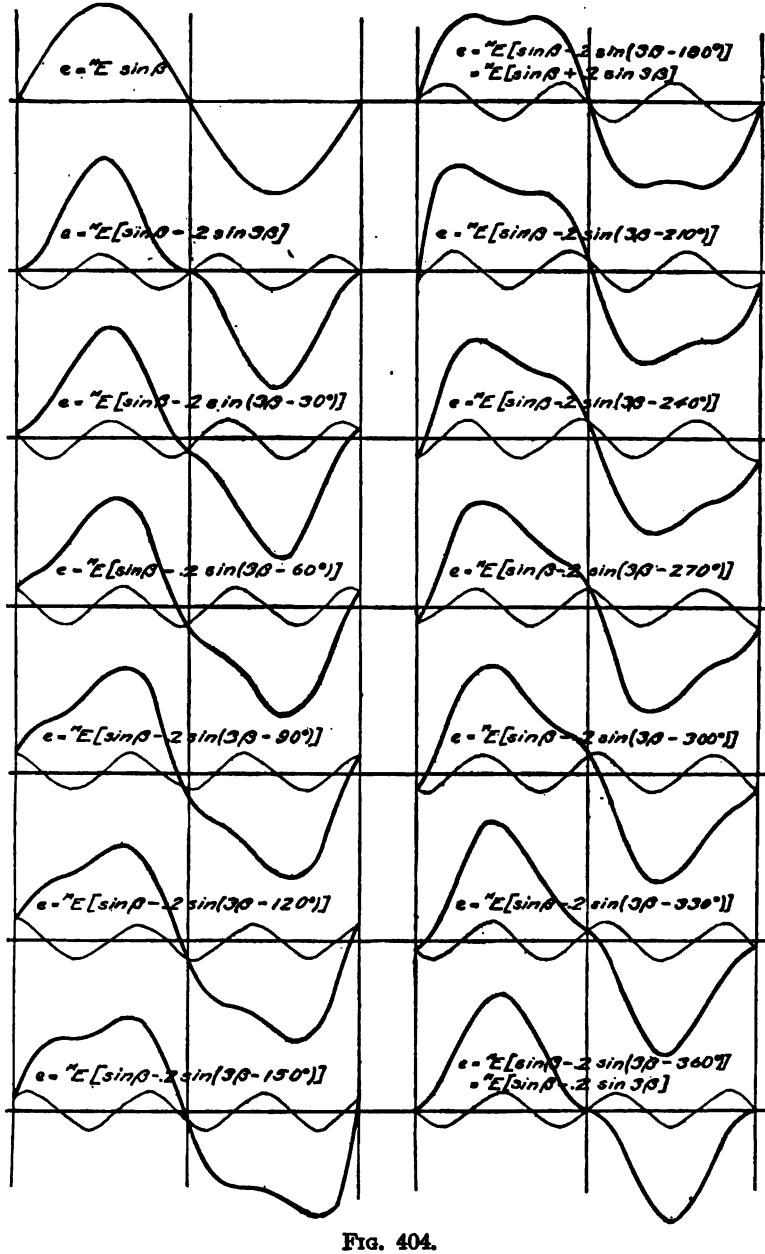

e = "E sinβ
e = "E[sinβ - 2 sin(3β - 180°)]
= "E[sinβ + 2 sin 3β]
e = "E[sinβ - 2 sin 3β]
e = "E[sinβ - 2 sin(3β - 210°)]
e = "E[sinβ - 2 sin(3β - 30°)]
e = "E[sinβ - 2 sin(3β - 240°)]
e = "E[sinβ - 2 sin(3β - 60°)]
e = "E[sinβ - 2 sin(3β - 270°)]
e = "E[sinβ - 2 sin(3β - 90°)]
e = "E[sinβ - 2 sin(3β - 300°)]
e = "E[sinβ - 2 sin(3β - 120°)]
e = "E[sinβ - 2 sin(3β - 330°)]
e = "E[sinβ - 2 sin(3β - 150°)]
e = "E[sinβ - 2 sin(3β - 360°)]
= "E[sinβ - 2 sin 3β]

FIG. 404.

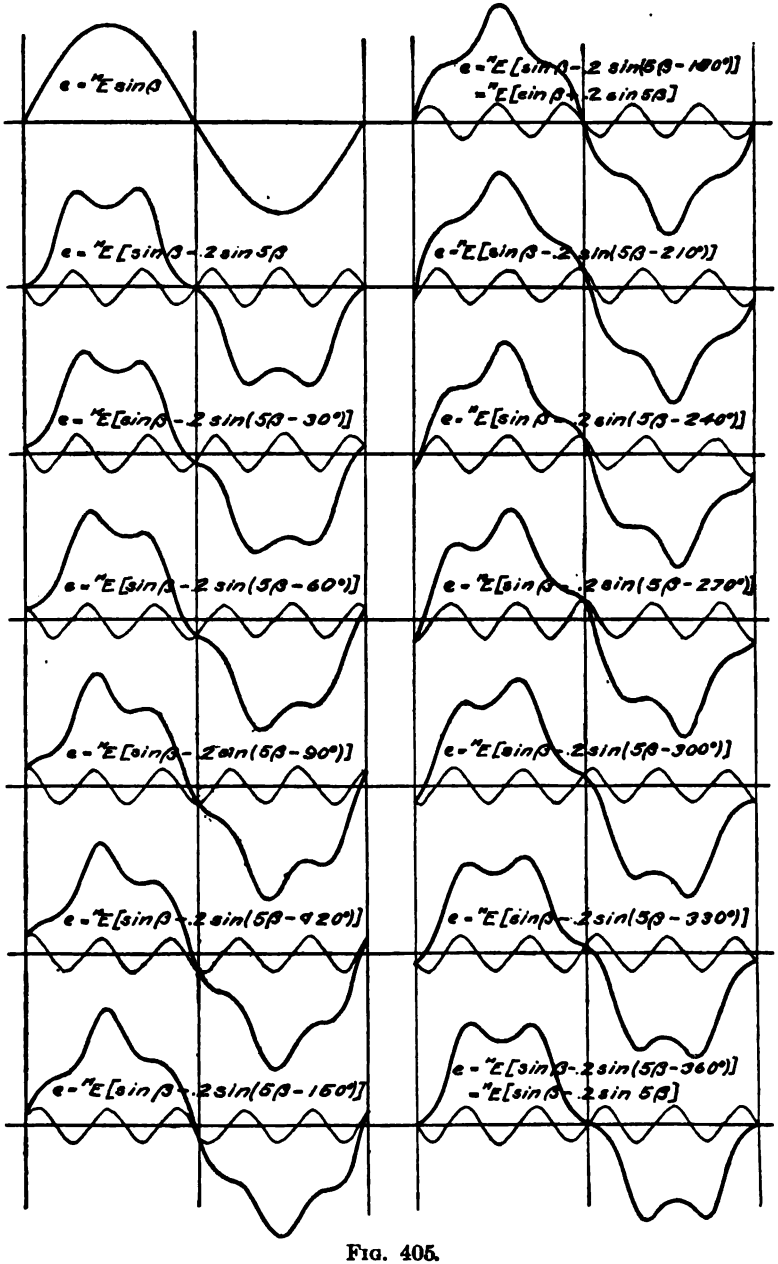

e = "E sinβ
e = "E [sinβ - .2 sin 5β
e = "E [sinβ - .2 sin(5β - 30°)]
e = "E [sinβ - .2 sin(5β - 60°)]
e = "E [sinβ - .2 sin(5β - 90°)]
e = "E [sinβ - .2 sin(5β - 120°)]
e = "E [sinβ - .2 sin(5β - 150°)]
e = "E [sinβ - .2 sin(5β - 180°)]
= "E [sinβ - .2 sin 5β]
e = "E [sinβ - .2 sin(5β - 210°)]
e = "E [sinβ - .2 sin(5β - 240°)]
e = "E [sinβ - .2 sin(5β - 270°)]
e = "E [sinβ - .2 sin(5β - 300°)]
e = "E [sinβ - .2 sin(5β - 330°)]
e = "E [sinβ - .2 sin(5β - 360°)]
= "E [sinβ - .2 sin 5β]

FIG. 405.

a very great variety of wave forms. Often the shape of the wave makes it quite evident what harmonics are present and an approximate solution may be found by inspection. In general a systematic analysis is required to determine the magnitude and phase relations of the harmonics. To the operating engineer it often becomes of great importance to know just what harmonics are present in the voltage or current waves. To gain this information is usually the first step in determining the cause of any periodic disturbance in the system. Knowing the relative magnitude of the harmonics present and noting which one is specially prominent the problem of finding the cause of the difficulty is simplified. It is of importance to distinguish between higher harmonics of voltage and higher harmonics of current. In general they are interdependent and the distorting effects of circuit conditions may affect one or both. Thus, if a sine voltage wave is impressed upon a transformer the hysteresis of the iron core distorts the current wave; and conversely, if a current of sine wave form is passed through the same transformer the hysteresis causes a distortion of the voltage wave.

(*c*) **Generated Voltage Waves.**—In any circuit the first source of voltage wave distortion is in the alternator. With no current flowing, that is, on open circuit, the instantaneous voltage at the generator terminals is directly proportional to the instantaneous rate of cutting lines of force. A simple conductor revolving in a uniform field (Fig. 14, Chap. II) at uniform velocity, generates a sine voltage wave. Distortions may therefore be caused by:

(1) Lack of uniformity or pulsation of the field;

(2) Variation in the speed; and

(3) The distribution of the armature conductors connected in series.

In present-day alternators the speed is always so nearly uniform that this possible source of wave distortion need not be considered.

The slotted armature necessarily affects the distribution of the magnetic flux and produces pulsations in the field that in turn cause distortions in the voltage wave. Thus, in Fig. 406, is shown a voltage wave having a pronounced twenty-fifth harmonic produced by the pulsations of the magnetic flux. In alternators with few slots and teeth per pole the movement of the slots across the field pole produces pulsations of the magnetic reluctance of

the field circuit and hence with constant excitation a pulsation in the magnetic flux. Therefore, in a machine having $s$ slots per pole the magnetic flux pulsates with $2s$ frequency.

Assuming that the pulsations follow a simple sine law and have $k^{\nu}\Phi$ amplitude, the instantaneous flux interlinked with the armature coil is:

$$\phi = {}^{\nu}\Phi \cos(\omega t)\,[1 + k \cos(2s\omega t - \gamma)] \tag{747}$$

The voltage generated is:

$$e = -n\frac{d\phi}{dt} \tag{748}$$

From (747) and (748):

$$e = 2\pi f n^{\nu}\Phi\{\sin(\omega t)[1 + k\cos(2s\omega t - \gamma)] + 2sk\cos(\omega t)\sin(2s\omega t - \gamma)\} \tag{749}$$

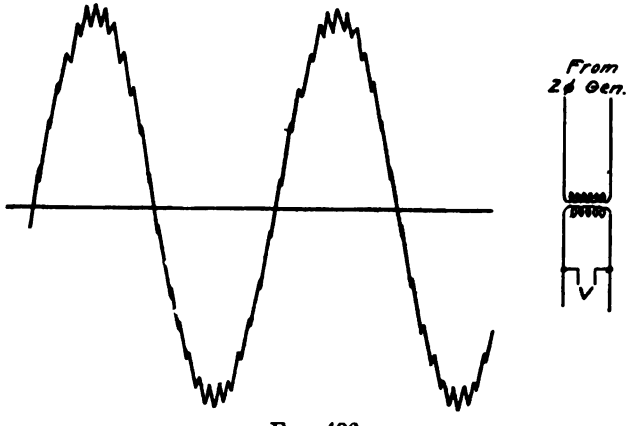



FIG. 406.

From trigonometry:

$$\left.\begin{aligned} 2\sin x\cos y &= \sin(x+y) + \sin(x-y \\ 2\cos x\sin y &= \sin(x+y) - \sin(x-y \end{aligned}\right\} \tag{750}$$

Hence,

$$e = 2\pi f n^{\nu}\Phi\{\sin\omega t + k\frac{2s-1}{2}\sin[(2s-1)\omega t - \gamma] + k\left(\frac{2s+1}{2}\right)\sin[(2s+1)\omega t - \gamma]\} \tag{751}$$

The pulsation of the magnetic flux due to the armature slots is therefore the source of two harmonics in the voltage wave of frequencies $2s + 1$ and $2s - 1$ times the fundamental.

For $s = 1$

$$e = 2\pi f n' \Phi[\sin(\omega t) + \frac{k}{2}\sin(\omega t - \gamma) + \frac{3k}{2}\sin(3\omega t - \gamma)] \quad (752)$$

For $s = 2$

$$e = 2\pi f n' \Phi[\sin(\omega t) + \frac{3k}{2}\sin(3\omega t - \gamma) + \frac{5k}{2}\sin(5\omega t - \gamma)] \quad (753)$$

Similarly for three slots the pulsations of the field flux introduce a fifth and a seventh harmonic into the generated voltage. While the pulsations of field flux to some extent distort the voltage wave, the main factors determining the wave shape are the flux distribution and the arrangement of the armature conductors. A single conductor or a bundle of conductors in a single slot produce a voltage wave of the same shape as the flux distribution curve. However, the voltage for each phase of an alternator armature is ordinarily made up of the vector sum of the electromotive forces of several conductors in series, since the adjacent slots are displaced by an angle corresponding to the slot pitch. The voltage wave at the terminals of the generator therefore differs from the flux distribution curve and depends upon:[1]

(1) The fractional pitch of the armature winding;
(2) The number of slots per pole;
(3) The number of coils per slot; and
(4) The angular span of the single-phase belt.

(*d*) **Distorted Current Waves. Constant R, L and C.**—In order to determine the current wave shape produced by any given impressed voltage wave the circuit conditions must be known. In many commercial circuits the so-called *circuit constants*, the resistance, inductance and condensance fluctuate or pulsate in a more or less irregular way during each cycle of the voltage or current waves. Thus in any circuit having iron, the inductance and hence the reactance varies, while the current changes from its zero to its maximum values. Moreover, the variation is not the same for decreasing as for increasing values of the current. Likewise, the resistance in the vapor of an arc lamp decreases with increase of the current. Similarly under conditions where corona is produced at the crest of the voltage wave the condensance pulsates during the voltage cycle. The discussion of current wave shapes therefore naturally falls into

[1] C. A. Adams, Wave Shape of Alternators, A. I. E. E., *Trans.*, XXVIII, 1053.

two groups: *first*, in circuits with constant $r$, $L$ and $C$; *second*, in circuits with $r$, $L$ or $C$ pulsating or changing in magnitude during each cycle.

In Chap. IV it was shown that the instantaneous value of the voltage in a series circuit was given by the differential equation:

$$e = ri + L\frac{di}{dt} + \frac{1}{C}\int idt \tag{754}$$

If the voltage is a sine wave, the current is likewise simple harmonic and of the same frequency but differing in magnitude and phase position.

For

$$e = {}^{\prime\prime}E \sin(\omega t),\ i = {}^{\prime\prime}I \sin(\omega t - \theta);$$

in which

$${}^{\prime\prime}E = z\,{}^{\prime\prime}I$$

and

$$\theta = \tan^{-1}\frac{{}_{\prime}x - {}_{\prime}x}{r}$$

Since the voltage equation is single valued, the law of superposition may be applied; and as each voltage wave produces a corresponding current of the same frequency, it follows that *in a complex voltage wave each harmonic produces its own current independent of the fundamental and all the other harmonics.*

Hence,

$$\begin{aligned} e &= e_1 + e_3 + e_5 + \ldots + e_{2n-1} \\ &= {}^{\prime\prime}E_1 \sin(\omega t + \gamma_1) + {}^{\prime\prime}E_3 \sin(3\omega t + \gamma_3) + {}^{\prime\prime}E_5 \sin(5\omega t + \gamma_5) \\ &\quad + \ldots + {}^{\prime\prime}E_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1}] \end{aligned} \tag{755}$$

and,

$$\begin{aligned} i &= i_1 + i_3 + i_5 + \ldots + i_{2n-1} \\ &= \frac{{}^{\prime\prime}E_1}{\sqrt{r^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin\left[\omega t + \gamma_1 - \tan^{-1}\left(\frac{\omega L}{r} - \frac{1}{r\omega C}\right)\right] \\ &\quad + \frac{{}^{\prime\prime}E_3}{\sqrt{r^2 + \left(3\omega L - \frac{1}{3\omega C}\right)^2}} \sin\left[3\omega t + \gamma_3 - \tan^{-1}\left(\frac{3\omega L}{r} - \frac{1}{3r\omega C}\right)\right] \\ &\quad + \frac{{}^{\prime\prime}E_5}{\sqrt{r^2 + \left(5\omega L - \frac{1}{5\omega C}\right)^2}} \sin\left[5\omega t + \gamma_5 - \tan^{-1}\left(\frac{5\omega L}{r} - \frac{1}{5r\omega C}\right)\right] \end{aligned}$$

$$+ \ldots + \frac{{}^{\varkappa}E_{2n-1}}{\sqrt{r^2 + \left[(2n-1)\omega L - \frac{1}{(2n-1)\omega C}\right]^2}} \sin\left\{(2n-1)\omega t + \gamma_{2n-1} - \tan^{-1}\left[\frac{(2n-1)\omega L}{r} - \frac{1}{(2n-1)r\omega C}\right]\right\} \quad (756)$$

In order to abbreviate the notation in equation (756), let,

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{r} - \frac{1}{nr\omega C}\right) \quad (757)$$

$${}^{\varkappa}I_n = \frac{{}^{\varkappa}E_n}{\sqrt{r^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}} \quad (758)$$

Then,

$$i = {}^{\varkappa}I_1 \sin(\omega t + \gamma_1 - \theta_1) + {}^{\varkappa}I_3 \sin(3\omega t + \gamma_3 - \theta_3) + {}^{\varkappa}I_5 \sin(5\omega t + \gamma_5 - \theta_5) + \ldots + {}^{\varkappa}I_{2n-1} \sin[(2n-1)\omega t + \gamma_{2n-1} - \theta_{2n-1}] \quad (759)$$

Comparing the expression for the voltage in equation (755) with the resulting current as expressed in either equation (756) or (759) it is seen that the current wave form differs from the voltage due to both a change in phase position and relative magnitude of the harmonic wave. Thus the phase position of the current for the third harmonic is $\theta_3$ behind the corresponding third harmonic of the voltage wave. Similarly the fifth harmonics differ by $\theta_5$ while $\theta_3$ is not equal to $\theta_5$. The relative magnitude of the harmonics in the current is not the same as for the voltage wave, for in each case the maximum current for each harmonic equals the maximum voltage divided by the corresponding impedance as shown in equation (760).

$${}^{\varkappa}I_n = \frac{{}^{\varkappa}E_n}{z_n} = \frac{{}^{\varkappa}E_n}{\sqrt{r^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}} \quad (760)$$

It is readily seen that the relative values of $r$, $L$ and $C$ are of great importance in determining the current wave form resulting from a given impressed voltage wave.

Thus in a circuit having resistance only,

$${}^{\varkappa}I_n = \frac{{}^{\varkappa}E_n}{r} \text{ and } \theta_n = 0 \quad (761)$$

The current is in phase with the voltage wave and has the same

wave form. This is illustrated in Fig. 407. The oscillogram shows a voltage wave $e$ having a pronounced third and fifth harmonic. This voltage was impressed upon a circuit having resistance only and the curve $i$ in the oscillogram shows the current wave.

In a circuit having both resistance and inductance the current wave is different from the impressed voltage.

$$^{n}I_n = \frac{^{n}E_n}{\sqrt{r^2 + (n\omega L)^2}}, \text{ and } \theta_n = \tan^{-1}\left(\frac{n\omega L}{r}\right) \qquad (762)$$

The higher the harmonic the less current flows for the same impressed voltage, since the reactance increases with the fre-

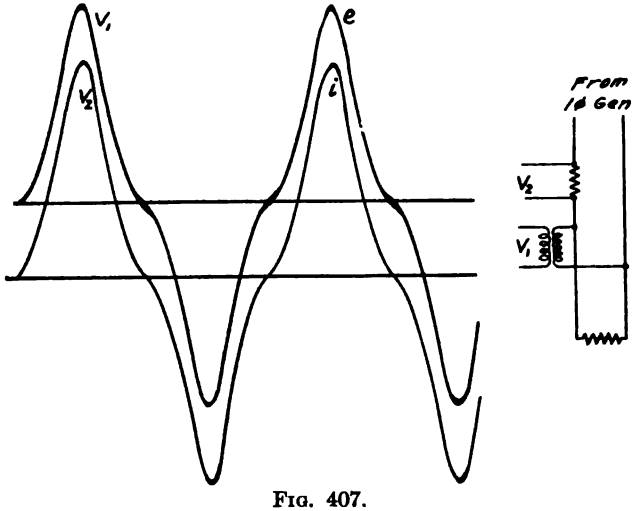



FIG. 407.

quency. Hence, the current harmonics are smaller than those in the voltage, and the current wave more nearly approaches the sine form. This is illustrated in Fig. 408. The same voltage wave $e$, as in Fig. 407, was impressed on the second circuit having both resistance and inductance. The current wave is less peaked and its maximum value lags behind the voltage. The relative magnitudes of the third and fifth harmonics of the current are reduced proportionately to the impedance and hence are less than the corresponding harmonics in the voltage. The phase displacements also cause a reduction in the peak as the maxima of the three component current waves do not coincide

as in the voltage waves. In a circuit having resistance and condensance the effect is just the reverse, since an increase in the frequency reduces the reactance and impedance and therefore causes a proportionate increase in the current.

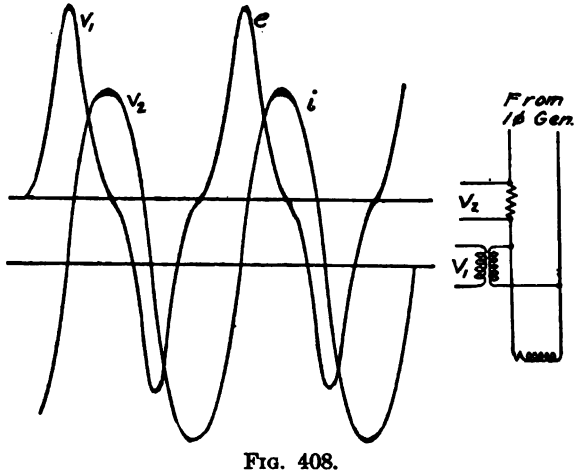



FIG. 408.

$$^{n}I_n = \frac{^{n}E_n}{\sqrt{r^2 + \left(\frac{1}{n\omega C}\right)^2}} \text{ and } \theta_n = \tan^{-1}\left(\frac{1}{rn\omega C}\right) \qquad (763)$$

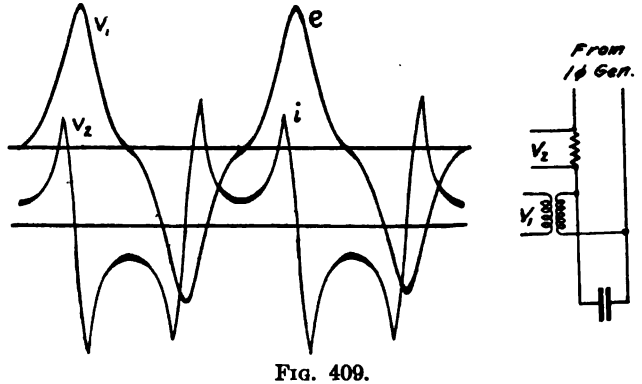



FIG. 409.

In Fig. 409 is shown the same voltage as in 407 and 408 impressed upon a circuit having resistance and condensance.

For the higher harmonics the effect of inductance or condensance is more marked. Thus, in Figs. 410 and 411 are shown the

voltage wave of a small generator having a strong eleventh harmonic. In the current, passing through a circuit having a constant resistance and inductance, Fig. 410, the eleventh harmonic

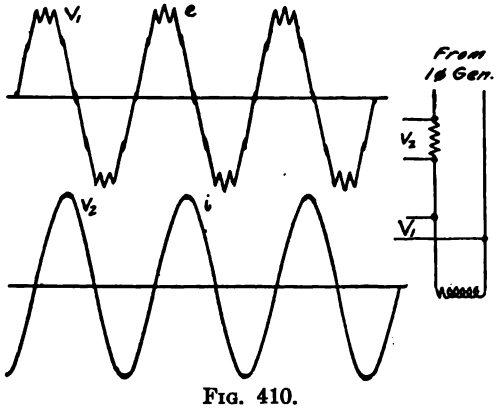



FIG. 410.

is practically eliminated; while the current flowing into a condenser, Fig. 411, from the same generator shows a very strong

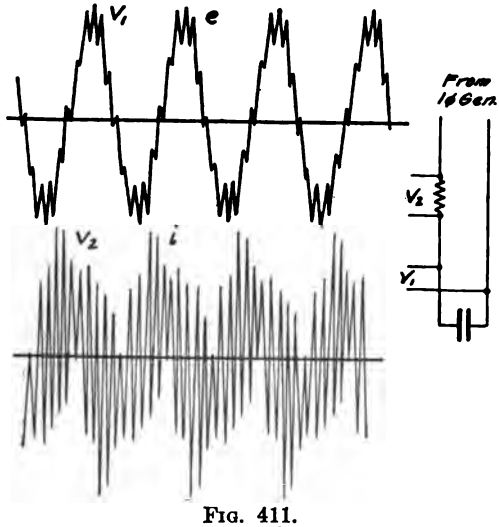



FIG. 411.

eleventh harmonic. The current distortion in Fig. 410 is only one-eleventh as large as for the voltage wave while in Fig. 411 the current distortion is eleven times as large as for a non-react-

ive circuit. By comparing the voltage waves in Figs. 410 and 411 it is seen that the lagging and leading currents produced different armature reactions in the generator and thus the terminal voltages were not quite the same in the two cases.

In Fig. 412 is shown the voltage wave of a large machine having a pronounced twenty-fifth harmonic and also the condenser current wave. The circuit diagrams in each case show the connections of the oscillograph vibrators in the circuits.

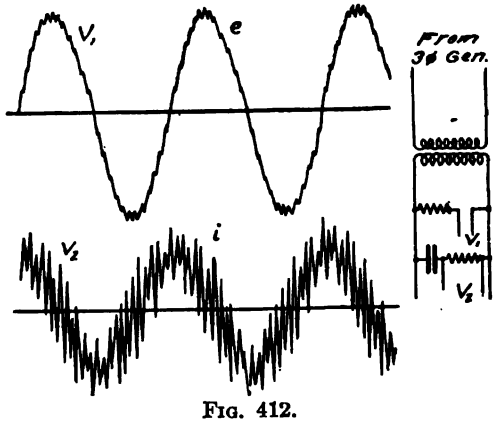



FIG. 412.

(*e*) **Distorted Current Waves in Circuits with Pulsating Inductance, Condensance and Resistance.**—1. *Pulsating Inductance.*—The main sources of pulsations in the inductive reactance in a circuit are: First, the variation of the reluctance around the armature conductors due to synchronous rotation; and second, the variation in the permeability and the hysteresis with the flux density in iron-clad circuits.

The pulsations of the generator field caused by the relative position of the armature slots are discussed in the first part of this chapter in relation to the distribution of the field flux and hence as a primary cause of distortion of the voltage wave. The relative position of the field pole and armature slots also affects the armature reactance, producing pulsations that cause distortions in the current wave as compared to the impressed voltage. It is evident that a similar effect is produced by a synchronous motor, or any other synchronous apparatus with slotted armature, in the circuit. Since the cycle of variations in magnetic reluctance is completed for each pole, the pulsations

of the inductive reactance complete a cycle for each half wave of the fundamental frequency. Moreover, as the distortion cycle is applied alternately to a positive and a negative fundamental half wave, the harmonics introduced have a frequency of $2s - 1$ and $2s + 1$; in much the same way as for the voltage wave, but differing in magnitude and phase position. With a large air gap, large numbers of slots per pole, proper shaping and spacing of the armature slots and by using fractional pitch in the winding, the distortions produced by pulsations in the synchronous reactance may be reduced to practically negligible values.

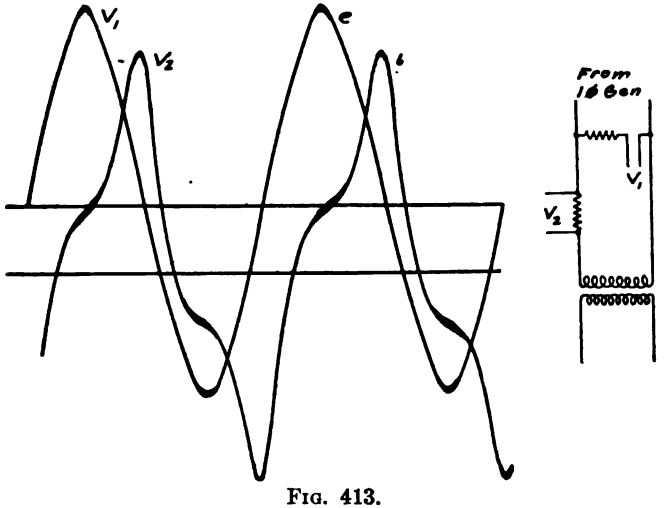



FIG. 413.

The second factor producing pulsations of the inductive reactance is present in all iron-clad circuits. In Chap. XI it is shown that with an impressed sine voltage wave the form of the current wave is determined by the shape of the hysteresis loop, Fig. 133. Conversely, the hysteresis loop of an iron-clad circuit may be found from the exciting current produced by a sine voltage wave. In Fig. 413 is shown an oscillogram of a sine voltage wave and the corresponding exciting current of a potential transformer.

$$E = 110; f = 60; I = 2.0;$$ 10 kw. transformer.

Since the upper and lower halves of the hysteresis curve are equal, the cycle of distortion is complete for each half of the fundamental wave. As the cycle of variation is applied alternately to the positive and negative halves of the fundamental

wave the distortion consists essentially of the 3rd and 5th harmonics. For this reason the distortion of the current wave

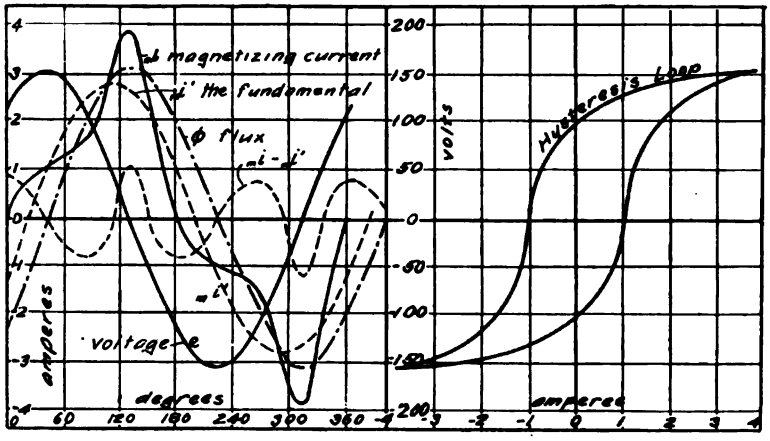



FIG. 414.

in Fig. 413 consists mainly of a third harmonic in combination with a smaller fifth harmonic. Analyzing the current wave in Fig. 413 the component fundamental, with the third and fifth

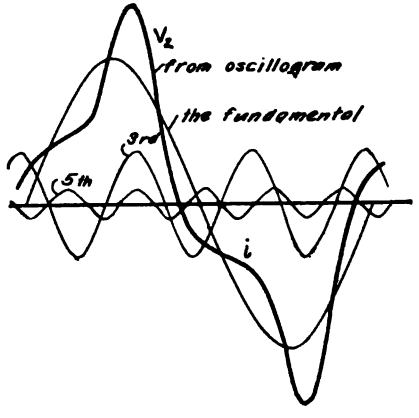



FIG. 415.

harmonics are represented graphically in Figs. 414 and 415 and analytically by equation (764).

$$i = 100 \sin(\omega t) + 37.0 \sin(3\omega t + 114°) + 10.7 \sin(5\omega t + 245°) \quad (764)$$

It is apparent that if a current of sine wave form is sent through an iron-clad circuit the distortions produced by the hysteresis of the iron will appear in the voltage wave. Thus, in Fig. 416 is shown the voltage wave produced by sine wave exciting current passing through an iron-clad circuit having a hysteresis loop of the shape shown in the figure. The third and fifth harmonics therefore appear in the voltage wave when a sine current is flowing in the circuit in precisely the same manner and for the same reason as the distortion of the current wave with an impressed sine voltage wave. In single-phase constant-potential systems the distortion produced by the hysteresis of the iron

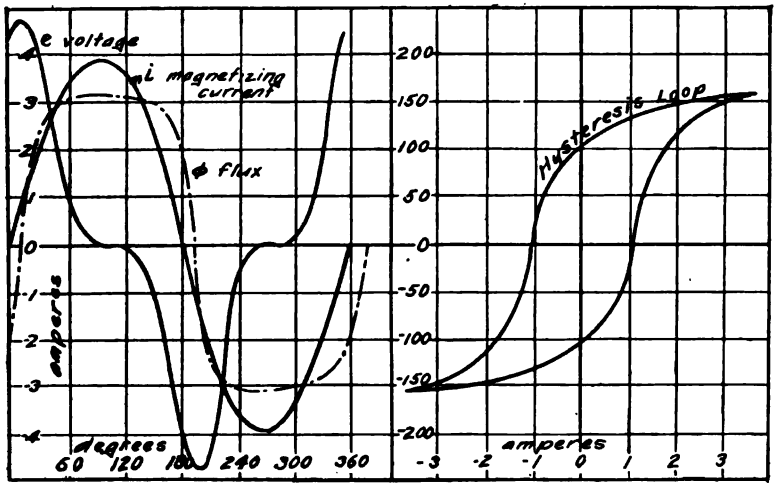



FIG. 416.

appears chiefly in the current wave. Secondary reactions of the distorted current wave upon the generator field may cause corresponding distortions in the voltage wave, but the current wave is primarily affected by the hysteresis distortion.

In polyphase circuits the manner of interlinking the component circuits, in a large measure, determines which part of the distortion shall appear in the current or voltage. The delta and star connections in three-phase circuits are of special importance in commercial systems. Since the fundamental waves in a three-phase system are 120° apart the third harmonics in the three phases are in phase with each other. This is shown in Fig. 417. Hence in a star connection the current can not have a third

harmonic and likewise in a delta connection the third harmonic must be absent from the voltage wave. The same phase rela-

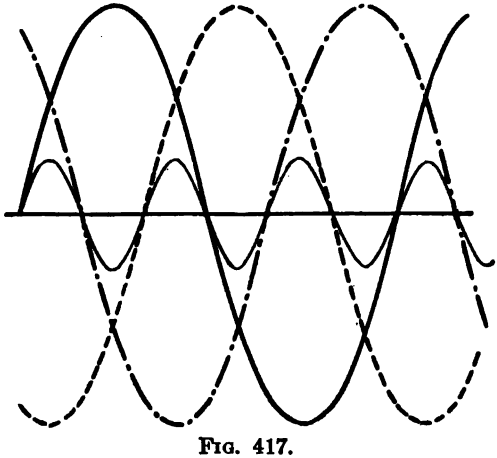


FIG. 417.

tions exist for any odd multiple of the third harmonic. Therefore, in a star connection the third, ninth, fifteenth. etc., harmonics

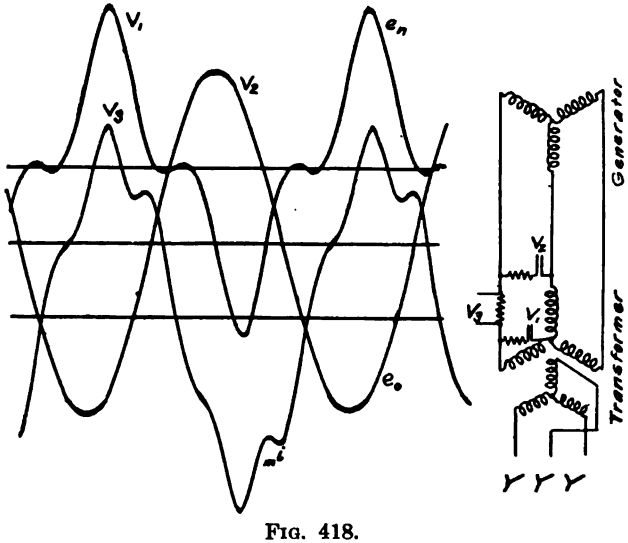



FIG. 418.

can not exist in the current wave; and similarly the third, ninth, fifteenth, etc., harmonics can not appear in the delta-connected

voltage wave. The fifth, seventh, eleventh, etc., harmonics, however, may exist in both current and voltage waves for both star and delta connections.

The effect of a star connection in distributing the harmonics produced by the hysteresis of the transformer iron is shown in Fig. 418. The circuit diagram shows the star connection of both the generator and the three transformers, and the locations in the circuit of the three vibrators of the oscillograph. The second-

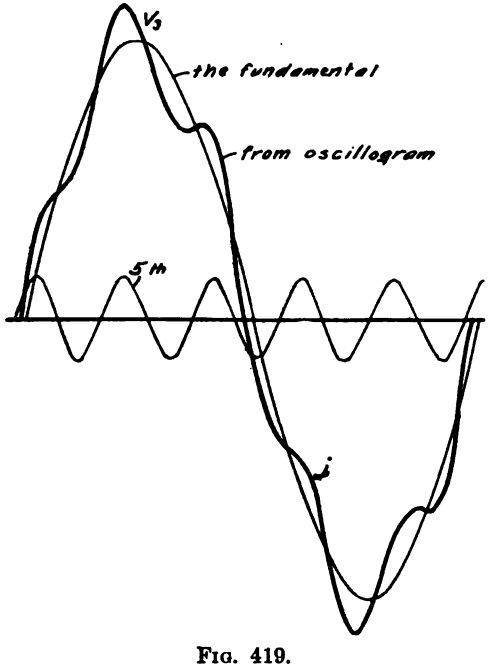



FIG. 419.

aries of the transformers are open so the transformer exciting current only is flowing in the circuits. Hence vibrator No. 1 gives the voltage from one main to neutral; vibrator No. 2 the voltage between mains; and vibrator No. 3 the exciting current in one transformer. The voltage between mains as impressed on the transformers closely approximates a sine wave. The exciting current consists of a fundamental and a fifth harmonic as shown in Fig. 419 and by equation (765).

$$_{\mu}i = 100 \sin (\omega t - 5°) + 15 \sin (5\omega t + 28°) \qquad (765)$$

The third harmonic produced by the hysteresis distortion

appears in the voltage wave to neutral. The components of the voltage to neutral wave are shown in Fig. 420 and expressed by equation (766).

$$_ne = 100 \sin (\omega t - 26°) + 44 \sin (3\omega t + 98°) \qquad (766)$$

In the star connection the third harmonic can not exist in the current and hence only the fifth harmonic appears in the exciting current while the third harmonic appears in the voltage to neutral wave. Since the voltage between the mains is the vector difference of the voltages of each pair of circuits the third harmonics in the circuits neutralize. This may be illustrated graphically

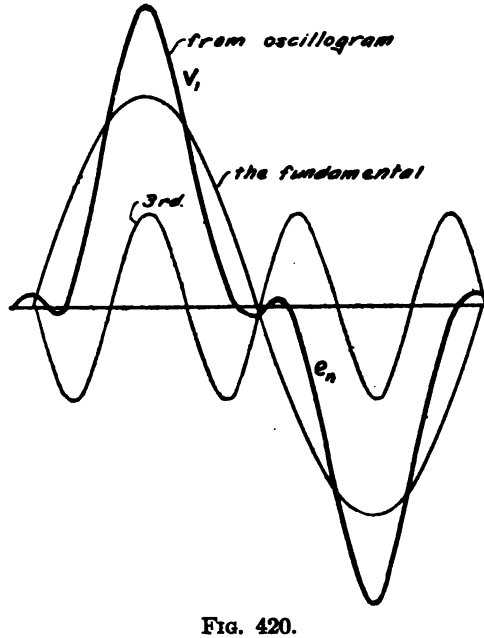



FIG. 420.

by the diagram in Fig. 421. Let the triangle $ABC$ represent the balanced voltage between the mains. If the voltage to neutral is also a sine wave the lines $OA$, $OB$, $OC$ with $O$ as the center represent the corresponding circuit voltages. With a third harmonic in the three circuits the neutral point may be represented by the point $N$ revolving around $O$ with a triple frequency. The corresponding magnetic flux wave and hysteresis cycle are shown in Fig. 422.

If the neutral of the generator be connected to the neutral point in the star-connected transformers then the point $N$ in Fig. 421

is fixed and coincides with the center *O*. Under these conditions there can be no third harmonic in the voltage to neutral, but a current of triple frequency flows in the neutral. In Fig. 423 is shown the circuit diagram and connections for the oscillograph,

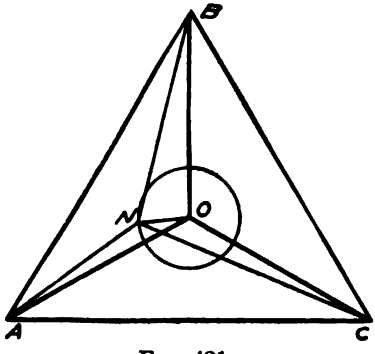



FIG. 421.

with the corresponding oscillograms. Vibrator No. 1 gives the voltage to neutral; vibrator No. 2 the voltage between mains; vibrator No. 3 the exciting current; and vibrator No. 4 the cur-

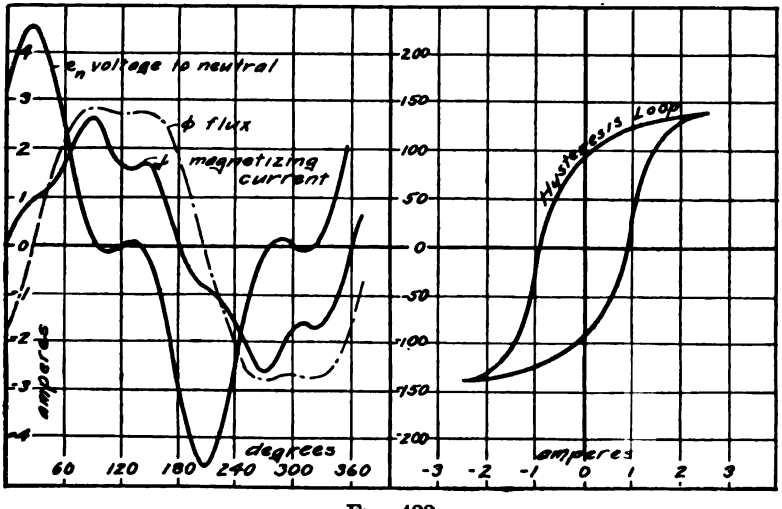



FIG. 422.

rent in the neutral. With the neutral connected, the hysteresis distortion produces a triple harmonic current in the neutral, which is the sum of the three triple harmonics in the exciting currents of the three transformers.

As a third illustration of the effect of transformation connections on the wave shapes in three-phase circuits take the star-star-delta connection as shown in the circuit diagram of Fig. 424. The star-connected generator is again connected to the low-tension side of the three star-connected transformers, and the neutral is left open. The high-voltage sides of the transformers are connected in delta and without load. As indicated in the dia-

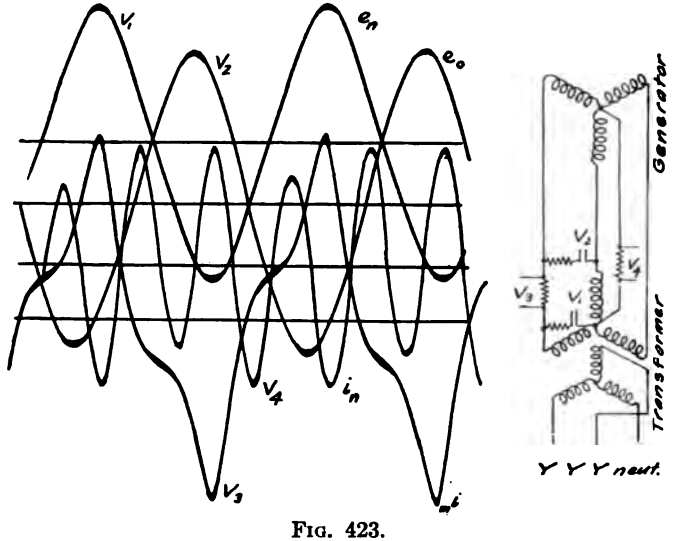



FIG. 423.

gram, Fig. 424, vibrator No. 1 gives the voltage to neutral; vibrator No. 2 the voltage between mains; vibrator No. 3 the line current; and vibrator No. 4 the delta current. The corresponding oscillograms show that the two voltages have only slight distortions while the fifth harmonic appears in the line current. The third harmonic required by the hysteresis distortion is supplied by the current of triple frequency flowing in the secondary delta connection of the transformers. From the above it is apparent that while the hysteresis of the iron introduces a third and a fifth harmonic the phase connections largely determine whether the distortion shall appear in the current or voltage waves.[1]

2. *Pulsating Condensance.*—Within ordinary voltages the properties of the generally used dielectrics remain fairly constant and do not show any marked variation of the condensance of the cir-

[1]A. I. E. E., Proc., XXXIII, 753, 771, 785, 791, 1153.

cuit. Leaky condensers, polarization cells and possibly dielectric hysteresis cause a distortion similar to that of magnetic hysteresis. The magnitude of such distortion is, however, comparatively small and of little commercial importance. With higher voltages, particularly under conditions producing the corona or brush discharge, air as a dielectric undergoes periodic changes that cause important pulsations in the condensive reactance as well as marked variations in the insulating properties of the material surrounding the conductor. The dielectric strength of air and the law of the corona are discussed in Chaps. XIX and XX.

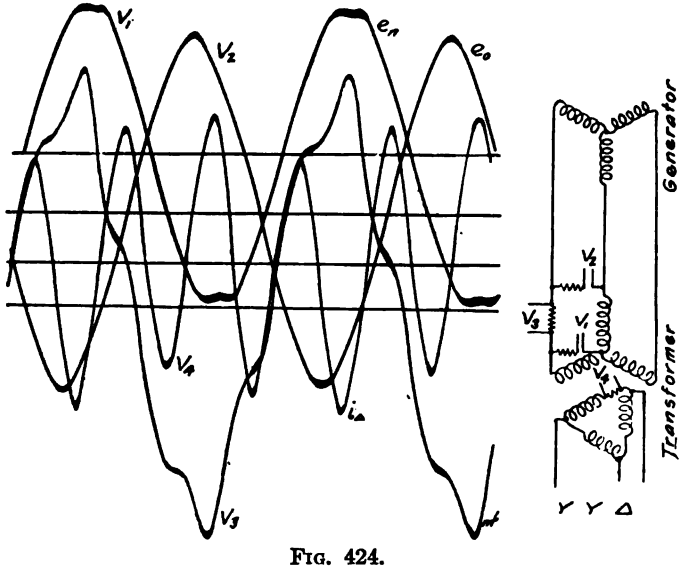



FIG. 424.

It was explained how the corona appeared only during that part of the voltage cycle when the instantaneous voltage exceeded the critical valve. It was also noted that the positive and negative halves of the voltage wave do not produce similar corona effects. Since the extent of the corona is affected by such factors as the shape, size and distance apart of the conductors, the temperature, barometric pressure and moisture of the atmosphere, etc., it is evident that the effects upon a commercial system operating under varying conditions may be difficult to determine. However, the presence of the corona increases the condensance and leakage loss of the system and hence causes a pulsation in the condensive reactance. As the variation is not the same for suc-

cessive half waves this pulsation may introduce even harmonics into the circuit.

In the same manner as magnetic hysteresis produces distortion primarily in the exciting current of iron-clad circuits, so the pulsations of the condensive reactance primarily introduce pulsations in the charging current of the circuit. If the condensance or charging current is of sufficient relative magnitude, secondary effects may cause distortions in the voltage wave. Moreover, the connections of the circuits in polyphase systems may shift the distortion in much the same manner as was explained for pulsations of inductive reactance.

3. *Pulsating Resistance.*—As a rule the pulsations in reactance are accompanied by similar pulsations in the resistance. This is

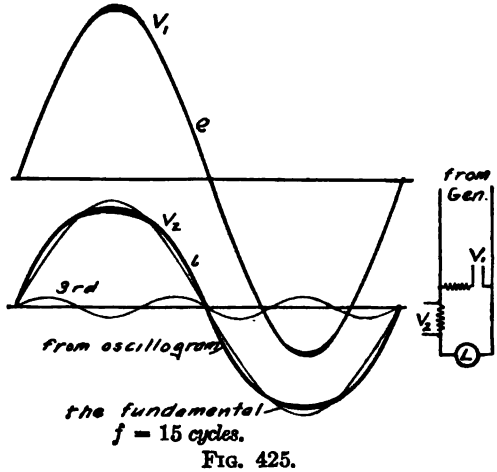



FIG. 425.

indicated by the *hysteretic angle of advance* in iron-clad circuits having a pulsation of inductive reactance; and similarly in the dielectric by pulsations in the leakage current coincident with the appearance of the corona.

Rapid changes in the temperature of a conductor, as for example, in an incandescent lamp filament, cause corresponding changes in the resistance. Thus in Fig. 425 are shown oscillograms of the sine voltage wave impressed upon the terminals of a tungsten incandescent lamp, and the current waves flowing in the lamp. Vibrator No. 1 gives the voltage wave and vibrator No. 2 the current in the lamp. The distortion consists mainly of a third harmonic. While periodic changes in temperature

introduce pulsations in the resistance of solid conductors, the distortion of the wave shapes is small under commercial conditions and as a consequence of little importance.

The most important example of a synchronously pulsating resistance is the alternating-current arc. In a gaseous conductor the resistance decreases with an increase of current, being high for small currents and low for large currents. The gaseous conductor between the carbons or metallic electrodes of an arc lamp has a rapidly varying resistance, pulsating at double the frequency of the current. Assuming that the current flowing in the arc is a sine wave, and that the fluctuations of the resistance vary inversely with the current, the circuit relations may be expressed as in equations (761) to (769).

$$i = {}^{\prime\prime}I \sin(\omega t) = \sqrt{2}I \sin(\omega t) \tag{767}$$

$$r = r_0(1 + k \cos 2\omega t) \tag{768}$$

$$e = ri = \sqrt{2}Ir_0 \sin(\omega t)(1 + k \cos 2\omega t)$$

$$= \sqrt{2}r_0 I\left[\left(1 - \frac{k}{2}\right)\sin(\omega t) + \frac{k}{2}\sin 3(\omega t)\right] \tag{769}$$

The distortion of the voltage wave therefore consists, under the given assumption, of a third harmonic in phase with the fundamental when passing zero value, and hence 180° out of phase at the maximum value of the fundamental. This is illustrated in Fig. 426 for the following values of current and resistance:

$$i = 10 \sin(\omega t) \tag{770}$$

$$r = 4(1 + 0.6 \cos 2\omega t) \tag{771}$$

$$e = 28(\sin \omega t + 0.43 \sin 3\omega t) \tag{772}$$

With a sine wave of current the voltage wave becomes double-peaked, having an abrupt rise near the zero value of the current. Conversely, if a sine wave of voltage is impressed across the terminals of the arc the resulting current wave in the arc is peaked at the maximum points and flat near the zero values of the voltage. While the assumption of a variation in the resistance inversely proportional to the current illustrates the main feature of the phenomenon, the relations in the actual arc are much more complex. Many factors affecting the resistance variation are difficult to determine. The material used in the electrodes radically modifies the resistance variation. A cored

or impregnated set of carbons gives an entirely different law of resistance variation than would apply to hard carbons or for carbides; similarly different metals or electrodes radically change the resistance of the arc. The temperature of the surrounding space, the vaporization point of the materials in the electrodes, the size and shape of the electrodes, enclosed or open condition of the arc, length of the arc, current density, frequency and wave shape of the current or of the impressed voltage and other factors affect the resistance and may produce an infinite variety

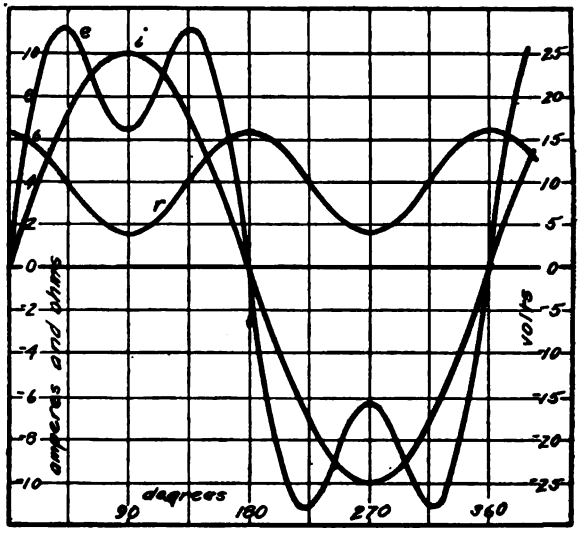



FIG. 426.

of resistance fluctuations. In all this diversity of electric arc phenomena the double-peaked nature of voltage wave is the most important. This characteristic is so marked that if in an investigation of an electric circuit by the oscillograph such wave shapes are found, the existence of an arc or arcing ground may be suspected. It is of special importance in high-tension systems since any arc or arcing ground is a most fruitful source for high-frequency oscillations producing dangerous voltages.

(*f*) **Analysis of Wave Forms.**—*First Method.*—A number of equally spaced ordinates are measured on the half wave to be analyzed. The number of ordinates used must be one greater than the highest harmonic. Thus, if the eleventh harmonic is the highest, twelve equally spaced ordinates are selected while

if the seventeenth harmonic is the highest, eighteen equally spaced ordinates must be measured. These measured values are tabulated and a series of operations performed as prescribed in the corresponding chart[1] for the number of ordinates measured. In this way the numerical values of the sine and cosine coefficients are derived for the fundamental and each of the harmonics. Having the values of the sine and cosine coefficients, the numerical values may be substituted in the equation. As this determines both the magnitude and phase position of the several component waves the curves may be plotted. The method is, however, laborious and requires painstaking care.

*Second Method.*—Several mechanical devices have been invented for analyzing complex waves, that operate in a more or less satisfactory manner. The Harmonic Analyzer by Dr. O. Mader



FIG. 427.

is an excellent example of a simple mechanical contrivance by which the complex waves may be rapidly analyzed. The instrument is shown in Fig. 427 in connection with an ordinary planimeter for obtaining quantitative values. It may be noted that the values of the sine and cosine coefficients are taken separately for each of the component waves. After the limits of the carriage movement have been adjusted to the size of wave to be analyzed the process is as follows: Select the proper cog wheel for the fundamental. Place the podium of the planimeter in the small hole marked $s$ on the cog wheel. Move the point $P$ to the left end of the wave, adjust the cog wheel to the zero position, and read the planimeter. Next let the point $P$ trace the complete wave and return to the starting point along the $X$-axis as indicated by the arrows in Fig. 427. Again read the

[1] U. S. Bureau of Standards, *Bull.* 203, BEDELL and PIERCE, "Direct and Alternating-current Manual," p. 331.

planimeter. The difference of the two planimeter readings gives directly the sine component of the coefficient for the fundamental wave.

To obtain the cosine component merely move the podium of the planimeter to the small hole marked $c$ on the cog wheel; read the planimeter; move the point $P$ through the complete cycle as before; again read the planimeter. The difference in the two planimeter readings gives directly the cosine component of the fundamental.

For any of the harmonics select the proper cog wheel and take similar readings for the sine and cosine components. The adjustments can be made quickly and the readings taken rapidly. With care the errors will be less than 5 per cent., which is about the accuracy that can be obtained by the first method.

(*g*) **Effective Values of Currents and Voltages of Distorted Wave Shapes.**—Since the effective value of voltage or current represents the square root of the mean square of the instantaneous values, and since the successive cycles are alike, the effective values may be expressed by (773).

$$E = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} e^2 d(\omega t)}$$

and

$$I = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} i^2 d(\omega t)} \qquad (773)$$

From equations (743) and (759):

$$e = {}^{\prime\prime}E_1 \sin(\omega t + \gamma_1) + {}^{\prime\prime}E_3 \sin(3\omega t + \gamma_3) + {}^{\prime\prime}E_5 \sin(5\omega t + \gamma_5) + \text{etc.} \qquad (774)$$

$$i = {}^{\prime\prime}I_1 \sin(\omega t + \gamma_1 - \theta_1) + {}^{\prime\prime}I_3 \sin(3\omega t + \gamma_3 - \theta_3) + {}^{\prime\prime}I_5 \sin(5\omega t + \gamma_5 - \theta_5) + \text{etc.} \qquad (775)$$

From calculus, for any positive integral values of $m$ and $n$:

$$\left.\begin{aligned} \int_{-\pi}^{+\pi} \cos mx \sin nx\,dx &= 0 \\ \int_{-\pi}^{+\pi} \cos mx \cos nx\,dx &= \begin{cases} 0 \text{ when } m \lesseqgtr n \\ 2\pi \text{ when } m = n = 0 \\ \pi \text{ when } m = n > 0 \end{cases} \\ \int_{-\pi}^{+\pi} \sin mx \sin nx\,dx &= \begin{cases} 0 \text{ when } m \lesseqgtr n \\ 0 \text{ when } m = n = 0 \\ \pi \text{ when } m = n > 0 \end{cases} \end{aligned}\right\} \qquad (776)$$

and it is evident that in the integration of $e^2dt$ and $i^2dt$, all terms except those that contain a sine squared are equal to zero

Hence,

$$E = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} e^2 d(wt)} = \sqrt{\tfrac{1}{2}({}^{\prime\prime}E_1^2 + {}^{\prime\prime}E_3^2 + {}^{\prime\prime}E_5^2} \ . \ . \ . \text{ etc.}$$

$$= \sqrt{E_1^2 + E_3^2 + E_5^2} + \ . \ . \ . \text{ etc.} \qquad (777)$$

and

$$I = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{+\pi} i^2 d(wt)} = \sqrt{\tfrac{1}{2}({}^{\prime\prime}I_1^2 + {}^{\prime\prime}I_3^2 + {}^{\prime\prime}I_5^2)} + \ . \ . \ . \text{ etc.}$$

$$= \sqrt{I_1^2 + I_3^2 + I_5^2} + \ . \ . \ . \text{ etc.} \qquad (778)$$

*Graphical Method.*—The effective value of a complex voltage or current wave may be found graphically without the necessity of analyzing the curve into the component harmonics. The half-wave cycle is plotted in polar coördinates and the area of the curve obtained by means of a planimeter. Draw a circle having the same area. Then the diameter of this circle is equal to the maximum value of the equivalent sine wave. The effective value is therefore $\sqrt{2}R$, where $R$ is the radius of the circle. Fairly accurate results may be obtained even without the aid of a planimeter, as a circle of approximately the same area can readily be drawn over the polar-wave diagram.

**(*h*) Form Factor, Curve Factor, Peak Factor, Harmonic Factor and Distortion Factor for Complex Waves.**—*Form Factor.*—The form factor is the ratio between the effective and average values. The average value may be found by plotting the wave in rectangular coördinates, obtaining the area by means of a planimeter and dividing the area by the base, thus obtaining an average ordinate. For sine waves the form factor = 1.11.

*Curve Factor.*—The curve factor is the ratio between the effective values of the complex wave and of the fundamental.

$$\text{Curve factor} = \frac{E}{E_1} = \frac{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ . \ . \ . \ .}}{E_1} \qquad (779)$$

*Peak Factor.*—The peak or crest factor is the ratio between the maximum and the effective values.

$$\text{Peak factor} = \frac{{}^{\prime\prime}E}{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ . \ . \ . \ .}} \qquad (780)$$

This ratio is of importance in the study of corona, insulation tests, etc.

*Harmonic Factor.*—The harmonic factor is the ratio of the effective value of all the harmonics to the effective value of the fundamental.

$$\text{Harmonic factor} = \frac{\sqrt{E_3^2 + E_5^2 + E_7^2 + \ldots}}{E_1} \tag{781}$$

*Distortion Factor.*—The distortion factor is the ratio of the effective value of the first derivative of the wave, with respect to time, to the effective value of the first derivative of the equivalent sine wave. If a condenser is connected across the terminals of a circuit having a complex voltage wave the current flowing, $I$, is greater than for a simple sine wave of effective value equal to the equivalent effective value of the complex wave. The true reactance of the condenser at the frequency of the fundamental is:

$$_cx_1 = \frac{E_1}{I_1} \tag{782}$$

The apparent reactance for the complex wave is:

$$_cx_0 = \frac{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}}{\sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots}} \tag{783}$$

$$\text{Distortion factor} = \frac{_cx_1}{_cx_0} \tag{784}$$

**(*i*) Power in Circuits with Distorted Current and Voltage Waves. Equivalent Sine Wave. Power Factor.**—The power delivered in a circuit is:

$$P = \frac{1}{T}\int_o^T eidt \tag{785}$$

Substituting the values of $e$ and $i$ from (774) and (775) and integrating the product, all terms vanish except those containing a sine squared, in the same manner as for the effective values of voltage and current.

$$\begin{aligned} P &= \tfrac{1}{2}({}^{\prime\prime}E_1{}^{\prime\prime}I_1 \cos\theta_1 + {}^{\prime\prime}E_3{}^{\prime\prime}I_3 \cos\theta_3 + {}^{\prime\prime}E_5{}^{\prime\prime}I_5 \cos\theta_5 + \ldots) \\ &= E_1I_1 \cos\theta_1 + E_3I_3 \cos\theta_3 + E_5I_5 \cos\theta_5 + \ldots \qquad (786) \\ &= P_1 + P_3 + P_5 + \text{etc.} \ldots \end{aligned}$$

It is important to note that as regards the power all the harmonics are independent of each other. The current of any harmonic is wattless with respect to the pressures of all other harmonics. Each harmonic has its own power factor and each follows independently the same laws as already explained for simple sine waves. The total power is the summation of the powers of the several harmonics. Hence each harmonic may be

treated by itself and the effects calculated separately by the laws and graphic constructions already deduced. In commercial work the wave shape is seldom known with any close degree of exactness. The analysis of waves is a tedious process and the power and power factor are usually determined by the wattmeter, voltmeter and ammeter readings. It is generally assumed that the actual conditions may be represented by an *equivalent sine wave* of current and voltage and that the *power factor* is the ratio of the watts to the volt-amperes.

$$P = EI \cos \theta = rI^2 \tag{787}$$

In equation (787) $r$ is the effective resistance, $I$ the *equivalent sine current wave* and $\cos \theta$ the measured *power factor.*

From (777), (778) and (789) and considering the effective resistance constant for all frequencies:

$$\cos \theta = \frac{rI}{E} = r \frac{\sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots}}{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{788}$$

$$= r \frac{\sqrt{\dfrac{E_1^2}{r^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} + \dfrac{E_3^2}{r^2 + \left(3\omega L - \dfrac{1}{3\omega C}\right)^2} + \dfrac{E_5^2}{r^2 + \left(5\omega L - \dfrac{1}{5\omega C}\right)^2} + \ldots}}{\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{789}$$

$$= \sqrt{\frac{E_1^2 \cos^2 \theta_1 + E_3^2 \cos^2 \theta_3 + E_5^2 \cos^2 \theta_5 + \ldots}{E_1^2 + E_3^2 + E_5^2 + \ldots}} \tag{790}$$

It should be noted that while the resistance is constant for all frequencies (in all cases to which the above equations apply) the reactance is affected by the frequency, and hence the relative value of the reactances for the several frequencies depends upon whether the circuit is inductive or condensive. If $x_n$ be the reactance for the $n$th harmonic, its value is given in equation (791) in terms of the fundamental frequency and the inductance and condensance of the circuit.

$$x_n = n\omega L - \frac{1}{n\omega C} = n_1 x_1 - \frac{x_1}{n} \tag{791}$$

Hence,

$$x_3 = 3_1 x_1 - \frac{x_1}{3}; \; x_5 = 5_1 x_1 - \frac{x_1}{5}; \text{ etc.} \tag{792}$$

$$\tan \theta_n = \frac{x_n}{r} = \frac{n\omega L}{r} - \frac{1}{rn\omega C} \tag{793}$$

$$\cos \theta_n = \frac{r}{\sqrt{r^2 + x_n^2}} = \frac{r}{Z_n} = \frac{rI_n}{E_n} \tag{794}$$

(*j*) **Resonance in Circuits with Distorted Current and Voltage Waves.**—Since any complex wave may be separated into a series of sine waves of multiple frequency, and each harmonic acts independently, the effect of a distorted voltage wave is equivalent to the resultant of the component sine waves. While the inductance and condensance are constant, the inductive and condensive reactances are different for the several harmonics. For resonance the inductive and condensive reactances or susceptances must be equal. Hence resonance can occur only for one frequency. This is illustrated in Fig. 428. A voltage wave repre-

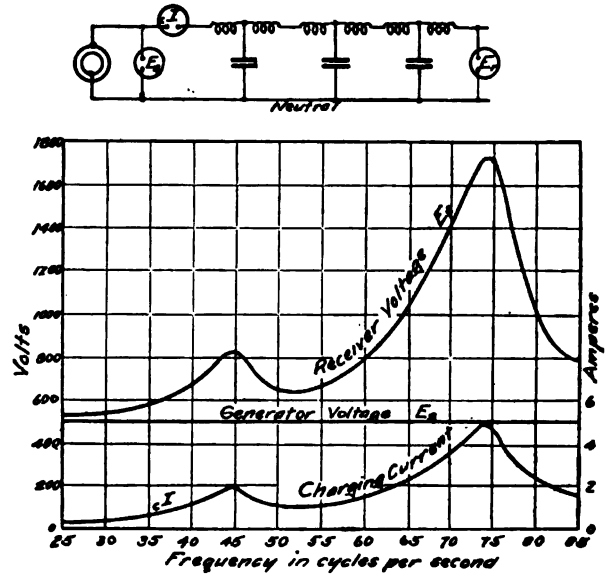



FIG. 428.

sented by equation (795) was impressed on one end of an artificial transmission line with the ammeter and voltmeters connected as shown in the circuit diagram.

$$e = 100 \sin(\omega t) - 28.2 \sin(3\omega t) + 5.4 \sin(5\omega t) \qquad (795)$$

By varying the frequency of the impressed voltage and reading the charging current at the generator end and the voltage at the receiver end the data for the two curves were obtained. The natural period of the line was 223 cycles. Note the increase in $_{s}I$ and $E_r$ when $f = 44.5$ and for $f = 74$. Also that $44.5 \times 5 = 222.5$; and $74 \times 3 = 222$. That is for $f = 44.5$, the circuit was

in resonance with the fifth harmonic and for $f = 74$, resonance was produced by the third harmonic. The increase is approximately proportional to the relative magnitude of the two harmonics in the impressed voltage wave.

It should also be noted that for some frequencies the current may be leading for one harmonic and lagging for another. Thus for frequencies giving resonance for the third harmonic,

$$x_3 = 3\omega L - \frac{1}{3\omega C} = 3_L x_1 - \frac{{}_C x_1}{3} = 0 \qquad (796)$$

At this frequency the fundamental has a leading current, since

$$x_1 = \omega L - \frac{1}{\omega C}$$

must be negative and therefore condensive; while the fifth, seventh, etc., harmonics must have lagging currents since

$$x_5 = 5_L x_1 - \frac{{}_C x_1}{5};\ x_7 = 7_L x_1 - \frac{{}_C x_1}{7};\ \text{etc.}$$

are positive and therefore inductive.

(*k*) **The Standard Wave Form.**—In small plants operating at low potential the current and voltage wave forms are of minor importance. As this was the general condition in the early development of the electrical industry little attention was given to wave forms. The vast extension of electrical distribution systems, the development of enormous central stations and the necessity of operating long-distance transmission lines at high potentials demand very careful attention to all factors that may produce wave distortion.

Two requirements may be considered as fundamental for the wave form in constant-potential systems:

(1) In parallel operation the waves generated should be equal at all instants so as to avoid cross-currents.

(2) The differentials and integrals of the curve should have the same shape as the generated wave.

For alternating currents, generated by rotating machinery, both requirements are met by the simple sine wave. Consider the two voltage waves in Fig. 429, an oscillogram taken from two small generators (35 and 60 kv.a.) operating in parallel in an isolated plant. Machine No. 1, $e_1$, gives almost a simple sine wave while in machine No. 2, $e_2$, both the third and fifth harmon-

ics are present. It is evident that for all instants during which the two waves are not equal the difference in the voltages cause cross-currents to flow through the two armatures. The addi-

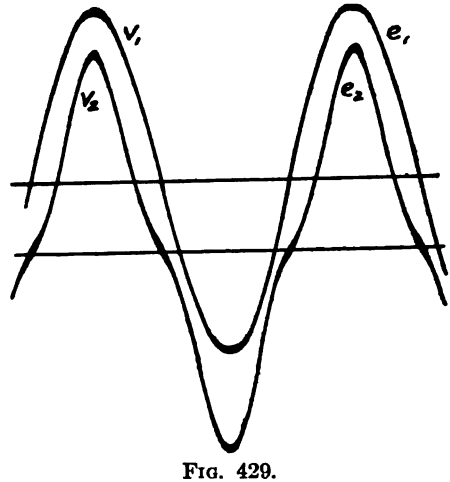



FIG. 429.

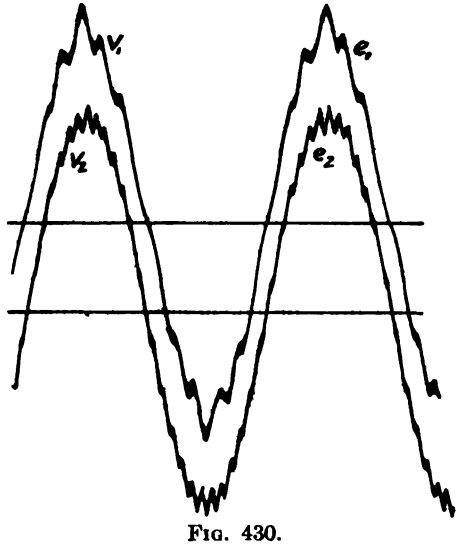



FIG. 430.

tional $RI^2$ losses reduce the efficiency and also raise the temperature above the value which would obtain for the same load if the machines operated independently. Since the permissible output, or rating, of well-designed electrical machines is based

on the temperature rise, the cross-currents reduce both the rating and the efficiency of the alternator. In Fig. 430 is shown an oscillogram of two larger units (500 and 1,500 kv.a.) operating in parallel on the same busbars. Machine No. 1, $e_1$, has a marked eleventh, and machine No. 2, $e_2$, has a strong twenty-fifth harmonic, and instantaneous values of the two waves differ at most points.

In the induction motor the harmonics are likewise undesirable. In a rotating field the fundamental alone produces the useful torque, while the harmonics increase the losses and may cause undesirable stresses on the dielectric.

As already explained the harmonics may produce entirely different effects from the fundamental while passing through an extensive distribution system. For instance, resonance conditions for one harmonic in some section of the system may greatly magnify the distortion and cause excessive pressures or abnormally large currents.

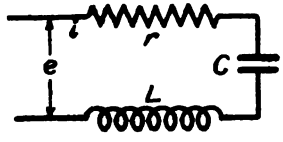


FIG. 431.

The basis for the second requirement is found in the fundamental law of magnetic and dielectric induction, as may be readily seen from equations (799) and (800). In a simple series circuit Fig. 431, the instantaneous voltage is:

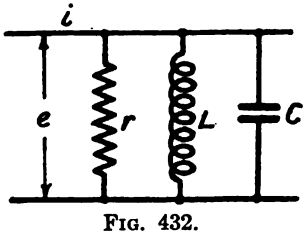


FIG. 432.

$$e = ri + L\frac{di}{dt} + \frac{\int idt}{C} \qquad (799)$$

For a simple parallel circuit, as in Fig. 432, the instantaneous current is:

$$i = \frac{e}{r} + \frac{\int edt}{L} + C\frac{de}{dt} \qquad (800)$$

Letting $i = {}^{\prime\prime}I \sin(\omega t)$ for equation (799), and similarly $e = {}^{\prime\prime}E \sin(\omega t)$ for equation (800), the total voltage and current waves are of the same simple sine form, of the same frequency and are merely displaced in time-phase position. For any other wave form there is distortion so that the total voltage wave in (799) is not of the same shape as the current; nor is the total current wave in (800) like the impressed voltage.

The chief unavoidable factor producing distortion in the wave forms is the variable permeability of iron. The effect is, of

course, greatest in iron-clad circuits. Many other causes, which offer a constant menace to satisfactory operation, are often present in commercial systems due to faulty design or construction. The generated voltage wave should approximate very closely the standard sinusoidal form, and the system should be so constructed as to eliminate as completely as possible all sources of wave distortion.

## PROBLEMS

**1.** A generator having an effective inductance of 0.01 henry and a resistance of 0.1 ohm is connected to a series circuit consisting of a 10 m.f. condenser, a 0.1 henry inductance and a 10 ohm resistance. The generated wave of the generator is represented by

$$e_0 = 100 \sin wt + 20 \sin (3wt + 30°) + 10 \sin (5wt + 15°)$$

The frequency is 60 cycles per second for the fundamental. Plot waves of current and voltage across the inductance in rectangular coordinates.

**2.** What are the effective voltages across each unit in the series circuit? What is the effective current? What is the average power? What is the relation between the cosine of the angle of phase difference between fundamental current and terminal voltage and the average power divided by the effective volt-amperes?

**3.** In Fig. 432 let $e_0 = 100 \sin wt + 30 \sin (5wt + 15°)$

$r = 10$ ohms

$L = 0.1$ henry

$C = 100$ m.f.

$f = 60$ cycles

(*a*) Find $E_0$, ${}_cI$, ${}_lI$, ${}_rI$, and $I_0$.

(*b*) Write the equation for $i_0$.

(*c*) Plot curves in rectangular coordination for $e$ and $i$.

For Experiments see page 551.

## CHAPTER XXIV

## PROTECTIVE APPLIANCES

In all electric plants safety devices are installed for protection against abnormal conditions. Even under normal operating conditions the stresses may vary greatly under changes of load, and especially while starting or stopping machines, may reach several times the average values. The extent of this variation may be calculated and the maximum stresses must be provided for in the design of the apparatus. Auxiliary apparatus, like starting autotransformers, are used to lower the impressed voltage; and circuit-breakers and fuses are inserted to protect against excessive currents. Speed control is centered in the governor of the prime mover. Any changes in the load must be balanced by a similar change in the amount of energy supplied by the prime mover. The action of the governor can not, however, be instantaneous and there will always be a certain lag in the adjustment of the energy delivered by the generator to the amount received by the load. Hence after a decrease in load the generators and motors receive more energy than is delivered; and conversely, for an increase in the load the amount delivered is greater than that received. The difference must be stored in or taken from the system and causes changes chiefly in the kinetic energy of the rotating machinery. Any change in the load must, therefore, be counteracted by rapid and effective action of the governor of the prime mover in order to obtain, as nearly as possible, the constant speed required for constant frequency.

Abnormal conditions are, of course, transient phenomena, and an extended discussion does not come within the scope of this book. However, a brief statement of two important groups of dangerous disturbances will be given in order that one may more readily understand the limitations imposed on operating conditions for alternating-current systems.

(*a*) Lightning.

(*b*) Short-circuits and arcing grounds.

(*a*) *Lightning.*—Lightning is a complex phenomenon and for

many of its manifestations no satisfactory explanations have been found. Several factors, however, have been determined with sufficient accuracy, so that apparatus can be constructed that effectively protect electric systems against damage from lightning disturbances, except in extreme cases. The thunder-cloud, the earth and the intervening air form a gigantic dielectric circuit, or condenser. The voltage difference between the earth and the cloud becomes very great, often reaching millions of volts before a disruptive spark, or lightning stroke, suddenly relieves the dielectric stress. The discharge is oscillatory with frequencies in the hundreds of thousands of cycles per second. The time duration of the stroke is extremely short, a small fraction of a second, while the current, and especially the power are very great, in some cases probably millions of kilowatts. The amount of energy released from the dielectric circuit, and suddenly transformed into light, heat, sound, mechanical work and chemical reactions, varies widely, and may be only a few kilowatt-hours. It is the *extremely rapid rate of transformation* in the stroke that makes the lightning phenomena so destructive, although in most cases the total amount of energy involved may be comparatively small. The sudden break in the stress on the electrically elastic air at any one point necessitates a readjustment of the energy stored dielectrically under the whole cloud. This, although a secondary effect, may produce violent disturbances, that may cause damage at a considerable distance from the actual stroke. This readjustment may cause damage far away from the lightning discharge, particularly in long-distance transmission lines, as the metallic conductors, insulated from the ground, provide special rapid-transit routes for equalizing the energy distribution. The lightning cycle may be described as consisting of three stages:

*First.*—A gradual increase in the difference in voltage between the earth and the thunder-cloud with a corresponding increase in the stress on the electrically elastic air, and in the amount of dielectrically stored energy.

*Second.*—When at any time and place the spark-over gradient is reached, a lightning stroke follows. A large portion of the dielectrically stored energy in a limited space near the point of discharge is transformed, at an extremely rapid rate, into light, heat, sound, mechanical work and chemical reactions.

*Third.*—A rapid readjustment of the energy distribution in all parts of the complex dielectric system. The rate of readjust-

ment is greatest nearest the point of discharge, and directly after or during the discharge.

The cycle then repeats: a gradual increase of the stress until the spark-over gradient is reached; a sudden release of the stress with energy transformation; and a rapid readjustment of the energy distribution in the dielectric system. In addition to the lightning discharges the continuous change in shape of the cloud and its motion relative to the earth produce a corresponding change in the dielectric circuit. This necessitates a continuous change in the energy distribution whether or not the voltage gradient be sufficient to produce a spark discharge. The rate of change in the form and in space position of the cloud is relatively slow, and hence the corresponding energy changes seldom cause any violent disturbances. The conductivity of the cloud, a variable factor, while greater than that of the intervening air, is much less than for metallic conductors. For clouds covering wide areas, only the energy stored in a limited space near the place of discharge is released by any one lightning stroke. So many variables, as the shape of the cloud, its distance from the earth, the contour of the earth's surface, the conductivity of the space filled by the cloud, the condition of the air between the cloud and the earth, variations and changes in the temperature, humidity, wind, precipitation, and similar factors make general quantitative calculations impossible. Hence, although the nature of lightning phenomena are in general, known, the several factors that enter into the problem may vary widely, and the local conditions of individual transmission lines may so modify any specific case, that no definite predictions can be made. In general, lightning and other atmospheric causes produce disturbances of very high frequency and high voltage, and may be of any degree of intensity according to the proximity of the discharge to the circuit in question.

(*b*) *Short-circuits. Arcing Grounds.*—Frequently the internal disturbances produce a breakdown of the insulation followed by a short-circuit, or an arcing ground. Other causes, such as the sudden operation of switches or circuit-breakers, or the occurrence of resonance may also produce disastrous results. Usually the disturbances from internal causes are of low voltage and low frequency but often of very large current. On constant-potential systems the generators and prime movers automatically tend to hold the voltage at the normal value, and hence under short-

circuit or arcing ground conditions enormous currents may flow. With the entire power of the station forcing energy into the break, the results may be very destructive. Intermittent arcing grounds are especially dangerous because they impose a succession of severe strains on the apparatus. The arc may start oscillations of almost any frequency, and it is therefore extremely dangerous since it may cause resonance or destructive surges in almost any system.

**Protective Apparatus.**—Whether the source of the disturbance be from external or internal causes, or a combination of both, the dangerous abnormal conditions consist of high voltages, excessively large currents or high frequencies. Since these factors all refer to a transient flow of energy the abnormal condition may be transferred from one factor to another in passing through parts of the electric apparatus. Thus a high-frequency current passing through an inductive reactance, as a transformer winding, generates in the first few turns an abnormally high voltage that may puncture the insulation. Although most of the dangerous disturbances are caused by external forces, as lightning and wind storms, and first affect the transmission line, the protective appliances are installed primarily to shield the station apparatus. One of three fundamental ideas underlies all protective installations:

(*a*) To provide a low-resistance path to ground by which the energy in the transient may be removed from the system;

(*b*) To automatically open the circuit and thus cut off the portion of the system having trouble; or

(*c*) To limit the energy flow between sections of the system, until the short-circuited portion may be isolated.

**A. Arresters.**—The first type deals chiefly with high-voltage and high-frequency disturbances, and in general may be classified as arresters. While a great variety of appliances have been tried only a few types are in general use: namely,

(*a*) Special ground wire.

(*b*) Horn-gap arresters.

(*c*) Reactors.

(*d*) Non-arcing metal arresters.

(*e*) Electrolytic arresters.

(*a*) *Special Ground Wire.*—Long-distance transmission lines are often provided with an extra wire or stranded cable supported on the tops of the poles above the regular transmission wires.

This conductor is grounded at frequent intervals, preferably at every pole. The *ground* is thereby brought above the transmission line and hence the dielectric stress in the air, due to lightning and other atmospheric disturbances, near the transmission wires and insulators is greatly reduced. Since the ground wire is parallel to the transmission wires it also dampens any impulses or surges that travel along the line, thus aiding the station arresters. Although the ground wire is on top of the transmission line, its chief service is to prevent external disturbances from reaching the station apparatus, the protection of the transmission line itself being of secondary importance.

(*b*) *Horn-gap Arresters.*—Two curved wires, bending away from each other, as shown in Fig. 433, form the horn-gap arrester. One side is connected to the line wire, and the other to ground, usually in series with a resistance or an electrolytic arrester. If the voltage gradient exceeds the spark-over value, a discharge occurs across the narrowest part of the gap and an arc is formed. The path of the current through the lower portion of the horn-shaped wires and the bridging arc is in the shape of an inverted U, and the magnetic field on the inside of this curve forces the arc upward, thereby increasing its length, until the arc breaks. The heat liberated by the arc produces air currents that, to some extent, aid the magnetic forces in breaking the arc. The cycle repeats until the voltage gradient in the gap is less than the spark-over value.

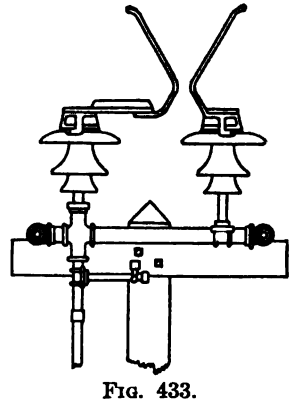


FIG. 433.

Since the arc resistance is very low, if one side of the gap is connected directly to ground, the discharge becomes a short-circuit. The simple horn gap is therefore not used alone except in special cases for protection against such violent disturbances that even a short-circuit is preferable. At places of special danger on long transmission lines, as where the line crosses a range of hills, the simple horn-gap arrester is sometimes installed to guard the system against direct strokes of lightning. For this service the horns are spaced further apart than at the station and therefore the arrester operates only for disturbances of exceptional violence. The horn gap is widely used in connec-

tion with a resistance and in combination with reactors and electrolytic arresters to give the desired protection.

(c) *Reactors.*—Electrically, reactors are merely low inductances that may be connected in series with the transmission-line wires where they enter the station. Mechanically the reactors consist of a few turns of wire arranged in a spiral or helical form with sufficient spacing to withstand high voltages. The chief function of the reactor is to prevent any high-frequency disturbance on the line from entering the station. Since the reactance of the reactor is directly proportional to the frequency, any high-frequency current from the line produces high

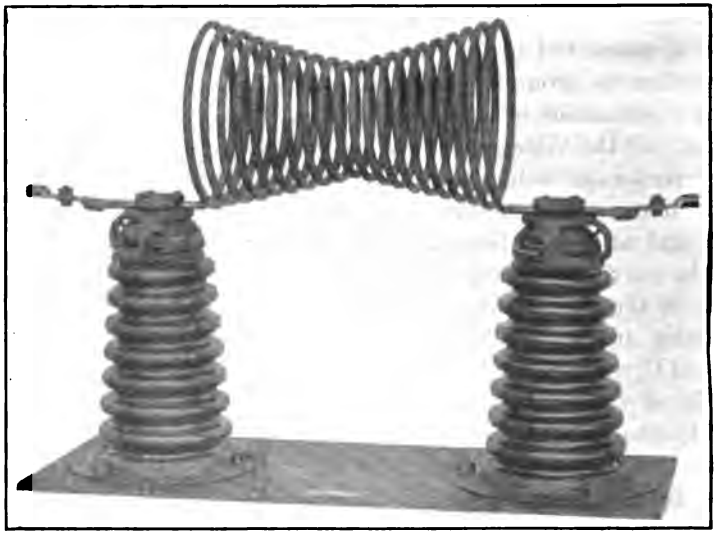


FIG. 434.—Hourglass type of reactor arrester. (*Gen. Elec. Co.*)

voltages across the reactor. An arrester is usually connected between the ground and a point on the line just outside the reactor. The high voltage produced by the high-frequency current in the reactor operates the arrester and the transient energy is shunted to ground.

The energy content of these transients is in most cases small but may be sufficient to produce a puncture in the insulation. In the air-insulated reactor the rupture is automatically repaired, but in the solid insulation of the transformer windings the breakdown persists and the normal busbar voltage may be

sufficient to enlarge the rupture and produce a short-circuit and burnout. If no reactors are installed it is customary to provide specially heavy insulation on the terminal turns of the transformer as a protection against the high voltages induced by the transient high-frequency currents. In systems having long-distance transmission lines reactors must be used in combination with extra heavy insulation on the terminal turns of the transformers.

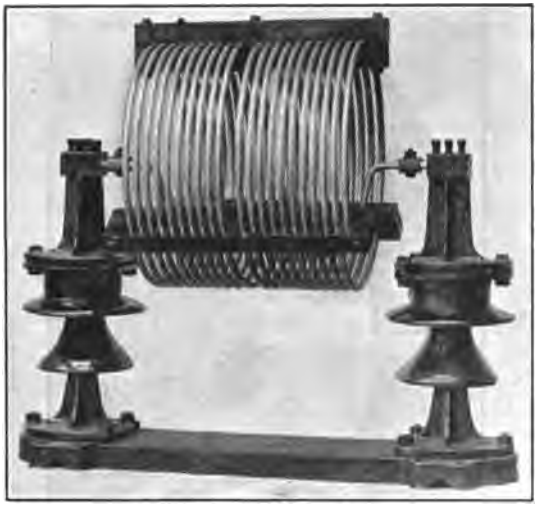


FIG. 435.—Cylindrical type of reactor arrester. (*Westinghouse Elec. & Mfg. Co.*)

Three types of commercial reactors are shown in Figs. 434, 435 and 436. In some designs the inductances of the three types are in the ratio of 1 : 5 : 50. Where still larger inductances are required, as in mountainous districts having severe thunder storms, the arresters, of the pancake type are enclosed in iron tanks and submerged in oil. Pancake types are at the present time not used on systems above 25,000 volts.

Reactors do not dissipate the energy in the high-frequency disturbance but merely reflect the transient back on the transmission line. Unless a path to ground or some means of dissipating the energy in the transient is provided, the disturbance is transmitted to other parts of the system, and may break through the insulation at some other point or surge to and fro in the line until dissipated by the $rI^2$ losses. In order to remove

the energy in the high-frequency disturbance from the system a path to ground is generally provided through a lightning arrester. The reactors, therefore, are merely magneto-induction buffers that reflect the high-frequency disturbance back on the line. When the transient is reflected the voltage rises, and, if high enough, the lightning arrester, connected to the line close to the reactor, operates and removes the energy of the transient from the system.



FIG. 436.—Pancake type of reactor arrester.
(*Westinghouse Elec. & Mfg. Co.*)

(*d*) *Non-arcing Metal Arresters.*—When a discharge occurs between copper electrodes, a vapor stream of low resistance bridges the gap. The high resistance of the air gap is thus changed into a fairly good conductor. Similar results follow if the electrodes are made of other metals. But if certain alloys of zinc are used as terminals, the vapor formed by spark-over has a rectifying property which tends to prevent the current from flowing in the reverse direction during the next half cycle.

This property of the so-called non-arcing metal tends to limit an alternating-current arc to one-half cycle. The *non-arcing metal arrester* consists essentially of a series of knurled cylinders of a non-arcing alloy separated by small air gaps. In general the series of air gaps is combined with resistances in order to extend the voltage range of the arrester. On the basis of these combinations the non-arcing metal arresters may be divided into four groups:

1. A series of gaps between non-arcing metal cylinders, Fig. 437.

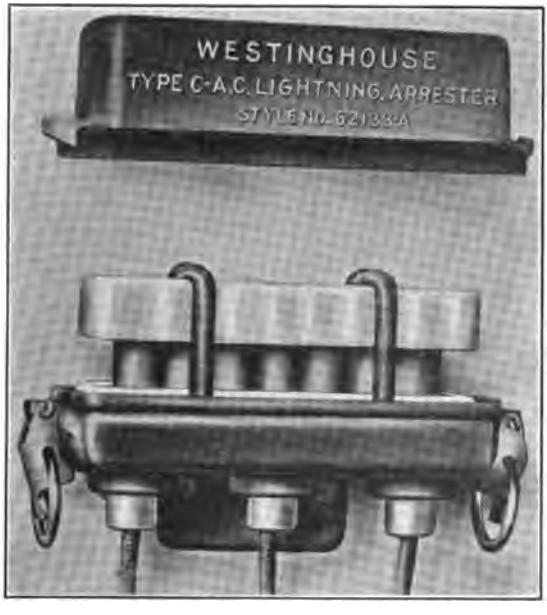



FIG. 437.—Non-arcing metal cylinders. Lightning arrester.

2. Series of gaps as in (1) connected in series with a resistance, Fig. 438.

3. Two series of gaps as in (1) connected in series, with one of the sets shunted by a resistance.

4. Combination of groups (2) and (3).

Group (1) is the simplest in design, but can be used only on small, low-voltage plants, upper limit 500 kw. and 2,500 volts. For higher voltage the rectifying property of the alloy used is not sufficient to break the arc if once formed. In group (2) the series resistance limits the current, and thereby aids the gaps in

breaking the circuit. This type may be used on plants of any capacity, and for a wide range in voltage, usually up to 22,000 volts. It may be used to advantage on higher voltages and has given satisfactory service on 33,000 volts. The series resistance must be increased with the voltage and this limits the initial discharge. For higher voltages the protection given by the arrester is reduced as the apparatus does not give a free path to ground for heavy discharges.



FIG. 438.—Non-arcing cylinders in series with resistance.

In group (3) the advantages of group (1) are extended to higher voltages and larger stations. A free path to ground is provided for the initial discharge when the voltage is high enough to arc across both sets of series gaps. The resistance forms a bypath and thereby the voltage across the shunted gaps is reduced until the arc goes out. The resistance then reduces the power current following the discharge until the arc is broken in the series gaps.

Group (4) is the complete form of the non-arcing metal arrester and is known as the multigap or low-equivalent type. It may

be used in stations of any capacity and for a wide voltage range. While this type is still used extensively it has been largely superseded by the electrolytic arrester on high-voltage systems. One difficulty in using the multigap arrester on high voltages is the unequal distribution of the voltage in the several gaps in series. The gap nearest the line will have the largest and the gap nearest the ground the least voltage on account of the combination of parallel and series dielectric circuits between the metal cylinders themselves and to ground. The combination of series and parallel dielectric circuits is similar to that explained in detail for string insulators in Chap. XX. The conditions are so variable, quantitatively, in different installations, due to the dimensions of the dielectric circuits to ground, that it is customary to provide an adjustable spark gap in addition to the series of non-arcing metal gaps, by which the spark-over voltage of the arrester may be adjusted to suit the conditions of each installation.

(*e*) *Electrolytic Arresters.*—The elemental unit in the electrolytic arrester is a cone-shaped aluminum tray immersed in an elec-

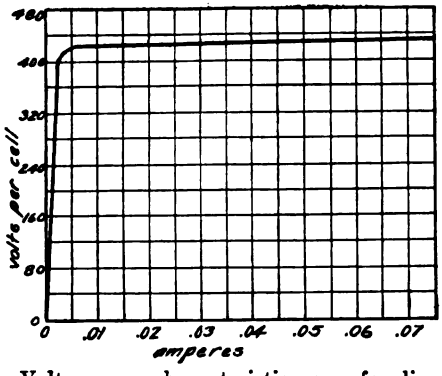



FIG. 439.—Volt-ampere characteristic curve for direct currents.

trolyte and so placed that the current flowing in the electrolyte passes through the tray. The current passing through the aluminum forms a thin surface layer of aluminum hydroxide having a very high resistance. For higher impressed voltages the hydroxide film becomes thicker and the resistance increases up to a *critical voltage.* For pressures above the critical voltage the hydroxide film breaks down and the high resistance is then removed from the circuit. While the range between partial and complete breakdown of the hydroxide layer is small the critical

voltage is a fairly definite value, as shown by the sharp bend in the curves in Figs. 439 and 440. It is affected by the electrolyte used, the wave shape and other factors. If the current passes through several trays in series, the total resistance and likewise the total breakdown voltage is increased in proportion to the number of trays. Arresters designed for service on high-voltage systems, therefore, have a sufficient number of trays arranged in series so that the voltage per tray is less than the critical voltage. For alternating currents it is customary to let the normal operating voltage be 300 volts or less per tray. Hence, for a system having 44,000 volts to neutral the electrolytic arrester should have at least 150 trays.

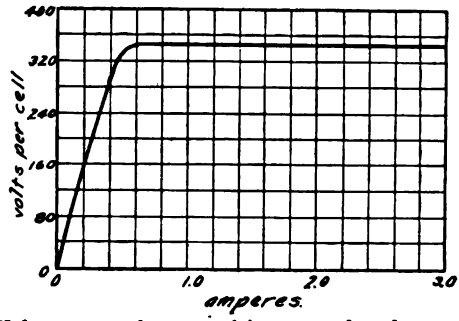



FIG. 440.—Volt-ampere characteristic curves for alternating currents.

The shape and arrangement of the aluminum trays, the space filled by the electrolyte, the oil insulation and terminal leads are shown in Figs. 441 and 442. The oil insulates the trays, conducts away the heat and prevents evaporation of the electrolyte. It also prevents the electrolyte from splashing out of the tray during discharges. The shape of the tray is usually conical or a double cone for large sizes, but always so designed that gas bubbles formed during the discharge may escape. To provide a path of sufficiently low resistance so as to afford the required freedom of discharge when the high-voltage disturbance disrupts the hydroxide films, the cross-sectional active tray area is from 90 to 100 sq. in.

With the arrester connected to the line a small current flows below the critical voltage as indicated by the curves in Figs. 439 and 440. For direct currents this is strictly a leakage current. For alternating currents the susceptance formed by the

aluminum trays takes a charging current in addition to the leakage through the resistance of the hydroxide films. The leakage current wastes energy which, if flowing continuously, heats the electrolyte to a temperature that may impair the effectiveness of the arrester. To eliminate both the energy loss and undesirable rise in temperature a horn gap is installed between the electrolytic arrester and the line as shown in Fig. 443*a*.

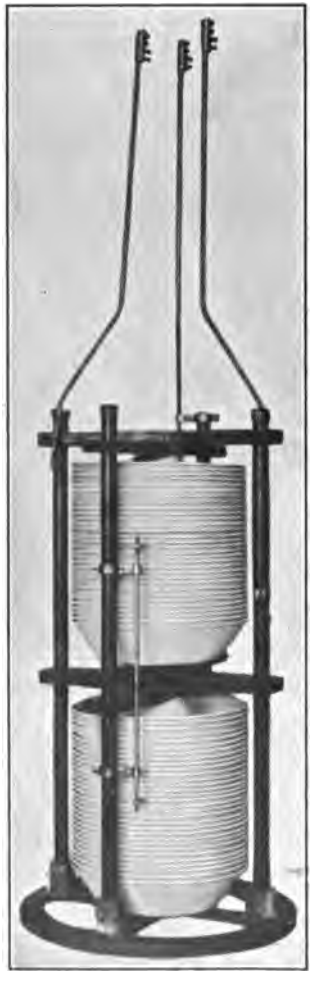


FIG. 441.—Aluminum trays removed from tank. (*Westinghouse Elec. & Mfg. Co.*)

The formation of a uniform and continuous hydroxide film requires great care and, moreover, the hydroxide film, once formed, is slowly dissolved by the electrolyte. To keep the film in good condition it is, therefore, necessary to send at intervals, a current through the arrester for a few seconds. This is called *charging the arrester*, and is accomplished by short-circuiting the horn gap through a resistance. One type of short-circuiting device is shown in Fig. 443*b*. For commercial systems it is customary to *charge the arresters* once every twenty-four hours. Under ordinary operating conditions the horn gaps prevent any current from flowing through the electrolytic arresters. The spacing of the horn gaps is adjusted so that any dangerous high-voltage disturbance on the line jumps across the gap. The arc thus formed has low resistance and the high voltage impressed on the electrolytic arrester disrupts the hydroxide films, thus forming a low-resistance path to ground through which the energy of the transient is conducted to ground. When the transient voltage drops below the critical voltage, the aluminum

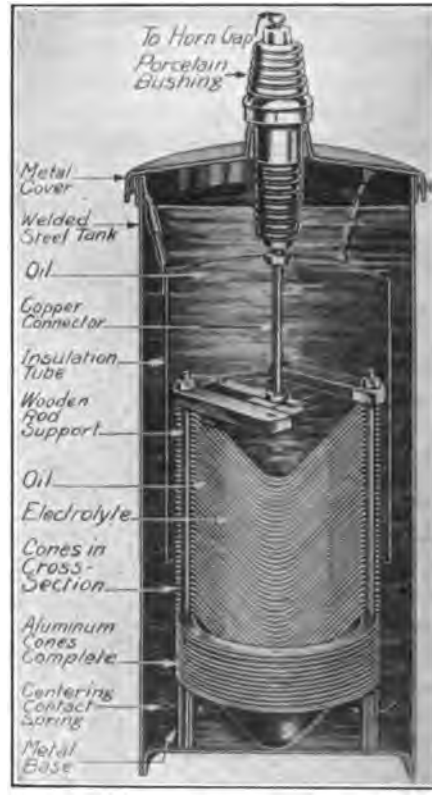



FIG. 442.—Cross-section of an aluminum arrester. (*General Electric Co.*)

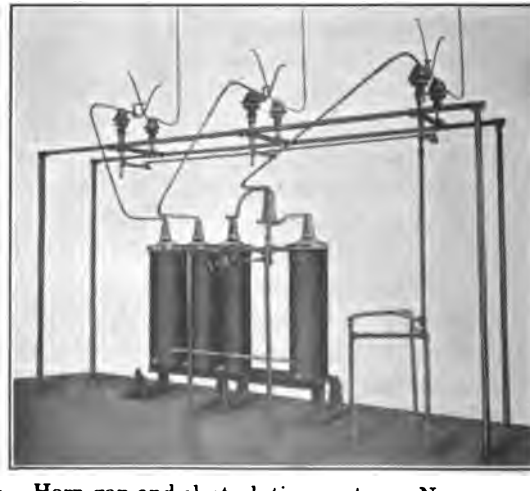


FIG. 443*a*.—Horn-gap and electrolytic arresters. Non-grounded system.

hydroxide films are formed and the high resistance thus automatically inserted cuts down the current until the arc in the horn gap breaks.

For low-voltage, high-frequency disturbances coming from the line, the reactors reflect the wave causing the voltage to rise until an arc breaks across the horn gap, and to disrupt the hydroxide resistance films in the electrolytic arrester. A low-resistance path is thus formed, through which the energy of the transient passes to ground.

The combination of reactors, horn gaps and electrolytic arresters effectively protects the plant for practically all types of

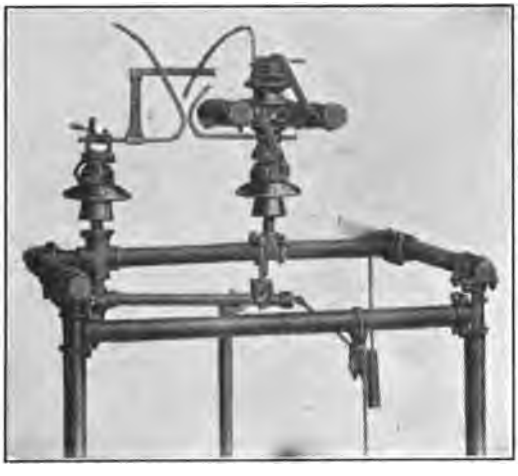


FIG. 443*b*.—Horn-gap short-circuited while charging electrolytic arrester.

high-voltage and high-frequency disturbances coming from the transmission line and is in general use on all long-distance transmission systems. For grounded systems three arresters are connected to a neutral point. For ungrounded systems the three arresters are also connected to a neutral point, and in addition a fourth arrester is placed between the neutral point and ground as shown in Fig. 443*a*.

The electrolyte generally used is a solution of ammonium-tetra-borate for installations where the temperature does not fall below 5°C. Below this temperature this solution freezes. In colder climates a mixture of ammonium-tetra-borate and ammonium-tartarate is often used. As the tartarates dissolve the

hydroxide film faster than the borates it is not desirable to use the mixture except under low temperature conditions.

(*f*) *Oxide Film Arrester.*—The arrester consists of a set of oxide film cells connected in series in a manner similar to the aluminum disks in the electrolytic arrester. The cells are disk-shaped, about 7½ in. in diameter and ⅝ in. thick. Each cell is made of two circular sherardized steel plates crimped firmly to the edges of an annular piece of porcelain. A powder, lead peroxide, which has very low resistance, compactly fills the space between the metal plates. The inside of the metal plates is covered with a varnish film which is an insulator. The number



FIG. 444*a*.—Oxide film arrester—indoor service—three-phase—15,000 to 25,000 volts. Type OF, form B. (*General Elect. Co.*)

of cells used in an arrester is such that the voltage per cell is approximately 300 volts.

When a lightning voltage sparks over the gaps it is impressed on the cells and breaks down the insulating coating on the metal plates. The breakdown occurs in the form of a small puncture of the film coating. The metal plates are not punctured. As soon as the film gives way, a discharge current flows through the cells to ground, thus relieving the lightning pressure. The flow of current through the cells immediately causes a chemical change, by heat, in the lead peroxide at the point of puncture. The lead peroxide is changed to red lead and litharge which have a very high resistance. Thus, following the lightning discharge, a

very high resistance amounting practically to insulation is automatically cut into the discharge path. This cuts off the flow of generator current that would otherwise follow the lightning discharge, and the arcs in the gaps die out. If the potential should still, or again, be sufficiently high to break down the gaps, the operation is repeated at some other point on the surface of the varnished plates.

At each discharge through the stack a minute area of the varnish film is thus replaced by a red lead or litharge film, which

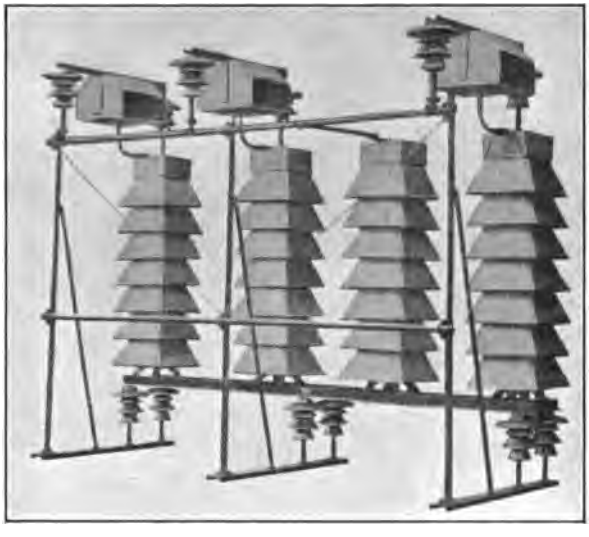


FIG. 444*b*.—Oxide film arrester—outdoor service—three-phase—69,300 to 73,000 volts. Type OF, form BO. (*General Elect. Co.*)

has a slightly higher insulating value. With continued service this action takes place over the active area of the plate so that the varnish film is more or less replaced by the litharge or red lead film and this in turn will act as the active film and be broken down by excess potentials with consequent sealing up action with a higher insulating value of film. Thus the film gradually thickens and becomes of higher breakdown voltage with resultant decrease in the protective value of the arrester. It is evident that the effective life of the cell, while not unlimited, is much longer than represented by the original active area of the plates.

All arresters are equipped with sphere gaps (Fig. 444*a*).

The 1000–7500-volt arresters actually have horn gaps but the curvature and size of the horns are such, with respect to the gap, that the effect is the same as if sphere gaps were used. The sphere gaps give the highest obtainable speed of discharge from the most dangerous kind of lightning disturbances—impulses of steep wave front.

For the outdoor arresters (Fig. 444*b*) the sphere gaps are covered so that the gap setting, not being affected by rain, can be made the same as for the indoor arresters.



FIG. 445.—Inverse time-element, overload relay.

**B. Relays.**—Aside from protection to life, continuity of service is of paramount importance in the operation of all electric systems. The larger the system the more important is the necessity for continuous service. Likewise, the larger the system the more frequent will be the occurrence of short-circuits that may cause shut-down. The enormous increase in the size of central stations during the past few years has in a measure been due to the successful operation of selective apparatus that automatically cut out any part of the system in which a short-circuit develops,

thus permitting the greater portion of the plant to operate without interruption. This automatic, selective action is obtained by means of some form of relay, which may be defined as an auxiliary device, that operates and transmits its action to another independent piece of apparatus on the occurrence of some predetermined condition in the electric circuit. Mechanically a relay consists: first, of a coil, or coils, connected to the circuit it controls; second, a movable part whose motion is determined by the current in the coils; and, third, a contact device operated by the movable part, controlling the circuit that operates a switch or circuit-breaker The relay must operate only for such disturbances for which it has been designed and only when the disturbances are in the part of the system under its control. Relays of great variety are in commercial use, and may be classified in several ways. The nature of the disturbance is a fundamental factor and may be used as a basis for classification; also, whether the switch or circuit-breaker controlled by the relay opens or closes the circuit. However, the most important factor in the service desired from the relay is the time elapsing between the occurrence of the disturbance and the operation of the contact device. Only the essential features determining the time elements of relays designed for protection of excessive currents or reversal of power due to short-circuits on the system will be mentioned.

(*a*) Instantaneous relays.
(*b*) Definite time relays.
(*c*) Inverse time-limit relays.
(*d*) Reverse power relays.

The nature of the disturbance and the general plan of the system determines which type should be installed at various places in the system. These factors are referred to only incidentally in explaining the characteristics of the relay mechanisms.

(*a*) *Instantaneous Relays.*—This is the simplest form and consists of a solenoid with a plunger acting on a trip mechanism. For currents in excess of a predetermined maximum or below a fixed minimum the relay operates as nearly instantaneously as the inertia of the moving parts permits.

(*b*) *Definite Time Relays.*—In most cases the disturbances do not cause a complete breakdown, and the excessive flow of current is of short duration causing no damage to the apparatus. The

continuity of service may be improved by using a relay so constructed that a definite time must elapse from the occurrence of the disturbance to the operation of the contact devices. While a number of devices have been used to give the desired time lag only three types are in general use.

(1) Clock mechanism.
(2) Leather bellows.
(3) Magnetic induction.

If two types of apparatus for the same service give satisfactory results the higher-priced machine will be forced from the market. On account of the relatively high cost of the clock type of relay its use is limited to special cases, since satisfactory services may be secured in most cases from the bellows and induction types.

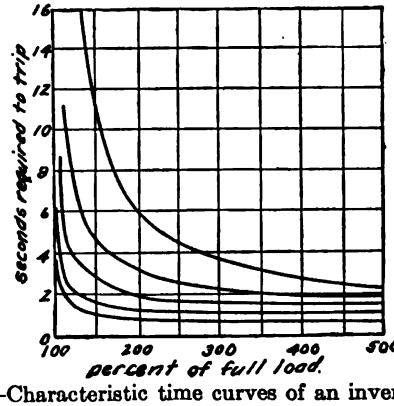



FIG. 446.—Characteristic time curves of an inverse time limit bellows-type relay.

The definite time limit is secured, in the second type, by means of leather bellows filled with air, under normal operating conditions. When the increased current operates the relay a spring is released which then presses against the bellows forcing the air out through a needle valve. The size of the opening in the needle valve and the pressure exerted by the spring may be adjusted so that it takes a predetermined length of time to empty the bellows. The third type is built on the same principle as an induction wattmeter. The rotating field produces the torque, and the *drag* is obtained by means of an aluminum disk moving between the poles of permanent magnets. A small transformer (compensator) is inserted between the series transformer and the relay. The core of this transformer is of such cross-section that the saturation point is reached at full-load current. Hence, for

overloads the current in the relay does not increase as fast as in the mains and the torque may be adjusted so that a definite time elapses before the relay operates.

(c) *Inverse Time-limit Relays.*—In the inverse time-limit relay of the bellows type, the core acts directly on the bellows; thus a pressure, proportional to the overload, forces the air through the needle valve. The time for compressing the bellows is, therefore

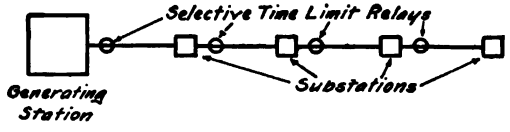



FIG. 447.

inversely proportional to the overload. Characteristic curves are shown in Fig. 446.

The location of the relays in the system depends on many factors. In general, inverse time-limit relays are placed nearest the generators, while definite time-limit relays control the substation feeders.

The time setting depends on the characteristics of the generators and the position in the system. Among the factors that must be known, the more important are:

(1) The instantaneous short-circuit current.

(2) The sustained short-circuit current.

(3) The safe circuit-opening capacity of the circuit-breakers.

(4) The time characteristics of the relays and the circuit-breakers.

(5) The position in the system.

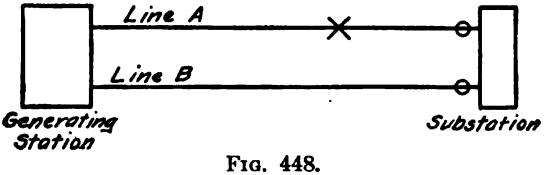



FIG. 448.

If several relays are in series on a system, as in Fig. 447, the one farthest from the generator must operate in the shortest time, with each relay in the series toward the generator requiring longer time than the one preceding it.

The characteristics of both the definite time and the inverse time-limit relays are sometimes combined in one apparatus for use in special cases.

(*d*) *Reverse Power Relays.*—If power is transmitted from a generating station to a substation over two lines connected to the same busbars Fig. 448, a short-circuit, as at $X$ on the line $A$, will cause power to flow not only from the generator on line $A$, but also on line $B$ through the substation and back on line $A$ to $X$. The excessive current should operate the relays and open the circuit of the line $A$ at the generator. In order to continue the

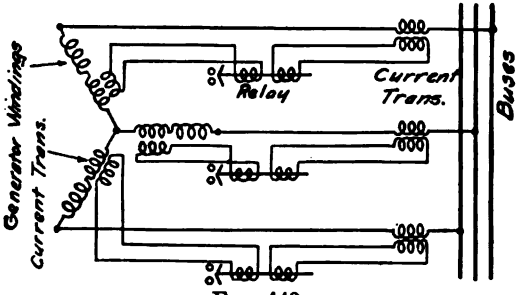



FIG. 449.

service it is also necessary that the line $A$ be disconnected at the substation. This is accomplished by means of reverse power relays. One form in common use is based on the dynamometer principle. The stationary coils are connected to series current transformers and the movable element to potential transformers connected to the busses. While the energy flows in the desired direction the contact lever operated by the shaft carrying the

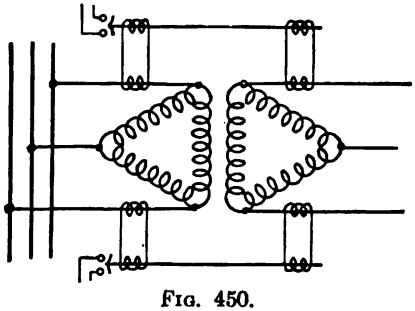


FIG. 450.

movable coil is held against a dead stop by a spring. When the direction of energy flow reverses, the movable coil tends to turn in the opposite direction and throws the contact lever against a contact completing the tripping circuit for the oil switch. It is evident that this apparatus is not affected by overloads and that it operates only on the reversal of the direction of the energy flow.

For the protection of generators or transformers against short-circuits inside the machine or for a short between the machine and the busbars a reverse-power relay, based on the differential principle, is used. The connections are shown in Figs. 449 and 450. Series transformers are connected in opposition as shown in the diagrams. If a short-circuit occurs between the two series transformers connected to the relay the flow of current in one is reversed and thus the sum of the two currents produces a heavy magnetomotive force that operates the relay. Various connections are used for special cases to give the desired action, but



FIG. 451.—119 kv.a., three-phase, 160 cycles, current limiting reactor. 200 amperes, 3 per cent. and 11,400 volt lines. (*Westinghouse Elect. & Mfg. Co.*)

fundamentally the reverse-power relays are based either on the dynamometric or the differential principles.

**C. Power-limiting Reactances.**—In large central stations apparatus are installed that will automatically limit the amount of power that can be developed under short-circuit conditions in order to save the machines from self-destruction.

As explained in Chap. XIV and illustrated by Fig. 258 the instantaneous short-circuit current is several times the value of the sustained short-circuit. Since the magnetic forces vary as

the square of the current, the mechanical forces due to the instantaneous short-circuit current may tear the alternator or the transformer windings apart and wreck the station.

Another factor that must be considered is the maximum current for which the circuit-breaking switches can safely operate. Under excessive currents the switch would be destroyed. A third factor is the necessity for limiting the amount of energy that can be transmitted from the station to the short-circuit. In stations of 100,000 kw. capacity or more the amount of power that could be concentrated in a short-circuit is enormous, probably several million kilowatts.

To protect the alternators, transformers and switches against excessive short-circuit currents and to limit the power that can be delivered by the station, power-limiting reactances are placed in the generator leads.

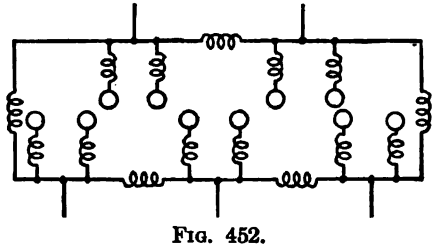


FIG. 452.

A typical set for a three-phase generator is shown in Fig. 451. The core is of concrete, and the winding consists of bare stranded copper cable held in place by wooden supports bolted to the core.

In addition to the protection given to the station machinery, power limiting reactances are also used to enable the large central station to give continuity of service. The busbars in the station are sectionalized and reactances are placed between the several sections, as illustrated for a typical case in Fig. 452. These reactances limit the power that flows between the sections so that if a short-circuit occurs on one section, only the circuit-breakers for that section shall operate. The generators in the other sections carry an overload until the operator isolates the section on which the short-circuit occurred. With 25 per cent. reactance between the sections neither the drop in voltage nor the phase displacement is large enough to cause synchronous motors or converters to drop their load. Similar reactances are sometimes placed in the feeders and in the tie lines between the stations.

## CHAPTER XXV

## POLYPHASE SYSTEMS

Any polyphase system may be analyzed by separating the network into its elemental branches or phases. The voltage generated in any one phase causes currents to flow in the network that may be calculated independently of the voltages in the other phases. The current flowing at any point and at any instant in an interconnected system is the vector sum or the resultant of the currents produced by all the voltages in the several phases. Ohm's and Kirchoff's laws apply to the complicated network in the same manner as for simple series or parallel circuits. In order to emphasize certain characteristics of commercial systems under operating conditions it is customary to make the following classifications:

(*a*) Symmetrical and unsymmetrical systems

(*b*) Independent and interlinked systems.

(*c*) Balanced and unbalanced systems.

(*d*) Systems having grounded or floating neutral.

(*a*) **Symmetrical and Unsymmetrical Systems.**—If a polyphase system is composed of $n$ phases having voltages of equal magnitude and differing in time phase successively by $1/n$th of a cycle the system is symmetrical; but if either the magnitude or the time-phase displacement of the voltages, or both, differ, the system is unsymmetrical.

For symmetrical systems:

$$
\begin{aligned}
e_1 &= {}^{\prime\prime}E \sin(\omega t) \\
e_2 &= {}^{\prime\prime}E \sin\left(\omega t - \frac{2\pi}{n}\right) \\
e_3 &= {}^{\prime\prime}E \sin\left(\omega t - 2\frac{2\pi}{n}\right) \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
e_n &= {}^{\prime\prime}E \sin\left[\omega t - (n-1)\frac{2\pi}{n}\right] \qquad (820)
\end{aligned}
$$

For $n = 1$, the single-phase, two-wire system.

$$e_1 = {}^{\max}E \sin(\omega t) \tag{821}$$

For $n = 2$, the single-phase, three-wire (Edison) system.

$$e_1 = {}^{\max}E \sin(\omega t) \text{ and}$$

$$e_2 = {}^{\max}E \sin(\omega t - 180°) \tag{822}$$

For $n = 3$, the three-phase, three-wire system.

$$e_1 = {}^{\max}E \sin(\omega t)$$

$$e_2 = {}^{\max}E \sin(\omega t - 120°) \text{ and}$$

$$e_3 = {}^{\max}E \sin(\omega t - 240°) \tag{823}$$

For $n = 4$, the four-phase, or quarter-phase four-wire system.

$$e_1 = {}^{\max}E \sin(\omega t)$$

$$e_2 = {}^{\max}E \sin(\omega t - 90°)$$

$$e_3 = {}^{\max}E \sin(\omega t - 180°) \text{ and}$$

$$e_4 = {}^{\max}E \sin(\omega t - 270°) \tag{824}$$

Similarly for $n = 6$ and $n = 12$, we have the six-phase and twelve-phase systems.

*Unsymmetrical Systems.*—The two-phase system, the three-phase open delta or $V$ connection and the inverted three-phase or monocyclic connection are the more common unsymmetrical systems.

(*b*) **Independent and Interlinked Systems.**—Each phase of an $n$-phase system may be arranged so as to form a closed circuit, the whole system, therefore, consisting of $n$ independent single-phase systems. The two phases of a four-wire two-phase system are often operated as two independent single-phase circuits. In most cases the several phases of commercial polyphase systems are interconnected, forming networks or interlinked systems through which the currents flow. In practice the phases are either star-connected or ring-connected, or the network consists of a combination of star and ring connections.

In a star-connected system the starting points of all the phases are connected together, forming the so-called neutral point. This common point is either connected to earth or to another neutral point in the same system. It is customary to consider

the neutral point as having zero potential. The voltage $e_n$ from the neutral to the line wire is the phase or circuit voltage. For any phase, $q$, of an $n$-phase system,

$$e_n = {}^{\nu}E_n \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] \tag{825}$$

Between the terminals of any two adjacent phases is the line voltage, $e_m$, or voltage between the mains. For any two adjacent phases, as $q$ and $q+1$,

$$\begin{aligned} e_m &= {}^{\nu}E_n \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] - {}^{\nu}E_n \sin\left(\omega t - q\frac{2\pi}{n}\right) \\ &= 2{}^{\nu}E_n \sin\frac{\pi}{n}\left\{\cos\left[\omega t - (2q-1)\frac{\pi}{n}\right]\right\} \end{aligned} \tag{826}$$

$$E_m = 2E_n \sin\frac{\pi}{n} \tag{827}$$

In any star-connected system the voltage between the mains is the vector difference of the voltages between the two adjacent circuits, and the current is the same as for the corresponding phase circuit.

In a ring-connected system the starting point of one phase is connected to the ending point of the next preceding phase. The sum of the voltages generated at any instant in the several phases connected in series should be zero, otherwise the resultant voltage causes currents to circulate in the ring connection. In symmetrical polyphase systems having simple harmonic voltage waves the sum of the voltages for all the phases is at all instants equal to zero and therefore either the ring or the star connections may be used. The voltage between successive mains is the same as the voltage in the corresponding circuit. The current in the mains is the vector difference of the currents in the adjoining phases. For the main between the two adjacent phases $q$ and $q+1$ the current is,

$$\begin{aligned} i_m &= {}^{\nu}I \sin\left[\omega t - (q-1)\frac{2\pi}{n}\right] - {}^{\nu}I \sin\left(\omega t - q\frac{2\pi}{n}\right) \\ &= 2{}^{\nu}I \sin\frac{\pi}{n}\cos\left[\omega t - (2q-1)\frac{\pi}{n}\right] \end{aligned}$$

$$I_m = 2I \sin\frac{\pi}{n} \tag{828}$$

The interlinked systems of commercial importance are described in Chaps. XII and XVI.

(*c*) **Balanced and Unbalanced Systems.**—Any polyphase system that develops a constant flow of energy is balanced, and any system in which the power varies or pulsates during each voltage or current cycle is unbalanced.

In single-phase circuits,

$$e = {}^{\prime\prime}E \sin(\omega t)$$

$$i = {}^{\prime\prime}I \sin(\omega t - \theta)$$

$$p = EI[\cos\theta - \cos(2\omega t - \theta)] \qquad (829)$$

The power pulsates with double the frequency of the voltage or current, and the system is unbalanced.

In two-phase circuits having the voltages, currents and power factors of the two circuits alike, the power,

$$p = p_1 + p_2 = 2EI\cos\theta, \text{ a constant} \qquad (830)$$

Under these conditions the two-phase system is balanced although unsymmetrical. However, if the voltage, current or power factor of phase $A$ is not equal to the corresponding factor in phase $B$ the flow of energy pulsates and the system is unbalanced.

In the three-phase system, either for delta or star connections, with the currents, voltages and power factors in the three phases of like value the power is constant and the systems are balanced.

$$p = p_1 + p_2 + p_3 = \sqrt{3}EI\cos\theta \qquad (831)$$

However, if the voltages, the currents or the power factors of the three phases differ, the power pulsates and the system is unbalanced; similarly for six-phase and twelve-phase or any symmetrical $n$-phase system. If the voltages, currents and power factors are alike for the several phases, the power is constant throughout the complete voltage cycle and the systems are balanced. For satisfactory operation the balanced condition is the most desirable. In commercial systems this is an ideal which is seldom attained.

The chief disadvantages resulting from unbalanced load conditions are: (*a*) poor voltage regulation, (*b*) unequal voltage stresses in the several phases, and (*c*) unequal heating in the circuits. The more common cause for unbalanced load conditions is the operation of essentially single-phase load, as incandescent lamps or single-phase motors, on polyphase systems.

With sufficient data given, the values of the currents and voltages in the several phases and their time-phase relations[1] may be found by the application of Ohm's and Kirchoff's laws. This may be illustrated by the solution of a few problems.

**Problem 1.**—Given a three-phase system, star-connected, floating neutral and having a non-reactive unbalanced load. Circuit diagram as in Fig. 453. $E_{A-D} = E_{B-A} = E_{D-B} = 220$ volts; $r_A = 3$; $r_B = 4$; $r_D = 5$ ohms.

Find (*a*) $E_A$, $E_B$, $E_D$.

(*b*) $I_A$, $I_B$, $I_D$.

(*c*) $\theta_{A_1A}$; $\theta_{B_1B}$; $\theta_{D_1D}$; time-phase angles between $E_A$ and $I_A$, etc.

(*d*) $\theta_{A-D_1A}$; $\theta_{B-A_1B}$; $\theta_{D-B_1D}$, time-phase angles between $E_{A-D}$ and $I_A$, etc.

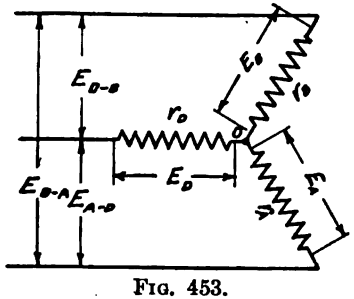



Fig. 453.

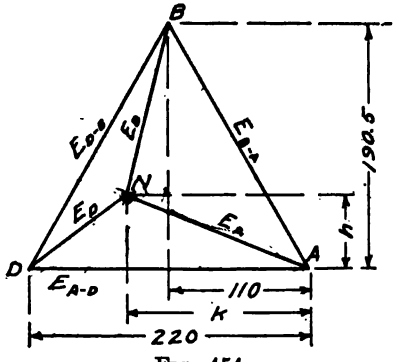



Fig. 454.

(*e*) The total kilowatts.

(*f*) Draw to scale the vector diagram for the currents and voltages.

Let the voltages between the mains be represented by the sides of an equilateral triangle, Fig. 454. Since the common point $O$ is not grounded and the resistances in the three circuits are unequal, the voltages from the mains to the common neutral point must be unequal. The vector difference of the voltages in any two circuits must be equal to the voltage between the corresponding pair of mains. Hence, by assuming some point $N$ in the diagram as the neutral position and drawing the vectors $NA$, $NB$ and $ND$ we have a graphical representation of the voltages in the circuits and in the mains. The actual position

[1] Three-phase power factor, by Austin Burt, *Trans.* A. I. E. E., XXVII, p. 801.

of the point $N$ may be found by the application of Kirchoff's laws. Though $N$ draw rectangular coördinates with the $X$-axis parallel to the line $DA$. Denote the coördinates of the point $A$ by $k$ and $h$. The following equations may then be derived directly from the diagram:

$$E_A = k - jh \tag{832}$$

$$E_B = (k - 110) + j(190.5 - h) \tag{833}$$

$$E_D = -(220 - k) - jh \tag{834}$$

$$I_A = \frac{k - jh}{3} \tag{835}$$

$$I_B = \frac{(k - 110) + j(190.5 - h)}{4} \tag{836}$$

$$I_D = \frac{(k - 220) - jh}{5} \tag{837}$$

From Kirchoff's laws the real and quadrature components may be separately equated to zero.

$$\frac{k}{3} + \frac{k - 110}{4} + \frac{k - 220}{5} = 0 \tag{838}$$

$$\frac{-h}{3} + \frac{190.5 - h}{4} + \frac{-h}{5} = 0 \tag{839}$$

Hence $k = 91.3$ and $h = 60.8$.

(*a*) $E_A = 110.0$ volts.
$E_B = 131.0$ volts.
$E_D = 142.2$ volts.

(*b*) $I_A = 36.7$ amp.
$I_B = 32.8$ amp.
$I_D = 28.5$ amp.

(*c*) $\theta_A = \theta_B = \theta_D = 0$ or the power factor is unity.

(*d*) The time-phase angles between the currents and the corresponding voltages between the mains may be found directly from the above equations.

The time-phase angle of the current in line $A$ and the voltage between the mains $A$ and $D$,

$$\theta_{A-D,A} = \tan^{-1}\frac{h}{k} = 33° 40'.$$

The time-phase angle of the current in line $B$ and the voltage between the mains $B$ and $A$,

$$\theta_{B-A,B} = 30° + \tan^{-1}\frac{-110+k}{190.5-h} = 21°\ 45'.$$

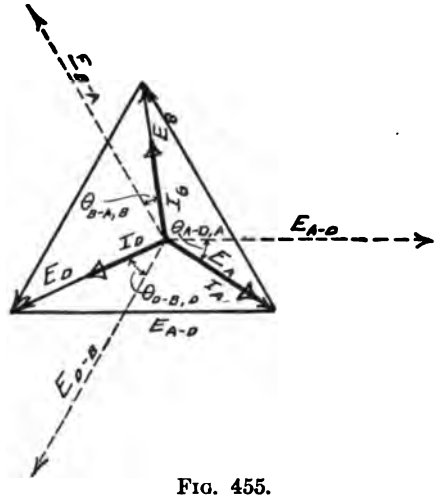



FIG. 455.

The time-phase angle of the current in line $D$ and the voltage between the mains $D$ and $B$,

$$\theta_{D-B,D} = 60° - \tan^{-1}\frac{-h}{-220+k} = 34°\ 40'.$$

(*e*) The total power $= E_AI_A + E_BI_B + E_DI_D = 12.35$ kw. (843)

(*f*) The vector diagram drawn to scale is shown in Fig. 455.

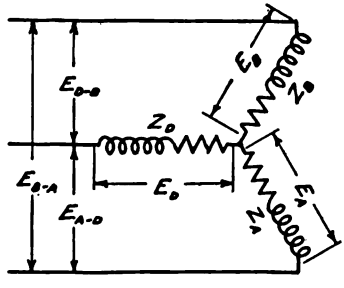



FIG. 456.

**Problem 2.**—Given a three-phase system having an inductive load, star-connected and with floating neutral, Fig. 456.

$E_{A-D} = E_{B-A} = E_{D-B} = 440$ volts; $z_A = 2 + j4$; $z_B = 3 + j2$; $z_D = 1 + j3$; $f = 60$ cycles.

Find (*a*) $E_A$, $E_B$, $E_D$.
(*b*) $I_A$, $I_B$, $I_D$.
(*c*) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.
(*d*) $\theta_{A-D,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.
(*e*) The total kilowatts.
(*f*) The total reactive power.
(*g*) Draw to scale the voltage and current vector diagram.

Following the same method as explained in problem 1, draw the equilateral triangle $ABD$, similar to Fig. 454; assume a neutral point, $N$, as before, for the origin and let $k$ and $h$ be the coördinates of the point $A$. The complex equations are:

$$E_A = k - jh \tag{840}$$

$$E_B = (k - 220) + j(381 - h) \tag{841}$$

$$E_D = (k - 440) - jh \tag{842}$$

$$I_A = \frac{k - jh}{2 + j4} \tag{843}$$

$$I_B = \frac{(k - 220) + j(381 - h)}{3 + j2} \tag{844}$$

$$I_D = \frac{(k - 440) - jh}{1 + j3} \tag{845}$$

Equating to zero the real and quadrature components of the currents passing through the common point, we have:

$$k - 1.52h - 83.7 = 0 \tag{846}$$

$$jh + j1.52k - j590 = 0 \tag{847}$$

Hence $k = 296$ and $h = 140$.

(*a*) $E_A = 328$ volts.
$E_B = 253$ volts.
$E_D = 201$ volts.

(*b*) $I_A = 73.3$ amp.
$I_B = 70.3$ amp.
$I_D = 63.3$ amp.

(*c*) $\theta_{A,A} = \tan^{-1}\frac{4}{2} = 63° 25'.$

$\theta_{B,B} = \tan^{-1}\frac{2}{3} = 33° 40'.$

$$\theta_{D,D} = \tan^{-1}\frac{3}{1} = 71° 35'.$$

(*d*) $\theta_{A-D,A} = 63°25' + \tan^{-1}\frac{h}{k} = 88° 45'.$

$$\theta_{B-A,B} = 33° 40' + \tan^{-1}\frac{k - 220}{381 - h} + 30° = 81° 10'.$$

$$\theta = 71° 35' + 60° - \tan^{-1}\frac{h}{440 - k} = 87° 25'.$$

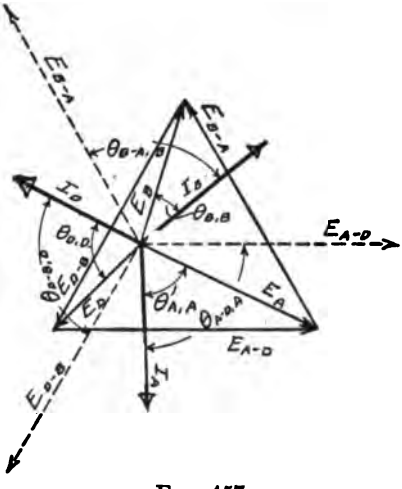



FIG. 457.

(*e*) The total power = $E_AI_A \cos \theta_{A,A} + E_BI_B \cos \theta_{B,B} + E_DI_D \cos \theta_{D,D} = 29.9$ kw.

(*f*) The total reactive power = $E_AI_A \sin \theta_{A,A} + E_BI_B \sin \theta_{B,B} + E_DI_D \sin \theta_{D,D} = 44.0$ kv.a.

(*g*) The vector diagram is drawn to scale in Fig. 457.

**Problem 3.**—Given the same data as in problem 2, except that the neutral is grounded with the voltage in each circuit to ground $= \frac{440}{\sqrt{3}}$ volts.

In the vector diagram the point $N$ is therefore fixed at the center of the equilateral triangle.

Find (*a*) $I_A$, $I_B$, $I_D$.

(*b*) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.

(*c*) $\theta_{A-D,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.

(*d*) The total power, in kilowatts.

(*e*) The total reactive power.

(*f*) Draw to scale the vector diagram for the currents and voltages. The voltage in each circuit is known, hence:

(*a*) $I_A = \dfrac{254}{2 + j4} = 56.8$ amp.

$I_B = \dfrac{254}{3 + j2} = 70.5$ amp.

$I_D = \dfrac{254}{1 + j3} = 80.4$ amp.

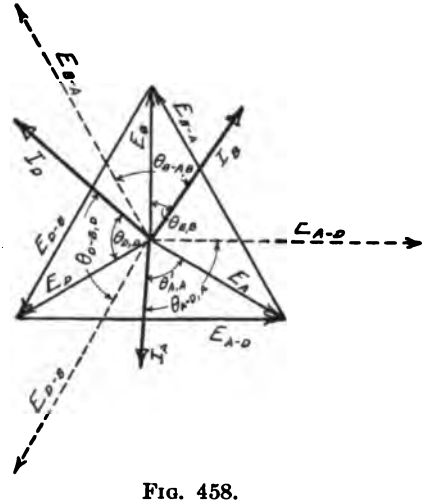


FIG. 458.

(*b*) $\theta_{A,A} = 63° 25'$.
$\theta_{B,B} = 33° 40'$.
$\theta_{D,D} = 71° 35'$.

(*c*) $\theta_{A-D,A} = 93° 25'$.
$\theta_{B-A,B} = 63° 40'$.
$\theta_{D-B,D} = 101° 35'$.

(*d*) Total power = 27.7 kw.

(*e*) The reactive power = 42.2 kv.a

(*f*) The vector diagram in Fig. 458.

**Problem 4.**—Given a three-phase circuit, star-connected, floating neutral, similar to Fig. 456.

$E_{A-D} = 120$ volts; $E_{B-A} = 115$ volts; $E_{D-B} = 118$ volts $z_A = 2 + j4$; $z_B = 3$; $z_D = 2 + j3$; $f = 60$ cycles.

Find (*a*) $E_A$, $E_B$, $E_D$.
(*b*) $I_A$, $I_B$, $I_D$.
(*c*) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.
(*d*) $\theta_{A-B,A}$, $\theta_{B-A,B}$, $\theta_{D-B,D}$.
(*e*) The total kilowatts.
(*f*) The total reactive power.
(*g*) Draw to scale the vector diagram for the voltages and currents.

From Fig. 459 the complex equations are:

$$E_A = k - jh \tag{848}$$

$$E_B = -(57.5 - k) + j(99.8 - h) \tag{849}$$

$$E_D = -(120 - k) - jh \tag{850}$$

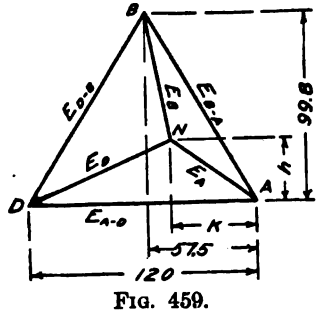



FIG. 459.

$$I_A = \frac{k - jh}{2 + j4} \tag{851}$$

$$I_B = \frac{-(57.5 - k) + j(99.8 - h)}{3} \tag{852}$$

$$I_D = \frac{-(120 - k) - jh}{2 + j3} \tag{853}$$

Equating real and quadrature components to zero, we have:

$$k - 0.733h - 64.5 = 0 \tag{854}$$

$$jk + j1.36h - j141.4 = 0 \tag{855}$$

Hence $k = 91.3$; $h = 37.0$.

(*a*) $E_A = 98.6$ volts.
$E_B = 71.5$ volts.
$E_D = 46.8$ volts.

(*b*) $I_A = 22.05$ amp.
$I_B = 23.8$ amp.
$I_D = 13.0$ amp.
(*c*) $\theta_{A,A} = 63° 25'$; $\theta_{B,B} = 0°$; $\theta_{D,D} = 56° 20'$.
(*d*) $\theta_{A-D,A} = 82° 47'$; $\theta_{B-A,B} = 58° 12'$; $\theta_{D-B,D} = 63° 8'$.
(*e*) The total power = 3.00 kw.

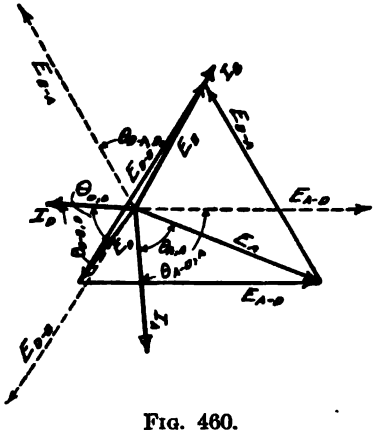


FIG. 460.

(*f*) The total reactive power = 2.45 kv.a.
(*g*) The corresponding vector diagram is shown in Fig. 460.

**Problem 5.**—Given a three-phase delta-connected system as in Fig. 461.
$E_A = E_B = E_D = 120$ volts; $f = 25$ cycles.
$z_A = 3 + j4$; $z_B = 4 + j2$; $z_D = 3$ ohms.

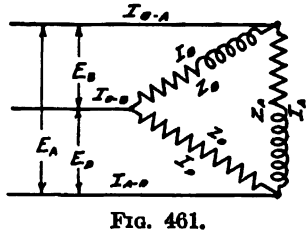


FIG. 461.

Find (*a*) $I_A$, $I_B$, $I_D$.
(*b*) $I_{A-D}$, $I_{B-A}$, $I_{D-B}$.
(*c*) $\theta_{A,A}$, $\theta_{B,B}$, $\theta_{D,D}$.
(*d*) $\theta_{A,A-D}$, $\theta_{B,B-A}$, $\theta_{D,D-B}$.
(*e*) The total kilowatts.

(*f*) The total reactive power.

(*g*) Draw the vector diagram.

Since the three voltages are equal the three circuit currents may be solved directly.

(*a*) $I_A = 24.0$ amp.; $I_B = 26.9$ amp.; $I_D = 40.0$ amp.

(*b*) The currents in the mains may be found graphically from the vector diagram or by applying the law of cosines.

$I_{A-D} = 37.6$ amp.; $I_{B-A} = 49.0$ amp.; $I_{D-B} = 64.0$ amp.

(*c*) $\theta_{A,A} = 53° 10'$; $\theta_{B,B} = 26° 34'$; $\theta_{D,D} = 0°$.

(*d*) $\theta_{A,A-D} = -24° 30'$; $\theta_{B,B-A} = 11° 10'$; $\theta_{D,D-B} = -13° 10'$

(*e*) The total power = 9.0 kw.

(*f*) The total reactive power = 3.6 kv.a.

(*g*) The vector diagram is drawn to scale in Fig. 462.

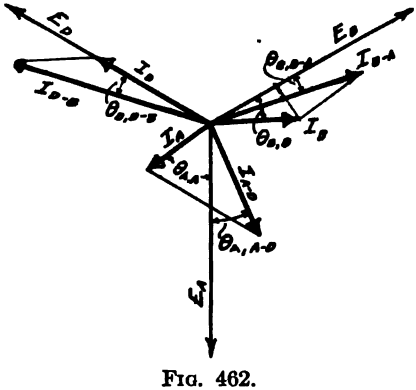


FIG. 462.

**Problem 6.**—Same circuit and data as in problem 2 except that the frequency of the impressed voltage = 25 cycles.

**Problem 7.**—Same circuit and data as in problem 4 except that the frequency of the impressed voltage = 40 cycles.

**Problem 8.**—Same circuit and data as in problem 5 except that the frequency of the impressed voltage = 50 cycles.

**Problem 9.**—A three-phase delta-connected distribution system has the following load:

(*a*) Three-phase induction motor, 50 kw., 85 per cent. power factor.

(*b*) Incandescent lamp load of 30 kw. on phase *A*, 22 kw. on phase *B*, and 20 kw. on phase *D*.

(*c*) Single-phase motor on phase *D*, 4 kw., 80 per cent. power factor. The line voltage is 2,200 volts. Find the line currents.

(*d*) **Grounded and Floating Neutral.**—While many factors enter into the problem of whether a high-voltage system should be

grounded or kept isolated, the main argument lies in the relative value of continuity of service as against an increased risk of damage to the apparatus. In any transmission system the apparatus operating on the lowest factor of safety needs most attention. Until the advent of the suspension insulator, line insulation was the weakest part of the system and on high-voltage transmission lines special care had to be taken in order that no excessive voltages should be impressed on the insulators. A partial protection is gained by grounding the neutral as then only the fixed $Y$ voltage (58 per cent. of the line voltage) can come on the line insulators; a ground on any of the lines produces a short-circuit and hence causes a shutdown. If the neutral is floating, *i.e.*, the line isolated, a ground on one of the lines does not produce a short-circuit, but shifts the neutral point to the grounded wire and therefore increases the stress across the insulators on the other two lines to full line voltage. If the factor of safety for the insulators is small, the increased stress may cause a breakdown with interruption of service. By means of the suspension type of insulator, sufficient insulation can be provided to give an ample factor of safety for the full line voltage.

Continuity of service is of very great importance in practically all electric service systems. Because an isolated system can operate with one line grounded while a system having the neutral grounded would be short-circuited and forced to shut down if one of the lines became grounded, it is generally assumed that more nearly continuous service is possible with the system isolated. However, if the increased stresses on the line insulators cause further breakdown, the reverse might be true. Many other factors like the location and number of power stations in a system, the number of parallel trunk lines, the possible mechanical and electrical stresses incident to short-circuit conditions, the importance of providing continuous service must also be taken into consideration. With a single power plant supplying energy over a single line, the isolated system is clearly desirable. With several power plants and several transmission lines forming a network, continuity may be secured even if one or more of the lines or plants are out of service. In the isolated system the single ground must be located and promptly removed as a second ground would cause a short-circuit. The location of grounds, particularly if of a partial or intermittent nature, is sometimes a difficult problem and as a consequence repairs must often be

delayed. Under such conditions the isolated system is virtually grounded and also the insulation to ground is subjected to full line voltage on two of the lines.

The chief advantage gained by having the neutral grounded is that the voltage to ground can not exceed the normal *Y* voltage of the system. In isolated systems having unbalanced loads the neutral shifts in position with changes in the load. In high-voltage lines the condensance of the system helps to keep the neutral fixed except if a ground develops on one of the lines. Short-circuit stresses become increasingly dangerous as the amount of power involved increases. In order to reduce the sudden flow of energy when a short-circuit occurs, the system is sometimes grounded through a resistance. While the short-circuit shock may be reduced by this device, two objections should be noted. First, it is very difficult to install and maintain a reliable grounding resistance. Second, the voltages between the lines and ground would rise above the normal *Y* voltage until the circuit-breakers operate. As explained in Chap. XXIV power-limiting reactances are installed in large systems in order to keep the effects of short-circuits within permissible limits.

The question of whether the neutral should be grounded or floating is independent of the use of star or delta transformer connections. However, isolated systems are generally delta-connected and the grounded neutral systems star-connected. A ground on a delta-connected system may be obtained by using a special grounding device or by letting one set of transformers be star-connected. The neutral should be grounded at one place only. If connected at two or more places, currents will flow through the ground even under normal operating conditions. These ground currents may cause serious interference with the telephone service.

(*e*) **Transmission-line Efficiency.**—Omitting the transformers and taking only the transmission line into consideration the efficiency of transmission depends practically on the $RI^2$ losses and the power delivered. For properly designed lines operating under normal conditions both the leakage and the corona losses are negligible. In order to make a comparison of the several systems it is necessary to first assume a basis. The power delivered is the product of the current, voltage and power factor, while the losses vary as the product of the resistance and the square of the current. As the power factor affects all systems alike it may, for

convenience, be taken as unity. Assuming the power delivered to be of the same magnitude in all cases, the comparison of line efficiencies may be made either by the losses for the same amount of copper or as to the copper required for the same losses under specified voltage conditions. As the voltage and the insulation are directly related, the assumptions on which the comparison should be made must be different in low-tension distribution systems from that which would be desirable for long-distance transmission lines. Likewise, the stresses on the insulation in a high-voltage system differ depending upon whether the neutral is floating or grounded. For application to commercial systems the comparison should be made under three voltage conditions.

1. The systems to have the same minimum effective voltage per circuit.

2. The systems to have the same maximum effective voltage between any conductor and ground.

3. The systems to have the same maximum effective voltage between any two conductors in each system.

In distribution networks ample insulation can be provided and hence the voltage per terminal circuit should be the basis for comparison. In high-tension distribution lines with grounded neutral the maximum voltage impressed on the insulators, or the voltage from conductor to ground should be the same in the several systems. In cables the voltage between conductors is the desirable basis for comparison.

**1. Low-potential Distribution Networks, Lighting Circuits, Etc.**—(*a*) *Single-phase, two-wire system.*

Let $r$ = the resistance per wire and $i$ = the current.

Power loss = $2ri^2$.

Copper required (two wires) = 100 per cent.

(*b*) *Single-phase, three-wire system. Neutral of same cross-section as either outside wire.*

Current in outside wire = $\frac{i}{2}$.

Copper required = 37.5 per cent.

(*c*) *Single-phase, three-wire system. Neutral of half the cross-section of one outside wire.*

Current in outside wire = $\frac{i}{2}$.

Copper required = 31.25 per cent.

(*d*) *Single-phase, three-wire system. Neutral of zero cross-section.*

Current in outside wire $= \frac{i}{2}$.

Copper required = 25 per cent.

(*e*) *Single-phase, five-wire system. All wires of the same cross-section.*

Current in outside wire $= \frac{i}{4}$.

Copper required = 15.67 per cent.

(*f*) *Single-phase, five-wire system. Each of the inside wires to have half the cross-section of one of the outside wires.*

Current in outside wire $= \frac{i}{4}$.

Copper required = 10.93 per cent.

(*g*) *Quarter-phase, four-wire system. All wires of the same cross-section.* Equivalent to two single-phase, two-wire systems. Same as (*a*).

Copper required = 100 per cent.

(*h*) *Quarter-phase, three-wire system. The third wire is the common return for the two phases. Its cross-section is* $\sqrt{2}$ *times that of either of the outside wires so as to have the same current density in the three wires.*

The resistance of one of the outside wires $= r_1$.

Current in each of the outside wires $= \frac{i}{2}$.

Current in the middle wire $= \frac{i}{\sqrt{2}}$.

Power loss in the middle wire $= \frac{r_1}{\sqrt{2}}\left(\frac{i}{\sqrt{2}}\right)^2 = \frac{r_1 i^2}{2\sqrt{2}}$.

Total loss in the three wires $= 2r_1\frac{i^2}{4} + \frac{r_1 i^2}{2\sqrt{2}} = r_1 i^2\left(\frac{2+\sqrt{2}}{4}\right)$.

Equating the total loss to that of the single-phase, two-wire system in (*a*)

$$r_1 i^2\left(\frac{2+\sqrt{2}}{4}\right) = 2ri^2$$

and solving for $r_1$, we have:

$$r_1 = \frac{8}{2+\sqrt{2}}\,r.$$

Hence each outside wire must have a cross-section of $\frac{2+\sqrt{2}}{8}$

and the middle wire $\frac{2+2\sqrt{2}}{8}$ times that of the equivalent single-phase wires.

$$\text{Copper required} = \frac{\frac{2(2+\sqrt{2})}{8} + \frac{2+2\sqrt{2}}{8}}{2} = 72.9 \text{ per cent.}$$

(*i*) *Quarter-phase, six-wire system. All wires of equal cross-section.*

Equivalent to two circuits of (*b*).

Copper required = 37.5 per cent.

(*j*) *Quarter-phase, five-wire system. The fifth wire to be used as neutral for both phases. All wires of the same cross-section.*

Equivalent to two circuits of (*c*).

Copper required = 31.25 per cent.

(*k*) *Three-phase, three-wire system. All wires of the same cross-section.*

Let $r_1$ = resistance for each wire.

Current in each main = $\frac{i}{\sqrt{3}}$.

Loss in each main = $\frac{r_1 i^2}{3}$; total loss = $r_1 i^2 = 2ri^2$.

The cross-section is inversely as the resistance, and $r_1 = 2r$.

Copper required = $3\frac{(\frac{1}{2})}{2} = 75$ per cent.

(*l*) *Three-phase, four-wire system. The load star-connected. All wires of same cross-section. Voltage from main to neutral equal to the voltage between lines in (a). Voltage between mains is $\sqrt{3}$ times voltage to neutral. For balanced load no current flows in the neutral.*

Current in mains = $\frac{i}{3}$.

Loss in each wire = $r_1 \frac{i^2}{9}$; total loss $\frac{r_1 i^2}{3} = 2ri^2$, and hence $r_1 = 6r$.

Copper required = $\frac{4(\frac{1}{6})}{2} = 33.3$ per cent.

(*m*) *Three-phase, four-wire system. Same as in (l) except that the neutral has only half the cross-section of one of the mains.*

Copper required = 29.16 per cent.

**2. High-voltage Circuits. Transmission Lines with Grounded Neutral.**—The maximum voltage from the line to ground is the basis of comparison.

On this basis the Edison three-wire and five-wire systems give no advantage over the two-wire system. Whether the ground connection is used merely for maintaining a constant potential difference between the ground and the line conductors, or as a return circuit carrying current, all grounded systems are equivalent to a single-phase, two-wire system with grounded neutral, or to a single-phase circuit having one wire and ground return.

**3. High-voltage Circuits. Isolated Transmission Lines, Cables, Etc.**—The maximum voltage between any two conductors is taken as the basis of comparison. On this basis the Edison three-wire and five-wire systems offer no advantage over the single-phase, two-wire system.

(*a*) *Quarter-phase, four-wire system. All wires of the same cross-section.* The system is equivalent to two single-phase circuits in time quadrature.

Copper required = 100 per cent.

(*b*) *Quarter-phase, three-wire system. The cross-section of the middle wire is $\sqrt{2}$ times that of either outside wire.*

Current in outside wire = $\frac{i}{\sqrt{2}}$.

Current in middle wire = $i$.

Power loss in outside wire = $\frac{r_1 i^2}{2}$; in middle wire = $\frac{r_1 i^2}{\sqrt{2}}$.

Total loss in the three wires = $2\left(\frac{r_1 i^2}{2}\right) + \frac{r_1 i^2}{\sqrt{2}} = r_1 i^2 \left(\frac{2+\sqrt{2}}{2}\right)$.

Copper required = $\dfrac{\frac{2(2+\sqrt{2})}{4} + \frac{\sqrt{2}(2+\sqrt{2})^2}{4}}{2}$ = 145.7 per cent.

(*c*) *Three-phase, three-wire system. All wires of the same cross-section.*

Current in each wire = $\frac{i}{\sqrt{3}}$.

Total loss = $3\left(\frac{i}{\sqrt{3}}\right)^2 r_1$.

Copper required = $\frac{3(\frac{1}{2})}{2}$ = 75 per cent.

For the same efficiency of transmission the three-phase line requires less copper and for this reason is most desirable for long-distance transmission lines.

# CHAPTER XXVI

## CERTAIN FORMS OF NOTATION

### TOPOGRAPHIC METHOD. ROOTS OF MINUS ONE AS OPERATORS. HYPERBOLIC SINES AND COSINES

It is often desirable to show in a single diagram the simultaneous time-phase relations of the currents and voltages at several points in an alternating-current system. It is especially advantageous to show both the time-phase and space-phase relations when discussing the electrical phenomena of long-distance transmission lines. Since the form of representation is somewhat like a topographic map it has been called the topographic method.

(*a*) **Topographic Method.**—For illustrative purposes, consider a single-phase circuit having resistances, inductances, condensers and leakage conductances as shown in Fig. 463. Let the following quantities be given, using the notation indicated in the figure:

$E_4$; $r_1$, $r_2$, $r_3$, $r_4$; ${}_{\iota}x_1$, ${}_{\iota}x_2$, ${}_{\iota}x_3$, ${}_{\iota}x_4$; ${}_{c}x_1$, ${}_{c}x_2$, ${}_{c}x_3$, ${}_{c}x_4$; ${}_{\imath}g_1$, ${}_{\imath}g_2$, ${}_{\imath}g_3$, ${}_{\imath}g_4$.

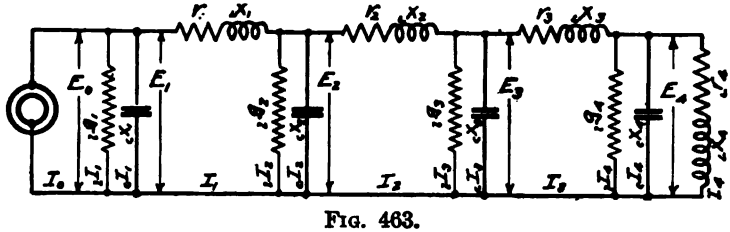


FIG. 463.

*Starting at the 4th station*, the point farthest from the generator, we have:

$$I_4 = \frac{E_4}{r_4 + j_{\iota}x_4} \text{ and } \theta_4 = \tan^{-1}\frac{{}_{\iota}x_4}{r_4} \tag{856}$$

Draw $E_4$ along the $X$-axis as reference vector, Fig. 464, and lay off the current, $I_4$, lagging $\theta^\circ_4$.

The components of $E_4$, $r_4I_4$ and ${}_{\iota}x_4L_4$, are drawn in phase and in quadrature, respectively, with $I_4$.

The charging current, $cI_4 = \frac{E_4}{cx_4}$, is drawn in quadrature with $E_4$, and the leakage current, $_iI_4 = {}_ig_4E_4$, in phase with $E_4$.

$$\dot{I}_3 = \dot{I}_4 + {}_c\dot{I}_4 + {}_i\dot{I}_4 \tag{857}$$

*At the 3d station:*

Add to the vector $E_4$ the voltage drop, $r_3I_3$ in phase and ${}_lx_3I_3$ in quadrature with $I_3$.

$$\dot{E}_3 = \dot{E}_4 + r_3\dot{I}_3 + {}_lx_3\dot{I}_3 \tag{858}$$

The charging current, ${}_cI_3 = \frac{E_3}{cx_3}$, and is drawn in quadrature

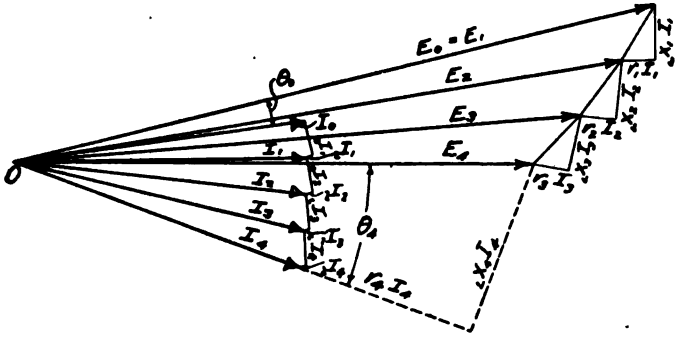



FIG. 464.

with $E_3$; the leakage current, ${}_iI_3 = {}_ig_3E_3$ is drawn in phase with $E_3$.

$$\dot{I}_2 = \dot{I}_3 + {}_c\dot{I}_3 + {}_i\dot{I}_3 \tag{859}$$

*At the 2d station:*

To the vector $E_3$ add the voltage drops $r_2I_2$ and ${}_lx_2I_2$ in phase and in quadrature, respectively, with $I_2$.

$$\dot{E}_2 = \dot{E}_3 + r_2\dot{I}_2 + {}_lx_2\dot{I}_2 \tag{860}$$

The charging current, ${}_c\dot{I}_2$, is in time quadrature and the leakage current, ${}_i\dot{I}_2$, in time phase with $E_2$.

$$\dot{I}_1 = \dot{I}_2 + {}_c\dot{I}_2 + {}_i\dot{I}_2 \tag{861}$$

*At the 1st station:*

To the vector $E_2$ add the voltage drops $r_1I_1$ and ${}_lx_1I_1$ in phase and in quadrature, respectively, with $I_1$.

$$\dot{E}_1 = \dot{E}_2 + r_1\dot{I}_1 + {}_lx_1\dot{I}_1 \tag{862}$$

The charging current, ${}_cI_1 = \frac{E_1}{{}_cx_1}$, is drawn in quadrature with $E_1$; the leakage current ${}_lI_1 = {}_lg_1E_1$ drawn in phase with $E_1$.

$$\dot{I}_0 = \dot{I}_1 + {}_c\dot{I}_1 + {}_l\dot{I}_1 \qquad (863)$$

$$\dot{E}_0 = \dot{E}_1.$$

From the diagram the magnitude and time-phase positions of currents and voltages may be found directly for all the points plotted. For example, the time-phase relation of $E_3$ with respect to $E_0$, $E_1$, $E_2$, $E_4$ or to any of the currents may be obtained, by measuring the corresponding angles on the diagram. In transmission lines the resistance, inductance, condensance and leakage

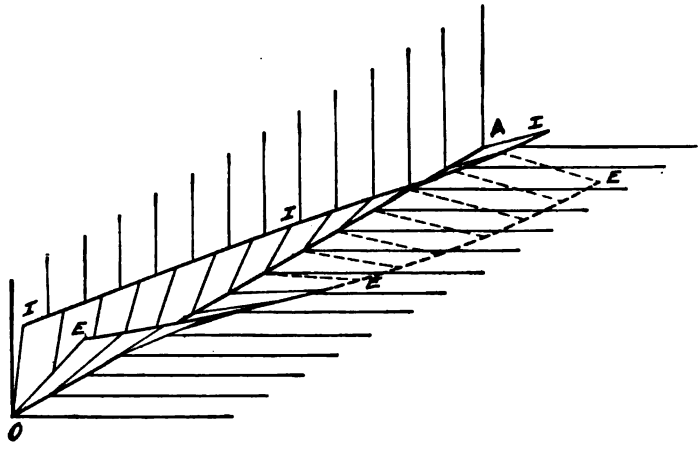



FIG. 465.

are uniformly distributed along the length of the line, and, therefore, the relative time-phase position of the current and voltage shifts continuously along the line. A typical perspective topographic diagram for a long line is shown in Fig. 465.

The time-phase relation of the current and voltage is given at any point on the line by the corresponding vectors at the given position on the line $OA$.

(*b*) **Roots of Minus One as Operators.**—In the preceding chapters the operator $j = \sqrt{-1}$ has been used as a coefficient to indicate a quadrature relation or a rotation of 90°. In the technical papers dealing with electrical phenomena the roots of minus one are frequently used as operators indicating time and

space-phase displacements, or a rotation of similar vectors. The operator $j$ is used both as coefficient and exponent and an understanding of the physical concepts involved is necessary.

That the operator $j$ indicates a rotation of 90° was shown in Chap. III.

Using the customary notation, and for a simple series circuit,

$$\begin{aligned}\dot{E} &= r\dot{I} + jx\dot{I} \\ &= z\dot{I}\cos\theta + jz\dot{I}\sin\theta \\ &= z\dot{I}\cos(\omega t) + jz\dot{I}\sin(\omega t) \\ &= z\dot{I}[\cos(\omega t) + j\sin(\omega t)] \end{aligned} \tag{864}$$

$$\begin{aligned}\dot{E} &= \dot{I}\sqrt{r^2 + x^2} = z\dot{I} \\ &= z\dot{I}\sqrt{\cos^2(\omega t) + \sin^2(\omega t)} = z\dot{I} \end{aligned} \tag{865}$$

Other roots of the factor $(-1)$ may also be used as operators indicating vector rotation.

Let $$u = \sqrt[3]{-1}, \text{ or } u^3 + 1 = 0 \tag{866}$$

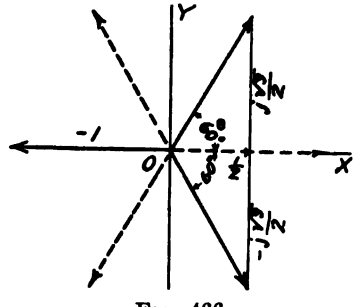



FIG. 466.

The three roots of the equation are:

$$u_1 = \tfrac{1}{2} + \tfrac{1}{2}\sqrt{3}\sqrt{-1} = \tfrac{1}{2} + j\frac{\sqrt{3}}{2} \tag{867}$$

$$u_2 = -1 \tag{868}$$

$$u_3 = \tfrac{1}{2} - \tfrac{1}{2}\sqrt{3}\sqrt{-1} = \tfrac{1}{2} - j\frac{\sqrt{3}}{2} \tag{869}$$

Letting $j$ indicate a rotation of 90°, as before, the three roots may be shown graphically as in Fig. 466, in which the first root indicates a rotation of 60°, the second 180°, and the third 300°. However, by the successive application of the first root the vector will rotate in 60° steps in the positive (counter-clockwise) direction.

$$u = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = u_1 = 60° \tag{870}$$

$$u^2 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^2 = -\left(\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -u_3 = 120° \tag{871}$$

$$u^3 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^3 = -1 = u_2 = 180° \tag{872}$$

$$u^4 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^4 = -\left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = -u_1 = 240° \tag{873}$$

$$u^5 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^5 = \left(\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = u_3 = 300° \tag{874}$$

$$u^6 = \left(\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)^6 = 1 = 360° \tag{875}$$

In a similar manner it may be shown that $\sqrt[4]{-1}$ indicates a rotation 45°; $\sqrt[6]{-1}$ a rotation of 36°; and in general $\sqrt[n]{-1}$ indicates a rotation in the positive direction of $\frac{\pi}{n}$.

The negative sign combined with these operators indicates a rotation of like amount in the negative direction. Thus:

$$-\sqrt{-1} = -90°$$

$$-\sqrt[3]{-1} = -60°$$

$$-\sqrt[n]{-1} = -\frac{\pi}{n} \tag{876}$$

From Fig. 466 it is evident that the operator (considered as a vector of unit length) may be expressed in terms of $j$ and the sine and cosine functions of the angle of rotation.

$$-1 = j^2 = \cos\pi + j\sin\pi$$

$$\sqrt[3]{-1} = j^{\frac{2}{3}} = \cos\frac{\pi}{3} + j\sin\frac{\pi}{3},$$

$$\sqrt[n]{-1} = j^{\frac{2}{n}} = \cos\frac{\pi}{n} + j\sin\frac{\pi}{n}$$

or in general,

$$-1 = j^2 = \cos(2m+1)\pi + j\sin(2m+1)\pi$$

$$\sqrt[n]{-1} = j^{\frac{2}{n}} = \cos\frac{(2m+1)\pi}{n} + j\sin\frac{(2m+1)\pi}{n} \tag{877}$$

Where $m$ may be any positive or negative whole number, and $n$ may be any positive or negative number, integral or fractional, constant or variable.

For given values let $\frac{\pi}{n} = \theta$.

$$\sqrt[\frac{\pi}{\theta}]{-1} = j^{\frac{2\theta}{\pi}} = \cos\theta + j\sin\theta \qquad (878)$$

As a function of time let $\frac{\pi}{n} = \omega t$.

$$\sqrt[\frac{\pi}{\omega t}]{-1} = j^{\frac{2\omega t}{\pi}} = \cos(\omega t) + j\sin(\omega t) \qquad (879)$$

Similarly for $\frac{\pi}{n} = -\omega t$:

$$\sqrt[-\frac{\pi}{\omega t}]{-1} = j^{-\frac{2\omega t}{\pi}} = \cos(\omega t) - j\sin(\omega t) \qquad (880)$$

The *product of two operators* indicates a rotation equal to the sum of the separate rotations.
Let

$$Q = \cos a + j\sin a,$$

and

$$V = \cos b + j\sin b.$$

$$\begin{aligned} QV &= (\cos a + j\sin a)(\cos b + j\sin b) \\ &= \cos a\cos b - \sin a\sin b + j(\sin a\cos b + \cos a\sin b) \\ &= \cos(a+b) + j\sin(a+b) \end{aligned} \qquad (881)$$

The *quotient of two operators* indicates a rotation equal to the difference of the separate rotations.

$$\begin{aligned} \frac{Q}{V} &= \frac{\cos a + j\sin a}{\cos b + j\sin b} \\ &= \frac{\cos a\cos b + \sin a\sin b + j(\sin a\cos b - \cos a\sin b)}{\cos^2 b + \sin^2 b} \\ &= \cos(a-b) + j\sin(a-b) \end{aligned} \qquad (882)$$

When $Q = V$,

$$(\cos a + j\sin a)^2 = \cos 2a + j\sin 2a$$

Stating De Moivre's theorem for $n$ factors,

$$(\cos a + j\sin a)^n = \cos na + j\sin na \qquad (883)$$

The exponential form is often a more convenient notation for expressing rotation of vectors than the trigonometric. By ex-

panding sin $u$, cos $u$ and $\epsilon^u$ into series of the powers of $u$ the relation between the two forms of notation is readily seen.

$$\sin u = \frac{u}{\lfloor 1} - \frac{u^3}{\lfloor 3} + \frac{u^5}{\lfloor 5} - \frac{u^7}{\lfloor 7} + \cdots\cdots (-1)^n \frac{u^{2n+1}}{\lfloor 2n+1} + \text{etc.} \tag{884}$$

$$\cos u = 1 - \frac{u^2}{\lfloor 2} + \frac{u^4}{\lfloor 4} - \frac{u^6}{\lfloor 6} + \cdots\cdots (-1)^n \frac{u^{2n}}{\lfloor 2n} + \text{etc.} \tag{885}$$

$$\epsilon^u = 1 + \frac{u}{\lfloor 1} + \frac{u^2}{\lfloor 2} + \frac{u^3}{\lfloor 3} + \frac{u^4}{\lfloor 4} + \cdots\cdots + \frac{u^n}{\lfloor n} + \text{etc.} \tag{886}$$

Substituting $ju$ for $u$,

$$\epsilon^{ju} = 1 + j\frac{u}{\lfloor 1} - \frac{u^2}{\lfloor 2} - j\frac{u^3}{\lfloor 3} + \frac{u^4}{\lfloor 4} - \cdots + (j)^n \frac{u^n}{\lfloor n} + \text{etc.} \tag{887}$$

Therefore,

$$\epsilon^{ju} = \cos u + j \sin u \tag{888}$$

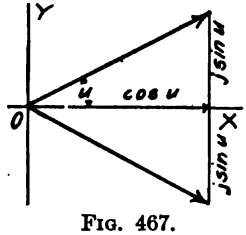



FIG. 467.

Likewise,

$$\epsilon^{-ju} = \cos u - j \sin u \tag{889}$$

The factor $\epsilon^{ju}$, therefore, indicates a rotation of $u°$ in the positive direction and $\epsilon^{-ju}$ a similar rotation of $u°$ in the negative direction. By adding and subtracting equations 888 and 889 or from the diagram in Fig. 467, we may obtain directly Euler's expressions for the sine and cosine.

$$\cos u = \frac{\epsilon^{ju} + \epsilon^{-ju}}{2} \text{ and } \sin u = \frac{\epsilon^{ju} - \epsilon^{-ju}}{2j} \tag{890}$$

Vector rotation can, therefore, be indicated by several symbols or operators. A rotation of $\theta°$ in the positive direction is indicated by any one of the following equal operators:

$$\cos\theta + j\sin\theta = \epsilon^{j\theta} = \sqrt[\frac{\pi}{\theta}]{-1} = j^{\frac{2\theta}{\pi}} \tag{891}$$

Similarly a rotation of $\theta°$ in the negative direction:

$$\cos\theta - j\sin\theta = \epsilon^{-j\theta} = \sqrt[-\frac{\pi}{\theta}]{-1} = j^{-\frac{2\theta}{\pi}} \tag{892}$$

For a continuous rotation with $\omega t$ as the variable,

$$\cos(\omega t) + j \sin(\omega t) = \epsilon^{j(\omega t)} = \sqrt[\frac{\pi}{\omega t}]{-1} = j^{\frac{2\omega t}{\pi}} \tag{893}$$

$$\cos(\omega t) - j \sin(\omega t) = \epsilon^{-j(\omega t)} = \sqrt[\frac{\pi}{\omega t}]{-1} = j^{-\frac{2\omega t}{\pi}} \tag{894}$$

(*c*) **Hyperbolic Sines and Cosines.**—If the problem involves vectors that vary in magnitude as well as in phase position the notation must be adjusted to comply with all the conditions. In long transmission lines, as explained in Chap. XXVII, the voltage and current vectors differ both in magnitude and in time phase along the line. The quantitative relations may be expressed by exponential equations having both "real and imaginary terms" in the exponents. As stated in the preceding paragraph the "imaginary" part of the exponent (containing the $j$ factor) indicates angular displacement or relative angular position and may be expressed in trigonometric terms. The real part of the exponent represents a variation in the length of the vector and may be expressed in similar form by means of hyperbolic sines and cosines.

By comparing equations (890) and (895) it is seen that the exponential expressions for the sinh and cosh are strikingly similar to those of the sine and cosine.

$$\sinh u = \frac{\epsilon^{u} - \epsilon^{-u}}{2}; \quad \cosh u = \frac{\epsilon^{u} + \epsilon^{-u}}{2} \tag{895}$$

Solving for the exponential terms in equation (895),

$$\epsilon^{+u} = \cosh u + \sinh u \tag{896}$$

$$\epsilon^{-u} = \cosh u - \sinh u \tag{897}$$

It is often more convenient to develop the equations in the exponential form and then convert into trigonometric form when the numerical values must be found. Thus in Chap. XXVII the expressions for the current and voltage are derived in the exponential form and then changed into trigonometric terms in order to use the tables of trigonometric functions when calculating the numerical results for any given transmission line.

(*d*) **Polar Expressions for Complex Quantities.**—As is noted on page 479 the product of two operators indicates a rotation equal to the sum of the separate rotations and the quotient indicates

the difference of these rotations, etc. When dealing with products, quotients, powers and roots of complex quantities it is convenient to keep the expressions in polar form indicating the scalar length and the angle of rotation.

$$\begin{aligned}\text{Let } \dot{C} &= A + jB \\ &= C(\cos\alpha + j\sin\alpha) \\ &= C(\cos\tan^{-1}B/A + j\sin\tan^{-1}B/A) \\ &= C/\underline{\tan^{-1}B/A} \\ &= C/\underline{\alpha}\end{aligned}$$

where $C$ is the scalar value and $\alpha$ or $\tan^{-1}B/A$ is the angle of rotation.

Likewise let

$$\begin{aligned}\dot{F} &= D + jE \\ &= F(\cos\beta + j\sin\beta) \\ &= F(\cos\tan^{-1}E/D + j\sin\tan^{-1}E/D) \\ &= F/\underline{\tan^{-1}E/D} \\ &= F/\underline{\beta}\end{aligned}$$

The following are some of the operations easily performed with complex quantities in this form:

$$\begin{aligned}\dot{C}\dot{F} &= CF/\underline{\alpha + \beta} \\ \dot{C}/\dot{F} &= C/F/\underline{\alpha - \beta} \\ \dot{C}^2 &= C^2/\underline{2\alpha} \\ \sqrt{\dot{C}} &= \sqrt{C}\ /\underline{\alpha/2} \\ \dot{C}^n &= C^n/\underline{n\alpha} \\ 1/\dot{C} &= 1/C/\underline{-\alpha} = 1/C/\underline{2\pi - \alpha} \\ \sqrt[n]{\dot{C}/\dot{F}} &= \sqrt[n]{C/F}\ /\underline{(\alpha - \beta)/n} \\ (\dot{C}\dot{F})^n &= C^nF^n/\underline{n(\alpha + \beta)}\end{aligned}$$

A numerical example is given in the calculations for "Line C." Chap. XXVII.

## CHAPTER XXVII

## LONG TRANSMISSION LINES

In transmission lines the condensance, inductance, resistance and leakage are uniformly distributed along the line; or the line may be considered as equivalent to an infinite number of infinitesimal series and parallel circuits uniformly distributed along its whole length. Each infinitesimal length of conductor may be represented by one of the series circuits in Fig. 468 and the corresponding infinitesimal portion of the dielectric between the line elements by a parallel circuit. The resistance and the inductance of the conductor element affect the voltage in pro-

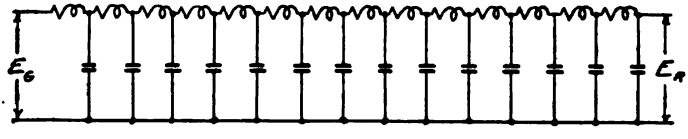



FIG. 468.

portion to the current flowing at that point. Similarly the conductance and susceptance in the elemental dielectric circuit affect the current in proportion to the voltage at the point selected. The relative phase positions of the voltages at successive points along the line differ in proportion to the resistance and the inductive reactance in each elemental series circuit. Likewise, the phase positions of the currents at successive points along the line vary in proportion to the conductance and susceptance of the parallel circuits. As a result both the current and the voltage change continuously in magnitude and phase position along the line.

(*a*) **Development of Transmission-line Equations.**—In Fig. 469 is shown an elemental vector diagram of the currents and voltages for two points at a distance *dl* apart, along the line,

with *OX* and *OY* as reference axes. In the original development of the general equations, Dr. C. P. Steinmetz used the polar vector diagram notation, and the same system has generally been used in the technical literature on long transmission lines. To facilitate comparison and to give the student an illustration of the reversal of signs in the complex equations, the polar vector diagram and the corresponding complex equations are used in this chapter. Therefore, the symbol for inductive reactance is $-jx$ and for condensive susceptance $-jb$.

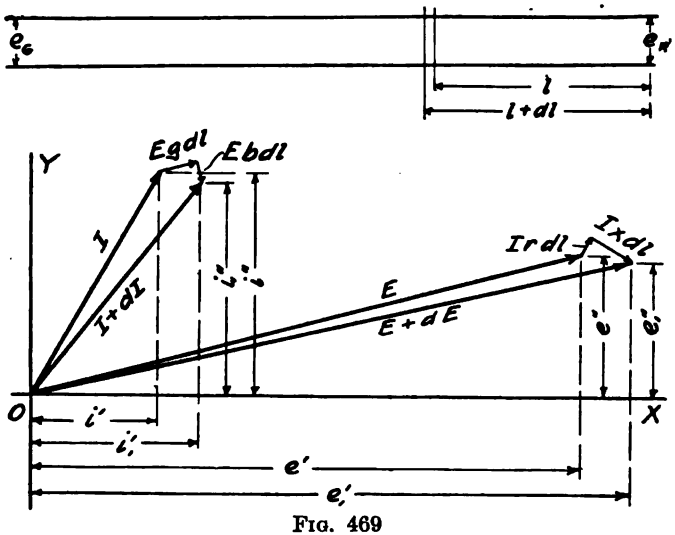



FIG. 469

For unit length of line, let:

$r$ = resistance, $x$ = reactance } for electric and magnetic (series) circuits.

$g$ = conductance, $b$ = susceptance } for leakage and dielectric (parallel or shunted) circuits.

At the distance $l$ from the receiver end of the line:

$$\dot{E} = e' + je''; \dot{I} = i' + ji''.$$

At the distance $l + dl$ from the receiver end of the line:

$$\dot{E} + d\dot{E} = e_1' + je''_1; \dot{I} + d\dot{I} = i'_1 + ji''_1,$$

From the vector diagram in Fig. 469,

$$\dot{E} + d\dot{E} = e' + i'rdl + i''xdl + j(e'' + i''rdl - i'xdl)$$

$$d\dot{E} = i'rdl + i''xdl + j(i''rdl - i'xdl)$$

$$\frac{d\dot{E}}{dl} = i'r + i''x + ji''r - ji'x$$

$$= (i' + ji'')(r - jx)$$

$$\frac{d\dot{E}}{dl} = \dot{I}Z \tag{901}$$

Similarly:

$$\dot{I} + d\dot{I} = i' + e'gdl + e''bdl + j(i'' + e''gdl - e'bdl)$$

$$\frac{d\dot{I}}{dl} = \dot{E}Y \tag{902}$$

Differentiating (902) with respect to $l$:

$$\frac{d^2\dot{I}}{dl^2} = Y\frac{d\dot{E}}{dl} \tag{903}$$

Substituting (901) in (903):

$$\frac{d^2\dot{I}}{dl^2} = YZ\dot{I} \tag{904}$$

Multiplying through by $2\frac{d\dot{I}}{dl}$:

$$2\frac{d\dot{I}}{dl}\frac{d^2\dot{I}}{dl^2} = 2YZ\dot{I}\frac{d\dot{I}}{dl}$$

Integrating:

$$\left(\frac{d\dot{I}}{dl}\right)^2 = YZ\dot{I}^2 + \dot{K}' \text{ or}$$

$$\left(\frac{d\dot{I}}{dl}\right)^2 = YZ(\dot{I}^2 + \dot{K}_1^2)$$

$$\frac{d\dot{I}}{dl} = (YZ)^{1/2}(\dot{I}^2 + \dot{K}_1^2)^{1/2}$$

$$\frac{d\dot{I}}{(\dot{I}^2 + \dot{K}_1^2)^{1/2}} = (YZ)^{1/2}dl$$

Integrating:

$$\text{Log}_e[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] + \dot{K}'' = (YZ)^{1/2}l \text{ or}$$

$$\text{Log}_e[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] + \text{Log}_e\dot{K}_2 = (YZ)^{1/2}l$$

$$\text{Log}_e\dot{K}_2[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}] = (YZ)^{1/2}l$$

$$\epsilon^{(YZ)^{1/2}l} = \dot{K}_2[\dot{I} + (\dot{I}^2 + \dot{K}_1^2)^{1/2}]$$

$$\frac{\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} - \dot{I} = (\dot{I}^2 + \dot{K}_1^2)^{1/2}$$

$$\frac{\epsilon^{2(YZ)^{1/2}l}}{\dot{K}_2^2} - \frac{2\dot{I}\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} = \dot{K}_1^2$$

$$\dot{I} = \tfrac{1}{2}\left[\frac{\epsilon^{(YZ)^{1/2}l}}{\dot{K}_2} - \frac{\dot{K}_1^2\dot{K}_2}{\epsilon^{(YZ)^{1/2}l}}\right]$$

$$\dot{I} = \dot{A}_1\epsilon^{(YZ)^{1/2}l} - \dot{A}_2\epsilon^{-(YZ)^{1/2}l} \qquad (905)$$

Where $\dot{A}_1 = \dfrac{1}{2\dot{K}_2}$ and $\dot{A}_2 = \dfrac{\dot{K}_1^2\dot{K}_2}{2}$

From (902):

$$\dot{E} = \frac{1}{Y}\frac{d\dot{I}}{dl} \qquad (906)$$

Differentiating (905) with respect to $l$,

$$\frac{d\dot{I}}{dl} = (YZ)^{1/2}(\dot{A}_1\epsilon^{(YZ)^{1/2}l} + \dot{A}_2\epsilon^{-(YZ)^{1/2}l}) \qquad (907)$$

Substituting (907) in (906):

$$\dot{E} = \frac{(YZ)^{1/2}}{Y}(\dot{A}_1\epsilon^{(YZ)^{1/2}l} + \dot{A}_2\epsilon^{-(YZ)^{1/2}l}) \qquad (908)$$

Equations (905) and (908) give the current and voltage at any point on the line. $\dot{A}_1$ and $\dot{A}_2$ are integration constants.

Equations (905) and (908) may now be made useful for actual computation of electrical line performances as follows:

The square root of the product of two vectors is also a vector:

$$(YZ)^{1/2} = \alpha - j\beta = [(r - jx)(g - jb)]^{1/2}$$
$$(\alpha - j\beta)^2 = (r - jx)(g - jb)$$
$$\alpha^2 - \beta^2 - j2\alpha\beta = rg - xb - j(rb + xg) \qquad (909)$$

From (909) it is evident that:

$$\alpha^2 - \beta^2 = rg - xb, \text{ and } 2\alpha\beta = rb + xg \qquad (910)$$

Squaring equations (910):

$$\alpha^4 + \beta^4 - 2\alpha^2\beta^2 = r^2g^2 + x^2b^2 - 2rxgb$$
$$4\alpha^2\beta^2 = r^2b^2 + x^2g^2 + 2rxgb \qquad (911)$$

Adding equations (911):

$$\begin{aligned} \alpha^4 + \beta^4 + 2\alpha^2\beta^2 &= r^2g^2 + x^2b^2 + r^2b^2 + x^2g^2 \\ (\alpha^2 + \beta^2)^2 &= (r^2 + x^2)(g^2 + b^2) \\ &= z^2y^2 \\ \alpha^2 + \beta^2 &= zy \end{aligned} \tag{912}$$

From (910) and (912):

$$\begin{aligned} \alpha &= [\tfrac{1}{2}(zy + rg - xb)]^{\frac{1}{2}} \\ \beta &= [\tfrac{1}{2}(zy - rg + xb)]^{\frac{1}{2}} \end{aligned} \tag{913}$$

Equations (905) and (908) now assume the forms:

$$\dot{I} = \dot{A}_1\epsilon^{+(\alpha - j\beta)l} - \dot{A}_2\epsilon^{-(\alpha - j\beta)l}$$

$$\dot{E} = \frac{\alpha - j\beta}{Y}\left[\dot{A}_1\epsilon^{+(\alpha - j\beta)l} + \dot{A}_2\epsilon^{-(\alpha - j\beta)l}\right] \tag{914}$$

Substituting for the exponential function with the $j$ factor exponent the trigonometric expression:

$$\epsilon^{\pm j\beta l} = \cos \beta l \pm j \sin \beta l$$

Equations (914) change in form to:

$$\begin{aligned} \dot{I} &= \dot{A}_1\epsilon^{+\alpha l}(\cos \beta l - j \sin \beta l) - \dot{A}_2\epsilon^{-\alpha l}(\cos \beta l + j \sin \beta l) \\ \dot{E} &= \frac{\alpha - j\beta}{Y}\Big[\dot{A}_1\epsilon^{+\alpha l}\ (\cos \beta l - j \sin \beta l) \\ &\qquad + \dot{A}_2\epsilon^{-\alpha l}\ (\cos \beta l + j \sin \beta l)\Big] \end{aligned} \tag{915}$$

Choosing as unit length the entire line, $l = 0$, gives $\dot{I}_R$ and $\dot{E}_R$ at the receiver end; and $l = 1$, gives $\dot{I}_G$ and $\dot{E}_G$ at the generator end of the line, and $Z$ is the total impedance, $Y$ the total shunted admittance of the entire line.

Neglecting the shunted conductance, that is, assuming the power consumed by leakage and dielectric losses in the line as negligible compared with that consumed by the effective line resistance $r$—as is always done for low frequencies—gives from equations (913):

$$\begin{aligned} \alpha &= \left[\tfrac{1}{2}b(z - x)\right]^{\frac{1}{2}} \\ \beta &= \left[\tfrac{1}{2}b(z + x)\right]^{\frac{1}{2}} \end{aligned} \tag{916}$$

and

$$Y = -jb$$

Substituting $l = 1$ in equations (915):

$$\dot{I}_G = \dot{A}_1\epsilon^{+\alpha}(\cos\beta - j\sin\beta) - \dot{A}_2\epsilon^{-\alpha}(\cos\beta + j\sin\beta)$$

$$\dot{E}_G = \frac{\alpha - j\beta}{-jb}\left[\dot{A}_1\epsilon^{+\alpha}(\cos\beta - j\sin\beta) + \dot{A}_2\epsilon^{-\alpha}(\cos\beta + j\sin\beta)\right] \tag{917}$$

Substituting $l = 0$ in equations (915):

$$\dot{I}_R = \dot{A}_1 - \dot{A}_2$$

$$\dot{E}_R = \frac{\alpha - j\beta}{-jb}(\dot{A}_1 + \dot{A}_2) \tag{918}$$

Solving equations (918) for $\dot{A}_1$ and $\dot{A}_2$:

$$\dot{A}_1 = \tfrac{1}{2}\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right) \tag{919}$$

$$\dot{A}_2 = -\tfrac{1}{2}\left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right)$$

Substituting equations (919) in (917):

$$\dot{I}_G = \tfrac{1}{2}\left[\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{+\alpha}(\cos\beta - j\sin\beta) + \left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{-\alpha}(\cos\beta + j\sin\beta)\right]$$

$$\dot{E}_G = \frac{\alpha - j\beta}{-jb}\tfrac{1}{2}\left[\left(\dot{I}_R - \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{+\alpha}(\cos\beta - j\sin\beta) - \left(\dot{I}_R + \frac{jb}{\alpha - j\beta}\dot{E}_R\right)\epsilon^{-\alpha}(\cos\beta + j\sin\beta)\right] \tag{920}$$

Since $\alpha$ is a small quantity even in very long lines, because in equation (916), $b$ is small at 25 cycles, while $z - x$ is small at 60 cycles, $\epsilon^{\pm\alpha}$ may be replaced by the first terms of its series:

$$\epsilon^{\pm\alpha} = 1 \pm \alpha + \frac{\alpha^2}{2} \pm \frac{\alpha^3}{6} + \frac{\alpha^4}{24} \pm \cdots$$

that is, by

$$\epsilon^{\pm\alpha} = 1 \pm \alpha \tag{921}$$

or, if greater accuracy is desired, by

$$\epsilon^{\pm\alpha} = 1 \pm \alpha + \frac{\alpha^2}{2} \tag{922}$$

with a possible error of $\frac{\alpha^3}{6}$.

Substituting (922) in (920) and separating $\dot{I}_E$ and $\dot{E}_E$ terms:

$$\dot{I}_G = \tfrac{1}{2}\left\{ \dot{I}_E\left(\left[1+\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right]+\left[1-\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta+j\sin\beta\right]\right) + \dot{E}_E\frac{jb}{\alpha-j\beta}\left(\left[1-\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta+j\sin\beta\right] - \left[1+\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta-j\sin\beta\right]\right)\right\} \quad (923)$$

$$\dot{E}_G = \tfrac{1}{2}\left\{ \dot{E}_E\left(\left[1+\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right]+\left[1-\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta+j\sin\beta\right]\right) + \dot{I}_E\frac{\alpha-j\beta}{-jb}\left(\left[1+\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta-j\sin\beta\right] - \left[1-\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta+j\sin\beta\right]\right)\right\} \quad (923)$$

Simplifying coefficients of $\dot{I}_E$ and $\dot{E}_E$:

$$\left[1+\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta - j\sin\beta\right] = \cos\beta + \alpha\cos\beta + \frac{\alpha^2}{2}\cos\beta - j\sin\beta - j\alpha\sin\beta - j\frac{\alpha^2}{2}\sin\beta \quad (924)$$

$$\left[1-\alpha+\frac{\alpha^2}{2}\right]\left[\cos\beta + j\sin\beta\right] = \cos\beta - \alpha\cos\beta + \frac{\alpha^2}{2}\cos\beta + j\sin\beta - j\alpha\sin\beta + j\frac{\alpha^2}{2}\sin\beta \quad (925)$$

Adding (924) and (925) gives:

$$2\cos\beta + \alpha^2\cos\beta - j2\alpha\sin\beta$$

or

$$2\left[\cos\beta\left(1+\frac{\alpha^2}{2}\right) - j\alpha\sin\beta\right] \quad (926)$$

Subtracting (924) from (925) gives:

$$-2\alpha\cos\beta + j2\sin\beta + j2\frac{\alpha^2}{2}\sin\beta$$

or,

$$2\left[-\alpha\cos\beta + j\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right] \quad (927)$$

Subtracting (925) from (924) gives:

$$2\alpha\cos\beta - j2\sin\beta - j2\frac{\alpha^2}{2}\sin\beta$$

or,

$$2\left[\alpha\cos\beta - j\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right] \quad (928)$$

Substituting (926), (927) and (928) in (923):

$$\dot{I}_G = \dot{I}_R\left[\cos\beta\left(1+\frac{\alpha^2}{2}\right)-j\alpha\sin\beta\right] + \dot{E}_R\frac{jb}{\alpha-j\beta}\left[-\alpha\cos\beta+j\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right]$$

$$\dot{E}_G = \dot{E}_R\left[\cos\beta\left(1+\frac{\alpha^2}{2}\right)-j\alpha\sin\beta\right] + \dot{I}_R\frac{\alpha-j\beta}{-jb}\left[\alpha\cos\beta-j\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right] \quad (929)$$

Rationalizing the denominators of the fractions:

$$\frac{jb}{\alpha-j\beta}=\frac{(\alpha+j\beta)jb}{\alpha^2+\beta^2}=\frac{(\alpha+j\beta)jb}{bz}=-\frac{\beta-j\alpha}{z}$$

and,

$$\frac{\alpha-j\beta}{-jb}=\frac{j(\alpha-j\beta)}{b}=\frac{\beta+j\alpha}{b} \quad (930)$$

Substituting equations (930) in (929) and separating $j$ terms:

$$\dot{I}_G = \dot{I}_R\left[\cos\beta\left(1+\frac{\alpha^2}{2}\right)-j\alpha\sin\beta\right]+\dot{E}_R\left\{\frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right]-j\frac{1}{z}\left[\beta\sin\beta\left(1+\frac{\alpha^2}{2}\right)+\alpha^2\cos\beta\right]\right\} \quad (931)$$

$$\dot{E}_G = \dot{E}_R\left[\cos\beta\left(1+\frac{\alpha^2}{2}\right)-j\alpha\sin\beta\right]+\dot{I}_R\left\{\frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\sin\beta\left(1+\frac{\alpha^2}{2}\right)\right]-j\frac{1}{b}\left[\beta\sin\beta\left(1+\frac{\alpha^2}{2}\right)-\alpha^2\cos\beta\right]\right\} \quad (932)$$

The coefficients of $\dot{I}_R$ in (931) and of $\dot{E}_R$ in (932) may be expressed by $a_1 - ja_2$, the coefficient of $\dot{E}_R$ in (931) as $c_1 - jc_2$, and the coefficient of $\dot{I}_R$ in (932) by $d_1 - jd_2$, and, finally, expressing $\dot{I}_R$ by $i'_R + ji''_R$ and $\dot{E}_R$ by $e'_R + je''_R$, equations (931) and (932) become:

$$\dot{I}_G = (i'_R + ji''_R)(a_1 - ja_2) + (e'_R + je''_R)(c_1 - jc_2)$$
$$\dot{E}_G = (e'_R + je''_R)(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2) \quad (933)$$

If in equations (935) the sign of $l$ is reversed, the distance along the line is measured in the opposite direction; that is, $l = 0$ gives generator voltage and current $\dot{E}_G$ and $\dot{I}_G$, $l = -1$ gives receiver voltage and current $\dot{E}_R$ and $\dot{I}_R$. Remembering that $\cos(-\beta) = +\cos\beta$ and $\sin(-\beta) = -\sin\beta$, the only change re-

sulting from this substitution is a reversal of the sign between the two products in equations (933):

$$\dot{I}_R = (i'_G + ji''_G)(a_1 - ja_2) - (e'_G + je''_G)(c_1 - jc_2)$$
$$\dot{E}_R = (e'_G + je''_G)(a_1 - ja_2) - (i'_G + ji''_G)(d_1 - jd_2) \qquad (934)$$

*Summary.*—With known current, voltage and power factor at the receiver end of the line and choosing the time when $e''_R = 0$, the current and voltage vectors at the generator end are found from equations:

$$\dot{I}_G = i'_G + ji''_G = (i'_R + ji''_R)(a_1 - ja_2) + e'_R(c_1 - jc_2)$$
$$\dot{E}_G = e'_G + je''_G = e'_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2) \qquad (935)$$

where: $e'_R$ = voltage, $i'_R$ = power current, $i''_R$ = reactive current } at receiver end of line.

With known current, voltage and power factor at the generator end of the line, the current and voltage vectors at the receiver end are found from equations:

$$\dot{I}_R = i'_R + ji''_R = (i'_G + ji''_G)(a_1 - ja_2) - e'_G(c_1 - jc_2)$$
$$\dot{E}_R = e'_R + je''_R = e'_G(a_1 - ja_2) - (i'_G + i''_G)(d_1 - jd_2) \qquad (936)$$

where: $e'_G$ = voltage, $i'_G$ = power current, $i''_G$ = reactive current } at generator end of line.

The factors $a_1 - ja_2$, $c_1 - jc_2$ and $d_1 - jd_2$ are line constants, which, when once computed, give the performance of the line, in equations (935) for all load conditions at the receiver end and in equations (936) for all conditions under which power may be sent into the line at the generator end. These constants are:

$$a_1 - ja_2 = \left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta$$

$$c_1 - jc_2 = \frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right],$$

$$d_1 - jd_2 = \frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right],$$

where: $\alpha = [\frac{1}{2} b(z - x)]^{\frac{1}{2}}$

$\beta = [\frac{1}{2} b(z + x)]^{\frac{1}{2}}$, in radians

$\beta$ in degrees $= 57.3\beta$

$z = [r^2 + x^2]^{\frac{1}{2}}$ = impedance
$b = 2\pi fC$ = shunted susceptance
$x = 2\pi fL$ = reactance
$r$ = resistance
$L$ = inductance
$C$ = condensance
} of entire line

$f$ = frequency.

Possible error of calculation is $\frac{\alpha^3}{6}$.

After the line constants have been computed and with the current, voltage and the power factor given at one end of the line the corresponding values at the other end of the line may be calculated by means of equations (935) and (936). For convenience the order for making the computations is given in tabular form.

GIVEN DATA

| Let the current, voltage and power factor be given at the: | | |
|---|---|---|
| Receiver end | Generator end | |
| For computing data at generator end | For computing data at receiver end | |
| $i'_G + ji''_G$ | $i'_R + ji''_R$ | Current Vector |
| $e'_G + je''_G$ | $e'_R + je''_R$ | Voltage Vector |
| $I_G = [(i'_G)^2 + (i''_G)^2]^{\frac{1}{2}}$ | $I_R = [(i'_R)^2 + (i''_R)^2]^{\frac{1}{2}}$ | Current |
| $E_G = [(e'_G)^2 + (e''_G)^2]^{\frac{1}{2}}$ | $E_R = [(e'_R)^2 + (e''_R)^2]^{\frac{1}{2}}$ | Voltage |
| $P_G = e'_G i'_G + e''_G i''_G$ | $P_R = e'_R i'_R + e''_R i''_R$ | Actual Power |
| $Q_G = E_G I_G$ | $Q_R = E_R I_R$ | Volt-amperes |
| $\cos \theta_G = P_G \div Q_G$ | $\cos \theta_R = P_R \div Q_R$ | Power Factor |
| $P_G - P_R$ | $P_G - P_R$ | Line Loss |

(*b*) **Transmission-line Equations with Hyperbolic Functions.**—Using hyperbolic functions, the approximation resulting from

the substitution of the first terms of its series for $e^{\pm\alpha}$ may be avoided.

$$\sinh\alpha = \frac{\epsilon^{\alpha} - \epsilon^{-\alpha}}{2}; \cosh\alpha = \frac{\epsilon^{\alpha} + \epsilon^{-\alpha}}{2} \tag{937}$$

Solving (937) for $\epsilon^{\alpha}$ and $\epsilon^{-\alpha}$ gives:

$$\epsilon^{+\alpha} = \cosh\alpha + \sinh\alpha \tag{938}$$

$$\epsilon^{-\alpha} = \cosh\alpha - \sinh\alpha.$$

Substituting (938) into (920), this equation becomes,

$$\dot{I}_G = \tfrac{1}{2}\Big\{\dot{I}_R([\cosh\alpha + \sinh\alpha][\cos\beta - j\sin\beta] - [\cosh\alpha - \sinh\alpha][\cos\beta + j\sin\beta]) + \dot{E}_R\frac{jb}{\alpha - j\beta}([\cosh\alpha - \sinh\alpha][\cos\beta + j\sin\beta] - [\cosh\alpha + \sinh\alpha][\cos\beta - j\sin\beta])\Big\} \tag{939}$$

$$\dot{E}_G = \tfrac{1}{2}\Big\{\dot{E}_B([\cosh\alpha + \sinh\alpha][\cos\beta - j\sin\beta] + [\cosh\alpha - \sinh\alpha][\cos\beta + j\sin\beta]) + \dot{I}_R\frac{\alpha - j\beta}{-jb}([\cosh\alpha + \sinh\alpha][\cos\beta - j\sin\beta] - [\cosh\alpha - \sinh\alpha][\cos\beta + j\sin\beta])\Big\}.$$

Simplifying:

$$\dot{I}_G = i'_G + ji''_G = (i'_R + ji''_R)(a'_1 - ja'_2) + (e'_R + je''_R)(c'_1 - jc'_2)$$
$$\dot{E}_G = e'_G + je''_G = (e'_R + je''_R)(a'_1 - ja'_2) + (i'_R + ji''_R)(d'_1 - jd'_2) \tag{940}$$

Following the same reasoning as that for equation (934):

$$\dot{I}_R = i'_R + ji''_R = (i'_G + ji''_G)(a'_1 - ja'_2) - (e'_G + je''_G)(c'_1 - jc'_2)$$
$$\dot{E}_R = e'_R + je''_R = (e'_G + je''_G)(a'_1 - ja'_2) - (i'_G + ji'_G')(d'_1 - jd'_2) \tag{941}$$

where in both (940) and (941):

$$a'_1 - ja'_2 = \cosh\alpha\cos\beta - j\sinh\alpha\sin\beta,$$

$$c'_1 - jc'_2 = \frac{-jb}{\alpha - j\beta}[\sinh\alpha\cos\beta - j\cosh\alpha\sin\beta]$$

$$d'_1 - jd'_2 = \frac{\alpha - j\beta}{-jb}[\sinh\alpha\cos\beta - j\cosh\alpha\sin\beta]$$

These constants correspond to those used in equations (935) and (936) and should be used in their stead if greatest possible accuracy is desired.

Constants $\alpha$ and $\beta$ are the same as for (935) and (936).

These equations are equivalent to the following expressions

and may be used in this form if tables of complex hyperbolic functions are available.

$$a'_1 - ja'_2 = \cosh(\alpha - j\beta)$$

$$c'_1 - jc'_2 = \frac{-jb}{\alpha - j\beta}\sinh(\alpha - j\beta) = \sqrt{\frac{Y}{Z}}\sinh(\alpha - j\beta)$$

$$d'_1 - jd'_2 = \frac{\alpha - j\beta}{-jb}\sinh(\alpha - j\beta) = \sqrt{\frac{Z}{Y}}\sinh(\alpha - j\beta)$$

It will be found convenient to keep the components of these expressions in the form illustrated in the problem on "Line C."

(*c*) **Transmission-line Equations Approximated for Preliminary Computation.**—Substituting $l = 1$ in equations (935) and (908):

$$\dot{I}_G = \dot{A}_1\epsilon^{(YZ)^{1/2}} - \dot{A}_2\epsilon^{-(YZ)^{1/2}}$$

$$\dot{E}_G = \frac{(YZ)^{1/2}}{Y}\left[\dot{A}_1\epsilon^{(YZ)^{1/2}} + \dot{A}_2\epsilon^{-(YZ)^{1/2}}\right] \tag{942}$$

Substituting $l = 0$ in equations (905) and (908):

$$\dot{I}_R = \dot{A}_1 - \dot{A}_2$$

$$\dot{E}_R = \frac{(YZ)^{1/2}}{Y}\left[\dot{A}_1 + \dot{A}_2\right] \tag{943}$$

Solving (943) for $A_1$ and $A_2$:

$$\dot{A}_1 = \tfrac{1}{2}\left[\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right]$$

$$\dot{A}_2 = -\tfrac{1}{2}\left[\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right] \tag{944}$$

Substituting (944) in (942):

$$\dot{I}_G = \tfrac{1}{2}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{(YZ)^{1/2}} + \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{-(YZ)^{1/2}}\right]$$

$$\dot{E}_G = \tfrac{1}{2}\frac{(YZ)^{1/2}}{Y}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{(YZ)^{1/2}} - \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\epsilon^{-(YZ)^{1/2}}\right] \tag{945}$$

In the converging series of:

$$\epsilon^{\pm(YZ)^{1/2}} = 1 \pm (YZ)^{1/2} + \frac{YZ}{2} \pm \frac{(YZ)^{3/2}}{6} + \frac{(YZ)^2}{24} \pm \cdots$$

the first four terms may be used to represent $\epsilon^{\pm(YZ)^{1/2}}$ with sufficient accuracy:

$$\dot{I}_G = \tfrac{1}{2}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 + (YZ)^{1/2} + \frac{YZ}{2} + \frac{(YZ)^{3/2}}{6}\right) + \left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{1/2}}\right)\left(1 - (YZ)^{1/2} + \frac{YZ}{2} - \frac{(YZ)^{3/2}}{6}\right)\right]$$

$$\dot{E}_G = \tfrac{1}{2}\frac{(YZ)^{\frac{1}{2}}}{Y}\left[\left(\dot{I}_R + \dot{E}_R\frac{Y}{(YZ)^{\frac{1}{2}}}\right)\left(1+(YZ)^{\frac{1}{2}}+\frac{YZ}{2}+\frac{(YZ)^{\frac{3}{2}}}{6}\right)\right.$$
$$\left. -\left(\dot{I}_R - \dot{E}_R\frac{Y}{(YZ)^{\frac{1}{2}}}\right)\left(1-(YZ)^{\frac{1}{2}}+\frac{YZ}{2}-\frac{(YZ)^{\frac{3}{2}}}{6}\right)\right] \quad (946)$$

Simplifying:

$$\dot{I}_G = \dot{I}_R\left(1+\frac{YZ}{2}\right) + \dot{E}_R Y\left(1+\frac{YZ}{6}\right)$$
$$\dot{E}_G = \dot{E}_R\left(1+\frac{YZ}{2}\right) + \dot{I}_R Z\left(1+\frac{YZ}{6}\right) \quad (947)$$

Since $Y = -jb$, and $Z = r - jx$,

$$\dot{I}_G = \dot{I}_R\left[1-\frac{b}{2}x - j\frac{b}{2}r\right] + \dot{E}_R\left[-\frac{b^2}{6}r - j\left(b-\frac{b^2}{6}x\right)\right]$$
$$\dot{E}_G = \dot{E}_R\left[1-\frac{b}{2}x - j\frac{b}{2}r\right] + \dot{I}_R\left[r\left(1-\frac{b}{3}x\right)\right.$$
$$\left. - j\left(x+\frac{b}{6}[r^2-x^2]\right)\right] \quad (948)$$

Final equations:

$$\dot{I}_G = i'_G + ji''_G = (i'_R + ji''_R)(a''_1 - ja''_2) + e'_R(c''_1 - jc''_2)$$
$$\dot{E}_G = e'_G + je''_G = e'_R(a''_1 - ja''_2) + (i'_R + ji''_R)(d''_1 - jd''_2) \quad (949)$$

Measuring distance from generator end:

$$\dot{I}_R = i'_R + ji''_R = (i'_G + ji''_G)(a''_1 - ja''_2) - e'_G(c''_1 - jc''_2) \quad (950)$$

$$\dot{E}_R = e'_R + je''_R = e'_G(a''_1 - ja''_2) - (i'_G + ji''_G)(d''_1 - jd''_2)$$

where for both equations (949) and (950):

$$a''_1 - ja''_2 = 1 - \frac{b}{2}x - j\frac{b}{2}r$$
$$c''_1 - jc''_2 = -\frac{b^2}{6}r - j\left(b-\frac{b^2}{6}x\right)$$
$$d''_1 - jd''_2 = r\left(1-\frac{b}{3}x\right) - j\left(x+\frac{b}{2}[r^2-x^2]\right).$$

In the above constants

$b = 2\pi fC$ = shunted susceptance
$x = 2\pi fL$ = reactance
$C$ = condensance
$L$ = inductance
$f$ = frequency.

} of entire line

In case it is desired to find the operating conditions at the low-tension buses on both ends of the line, the equations must include the constants of the transformers. Sufficiently accurate results may be obtained by adding the equivalent reactance and resistance of the transformers to the reactance and resistance, respectively, of the line. This procedure would, of course, distribute the transformer reactance and resistance uniformly along the whole line, whereas for greatest accuracy they should be treated as concentrated on both ends.

The actual low bus potential and current are obtained from the known ratios of the transformers.

(*d*) **Arrangement of Conductors.**—The economy in copper of the three-phase system for long transmission lines was shown in

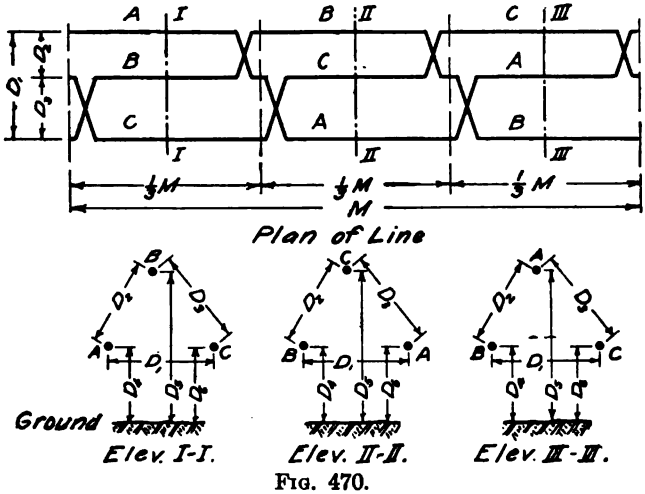



FIG. 470.

Chap. XXV. In America this system is so generally used that no other need be considered. Assuming a balanced three-phase system, with the corresponding neutral as explained in Chap. XXV, letting $e_G$ and $e_R$ in the line equations represent the voltages to neutral and using in the line constants the inductance and condensance to neutral, the transmission line may be considered as three single-phase circuits. The complete performance curves for the line may be found by analyzing one phase to neutral since the system is balanced.

In order that the three phases shall have identical performance curves the electrical constants must be the same: that is,

1. The three conductors must be of the same length and cross-section and be of the same material so that the resistances shall be equal.

2. The inductance per conductor to neutral must be the same for the three phases.

3. The condensance per conductor to neutral must be the same for the three phases.

The first requirement is always met in long-distance lines. The inductance and condensance of the three phases are equal only if the wires are properly transposed. Two complete transpositions dividing the line into three sections of equal length give a balanced arrangement. In Fig. 470 the interaxial spacing for the one-third of the line for any pair of conductors is $D_1$, for the second one-third, $D_2$, and the last one-third, $D_3$.

In order to apply the equation for inductance derived in Chap. XXI, it is desirable to find the equivalent equidistant spacing which gives the same inductance per conductor for the whole length of the line as that given by the sum of the inductances, each for one-third of the length, of the spacings $D_1$, $D_2$ and $D_3$. Let $D$ = the equivalent equidistant spacing.

$d$ = the diameter of the line wire in the same units as $D$.
$L$ = the inductance of one conductor in henrys.
$l$ = the length of the line conductor in miles.
$k_1 = 74.1 \times 10^{-5}$
$k_2 = 8.05 \times 10^{-5}$

$$L = l\left(k_1 \log_{10} \frac{2D}{d} + k_2\right) \text{ henrys (from equation (624))}$$

$$= \tfrac{1}{3}l\left(k_1 \log_{10} \frac{2D_1}{d} + k_2\right) + \tfrac{1}{3}l\left(k_1 \log_{10} \frac{2D_2}{d} + k_2\right) + \tfrac{1}{3}l\left(k_1 \log_{10} \frac{2D_3}{d} + k_2\right) \quad (952)$$

Eliminating common factors and solving for $D$,

$$\begin{aligned} 3 \log_{10} 2D &= \log_{10} 2D_1 + \log_{10} 2D_2 + \log_{10} 2D_3 \\ \log_{10} (2D)^3 &= \log_{10} (2^3 D_1 \times D_2 \times D_3) \\ D^3 &= (D_1 \times D_2 \times D_3) \\ D &= (D_1 \times D_2 \times D_3)^{\frac{1}{3}} \end{aligned} \quad (953)$$

Therefore, if the conductors are properly transposed and a spacing, $D$, is used equal to the cube root of the product of the three unequal spacings, the equation for the inductance (952) may be

used on all practical transmission lines whatever be the arrangement of the conductors.

Similarly the condensance of one wire to neutral for lines having unequal spacings may be expressed by equation (634) (derived in Chap. XXI) provided the line conductors are properly transposed and an equivalent spacing, $D$, is used, equal to the cube root of the product of the three unequal spacings.

$C$ = the condensance of one conductor to neutral in farads.

$k = 0.0388 \times 10^{-6}$

$D$, $D_1$, $D_2$, $D_3$, $d$ and $l$ as already defined.

$$C = l\frac{k}{\log_{10}\frac{2D}{d}}$$

$$= \frac{l}{3}\frac{k}{\log_{10}\frac{2D_1}{d}} + \frac{l}{3}\frac{k}{\log_{10}\frac{2D_2}{d}} + \frac{l}{3}\frac{k}{\log_{10}\frac{2D_3}{d}} \qquad (954)$$

Eliminating the common factors, $l$ and $k$

$$\frac{1}{\log_{10}\frac{2D}{d}} = \frac{1}{3\log_{10}\frac{2D_1}{d}} + \frac{1}{3\log_{10}\frac{2D_2}{d}} + \frac{1}{3\log_{10}\frac{2D_3}{d}}.$$

Hence:

$$\log_{10}\frac{2D}{d} = \frac{3\log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_2}{d}\log_{10}\frac{2D_3}{d}}{\log_{10}\frac{2D_2}{d}\log_{10}\frac{2D_3}{d} + \log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_3}{d} + \log_{10}\frac{2D_1}{d}\log_{10}\frac{2D_2}{d}}$$

$= \frac{1}{3}\log_{10} 2D_1 + \frac{1}{3}\log_{10} 2D_2 + \frac{1}{3}\log_{10} 2D_3 - \log_{10} d$, plus a quantity which is zero for equal spacings and so small for any unsymmetrical spacing used in practical transmission lines that for numerical calculations it may be neglected.

Therefore:

$$\log_{10}\frac{2D}{d} = \frac{1}{3}\log_{10} 2D_1 + \frac{1}{3}\log_{10} 2D_2 + \frac{1}{3}\log_{10} 2D_3 - \log_{10} d$$

$$\log_{10} 2D = \frac{1}{3}\log_{10}(2^3 D_1 \times D_2 \times D_3)$$

$$D = (D_1 \times D_2 \times D_3)^{\frac{1}{3}} \qquad (955)$$

Likewise the condensance to ground (Fig. 470) is,

$$C = l\frac{0.0388 \times 10^{-6}}{\log_{10}\frac{4D'}{d}} \qquad (956)$$

where:

$$D' = (D_4 \times D_5 \times D_6)^{1/3}$$

(*e*) **Voltage Regulation by Synchronous Condenser (Power-factor Regulator).**—The most important problem in the operation of long-distance transmission lines is the voltage regulation, which must be automatic to give satisfactory results. The ratio between the generator and receiver voltages is determined by the line constants and the nature of the load, but the equations, already derived, are complicated. As in most alternating-current phenomena it is essential to first understand the energy changes involved. The amount of energy stored inductively in the magnetic field along the line depends on the current in the line while the energy stored condensively in the dielectric depends on the voltage. Hence the energy stored magnetically is very small at no load and increases up to a maximum for the maximum current in the line. The energy stored dielectrically, on the other hand, is approximately constant for all loads for systems operating at practically constant potentials. In long transmission lines, operating at high voltage the condensance as well as the inductance is of considerable magnitude. The energy stored dielectrically is of such quantitative value in comparison to the amount stored magnetically that they balance or offset each other at some load between no load and full load. At no load the condensive action predominates, while at full load, and particularly for the maximum current in the line the inductance becomes the controlling factor. Variations in the receiver load during the daily cycle combined with the ever-changing power factor of the commercial load continually change the relative effect on the voltage regulation of the inductance and condensance of the line.

In constant-potential systems it is imperative that the receiver voltage, $e_R$, be kept constant at all loads. This eliminates one possible variable in each voltage equation. It is also highly desirable that the generator voltage, $e_G$, should be a constant for all loads. To secure satisfactory voltage regulation it is, therefore, necessary to provide some means by which the voltage at both ends of the line can be automatically kept constant for all variations in the load, from no load to the maximum value for which the line is designed. As mentioned in Chap. XV all the conditions required for automatic voltage regulation are met by

the use of synchronous condensers placed at the receiver end of the line. The field excitation of the synchronous condenser is adjusted by some automatic device, as the *TA* regulator described in Chap. XIV, so that the current in the machine is lagging at no load and leading at maximum load. When properly designed and installed, this arrangement gives the required automatic voltage regulation for all loads. It is not necessary that the receiver and generator voltages be equal but only that they shall remain constant at all loads. With a given generator voltage the receiver voltage should be selected so as to require a synchronous condenser of minimum capacity.

As the function of the synchronous condenser is to provide the required reactive kv.a. for all loads the ratio of the generator and receiver voltages should be so selected that the maximum lagging current (at no load) in the synchronous condenser should be equal to the maximum leading current (at maximum load) in order that the capacity or size of the machine may be a minimum. By solving the general voltage equation (933) for the reactive current, $i''_R$, the amount of reactive power (lagging or leading current) that will be required to give constant receiver voltage may be calculated. For $e''_R = 0$, $e'_R = e_R$ from equation (933):

$$\dot{E}_R = e_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2)$$

$$= e_R a_1 + i'_R d_1 + i''_R d_2 + j(-e_R a_2 + i''_R d_1 - i'_R d_2)$$

$$e^2_G = (i''_R)^2(d_1^2 + d_2^2) + 2i''_R(e_R a_1 d_2 - e_R a_2 d_1) + (i'_R)^2(d_1^2 + d_2^2)$$
$$+ 2i'_R(e_R a_1 d_1 + e_R a_2 d_2) + e_R^2(a_1^2 + a_2^2)$$

$$i''_R = \frac{e_R(-a_1 d_2 + a_2 d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \frac{e_R^2(a_1^2 + a_2^2)}{d_1^2 + d_2^2}\right.$$
$$\left. + \frac{e_R^2(a_1 d_2 - a_2 d_1)^2}{d_1^2 + d_2^2} - (i'_R)^2 - \frac{2e_R i'_R\,(a_1 d_1 + a_2 d_2)}{d_1^2 + d_2^2}\right]^{1/2}$$

$$= \frac{e_R(-a_1 d_2 + a_2 d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \left(\frac{e_R(a_1 d_1 + a_2 d_2)}{d_1^2 + d_2^2} + i'_R\right)^2\right]^{1/2} \tag{951}$$

By referring to Figs. 471 and 472 for quantitative values, the method for determining the most desirable receiver voltage may be more easily understood. For known line constants and given constant generator voltage, plot curves for the reactive power, equation (951), as ordinates and the kw. receiver load as abscissæ for three values of the receiver voltage: of $e_R$ = 90 per cent. of $e_G$; $e_R = e_G$; and $e_R$ = 110 per cent. of $e_G$. On the same diagram,

Figs. 471 and 472, draw the line $OD$ representing the kv.a. load for the specified power factors, 85 per cent. and 90 per cent. respectively. For any given load the corresponding ordinate to the line $OD$ represents the reactive power in the receiver load. Likewise the ordinate to the curves for the receiver voltage selected gives the amount of reactive power that must be provided in order to give the desired regulation. The function of the synchronous condenser is therefore to supply the difference.

By interpolation that receiver voltage may be found which will require an amount of inductive reactive power at no load, ordinate $OA$, equal in magnitude to the condensive reactive power at maximum load, ordinate $BD$.

The receiver voltage thus determined can be maintained automatically by a synchronous condenser of minimum capacity for the given line, load and generator voltage. Thus for line $A$, Fig. 471, for the given generator voltage, $e_G = 135{,}000$ volts and power factor of load = 85 per cent., the capacity of the synchronous condenser is a minimum if the receiver voltage $e_R = 123{,}000$ volts. Similarly for line $B$, Fig. 472, for $e_G = 160{,}000$ volts and load power factor = 90 per cent., the required capacity of the synchronous condenser is a minimum if the receiver voltage $e_R = 155{,}500$ volts.

The effect of varying the power factor of the load is apparent from reactive power curves, Figs. 471 and 472. For loads having power factors nearer unity the receiver voltage would be correspondingly increased while the reverse would be the case for loads with lower power factors.

In order to illustrate the application of the theoretical equations to practical transmission lines the complete solution is given for two typical high-tension systems. The complete operating characteristics of the two lines are shown in Figs. 473 and 474.

(*f*) **Performance Computations of Long-distance Transmission Lines.**—Below are given the performance computations of two transmission lines called $A$ and $B$. The computations are given in parallel columns for the two lines in order to facilitate comparisons; the pages are divided so that the treatment of line $A$ appears on the left-hand side and of line $B$ on the right. Statements which hold true for both lines are carried across both divisions.

| —LINE *A*— | —GIVEN DATA— | —LINE *B*— |
|---|---|---|
| 120 miles | Distance of transmission | 240 miles |
| 30,000 kw. | Normal load per line | 30,000 kw. |
| 45,000 kw. | Maximum load per line | 45,000 kw. |
| 3,000 ft. | Elevation of power house | 5,000 ft. |
| 2,000 ft. | Elevation of receiving station | 200 ft. |
| Eastern Washington | Location of line | Central California |
| 75°F. | Average operating temperature | 100°F. |
| 135,000 volts | Transmission voltage at generator end | 160,000 volts |
| 500,000 cm. hard-drawn copper stranded cable, diameter 0.819 in. | Conductor used | 850,000 cm. hard-drawn aluminum stranded cable, diameter 1.062 in. |
| 14 ft. 6 in. | Spacing of conductors | 16 ft. 9 in. |
| 60 cycles | Frequency | 50 cycles |
| 0.85 | Power factor at receiver end | 0.90 |

—LINE *A*— —LINE *B*—

Conductors arranged in a horizontal plane.

Line star-connected at both ends with the neutral grounded, but means provided to disconnect neutral from ground whenever necessary.

Voltage at generator and receiver ends to be kept constant for all loads by the installation of synchronous condensers, of adequate capacity, at the receiver end.

Equations (935) have been used in the computations for both lines.

—LINE CONSTANTS—

Resistance

| For Copper | For Aluminum |
|---|---|
| $\rho = 9.35[1 + 0.0042\ (t - 32)0.556]$ | $\rho = 15.4[1 + 0.00435(t - 32)0.556]$ |
| $t = 75°\text{F.}$, | $t = 100°\text{F.}$, |
| $\rho = 9.35[1 + 0.0042(75 - 32)0.556] = 10.29$ ohms | $\rho = 15.4[1 + 0.00435(100 - 32)0.556] = 17.933$ ohms |

where $\rho$ = resistance per mil-foot of conductor,

$t$ = temperature, at which resistance is desired, in degrees F.

Total resistance of conductor

| | |
|---|---|
| $r = \frac{120 \times 5{,}280 \times 10.29}{500{,}000} = 13.04$ ohms | $r = \frac{240 \times 5{,}280 \times 17.933}{850{,}000} = 26.74$ ohms |

| —LINE *A*— | —LINE *B*— |
|---|---|

**Inductance**

$$L = l(0.00074 \log_{10} \frac{2D}{d} + 0.0000805)$$

where $L$ = inductance in henrys
$l$ = length of the line in miles
$D$ = equivalent interaxial spacing in inches between conductors symmetrically arranged in triangle.
$D = (D_1 \times D_2 \times D_3)^{1/3}$
$d$ = diameter of conductor in inches

} See formulæ (952) and (953)

| | |
|---|---|
| $L = 120(0.00074 \log_{10} \frac{2 \times 174 \times 2^{1/4}}{0.819} + 0.0000805)$ | $L = 240(0.00074 \log_{10} \frac{2 \times 201 \times 2^{1/4}}{1.062} + 0.0000805)$ |
| $= 120(0.00074 \times 2.7285 + 0.0000805)$ | $= 240(0.00074 \times 2.6782 + 0.0000805)$ |
| $= 0.252$ henry | $= 0.495$ henry |

**Reactance**

$$x = 2\pi f L$$

| | |
|---|---|
| $x = 2 \times 3.1416 \times 60 \times 0.252 = 95.0$ ohms | $x = 2 \times 3.1416 \times 50 \times 0.495 = 155.5$ ohms |

**Impedance**

$$z = (r^2 + x^2)^{1/2}$$

| | |
|---|---|
| $z = (\overline{13.04}^2 + \overline{95.0}^2)^{1/2} = 95.885$ ohms | $z = (\overline{26.74}^2 + \overline{155.5}^2)^{1/2} = 157.78$ ohms |

**Condensance**

$$C = l \times \frac{0.0388 \times 10^{-6}}{\log_{10} \frac{2D}{d}}$$

where $C$ = condensance in farads.
Other units same as for inductance.

} See formulæ (954) and (955)

| —LINE *A*— | —LINE *B*— |
|---|---|
| $C = 120 \times \dfrac{0.0388 \times 10^{-6}}{\log_{10}\dfrac{2 \times 174 \times 2^{1/2}}{0.819}} = 1.7065 \times 10^{-6}$ farads | $C = 240 \times \dfrac{0.0388 \times 10^{-6}}{\log_{10}\dfrac{2 \times 201 \times 2^{1/2}}{1.062}} = 3.4772 \times 10^{-6}$ farads |

Shunted line susceptance

$$b = 2\pi fC$$

| | |
|---|---|
| $b = 2 \times 3.1416 \times 60 \times 1.7065 \times 10^{-6}$ | $b = 2 \times 3.1416 \times 50 \times 3.4772 \times 10^{-6}$ |
| $= 0.6433 \times 10^{-3}$ mhos | $= 1.1068 \times 10^{-3}$ mhos |

$$\alpha = [\tfrac{1}{2}b(z - x)]^{1/2}$$

| | |
|---|---|
| $\alpha = [\tfrac{1}{2} \times 0.6433 \times 10^{-3}(95.885 - 95.0)]^{1/2}$ | $\alpha = [\tfrac{1}{2} \times 1.1068 \times 10^{-3}(157.78 - 155.5)]^{1/2}$ |
| $= 0.016872$ radian | $= 0.03552$ radian |
| $\alpha^2 = 0.00028465$ | $\alpha^2 = 0.0012616$ |
| $1 + \dfrac{\alpha^2}{2} = 1.00014233$ | $1 + \dfrac{\alpha^2}{2} = 1.0006308$ |

$$\beta = [\tfrac{1}{2}b(z + x)]^{1/2}$$

| | |
|---|---|
| $\beta = [\tfrac{1}{2} \times 0.6433 \times 10^{-3}(95.885 + 95.0)]^{1/2}$ | $\beta = [\tfrac{1}{2} \times 1.1068 \times 10^{-3}(157.78 + 155.5)]^{1/2}$ |
| $= 0.2478$ radian | $= 0.41638$ radian |
| $\beta^\circ = 0.2478 \times 57.3 = 14^\circ\ 11'\ 53''$ | $\beta^\circ = 0.41638 \times 57.3 = 23^\circ\ 51'\ 22''$ |
| $\sin \beta = 0.2453$ | $\sin \beta = 0.40443$ |
| $\cos \beta = 0.96945$ | $\cos \beta = 0.91456$ |
| $\beta \sin \beta = 0.06078$ | $\beta \sin \beta = 0.16838$ |
| $\beta \cos \beta = 0.2404$ | $\beta \cos \beta = 0.38076$ |

| —LINE *A*— | —LINE *B*— |
|---|---|

—CONSTANTS OF LINE EQUATIONS—

$$a_1 - ja_2 = \left(1 + \frac{\alpha^2}{2}\right)\cos\beta - j\alpha\sin\beta$$

$$a_1 - ja_2 = 1.00014233 \times 0.96945 - j0.016872 \times 0.2453 = 0.96958 - j0.0041387$$

$$a_1 - ja_2 = 1.0012616 \times 0.91456 - j0.03552 \times 0.40443 = 0.91571 - j0.014365$$

$$c_1 - jc_2 = \frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right]$$

$$c_1 - jc_2 = \frac{1}{95.885}[0.016872(0.2404 - 1.00014233 \times 0.2453)] - \frac{j}{95.885} \times [1.00014233 \times 0.06078 + 0.00028465 \times 0.96945] = -0.000000868 - j0.0006369$$

$$c_1 - jc_2 = \frac{1}{157.78}[0.03552(0.38076 - 1.0006388 \times 0.40443)] - \frac{j}{157.78}[1.0006308 \times 0.16838 + 0.0012616 \times 0.91456] = -0.000005385 - j0.0010752$$

$$d_1 - jd_2 = \frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right]$$

$$d_1 - jd_2 = \frac{1}{0.6433 \times 10^{-3}}[0.016872(0.2404 + 1.00014233 \times 0.2453)] - j\frac{1}{0.6433 \times 10^{-3}}[1.00014233 \times 0.06078 - 0.00028465 \times 0.96945] = 12.74 - j94.07$$

$$d_1 - jd_2 = \frac{1}{1.1068 \times 10^{-3}}[0.03552(0.38076 + 1.0006308 \times 0.40443)] - j\frac{1}{1.1068 \times 10^{-3}}[1.0006308 \times 0.16838 - 0.0012616 \times 0.91456] = 25.206 - j151.2.$$

The above constants are independent of voltage or current in the line and are the numerical coefficients of the physical properties of the line. The performance of the line depends upon the magnitude and fre-

—LINE *A*— —LINE *B*—

quency of the voltage impressed at the generator end, the above line constants, and the nature of the load at the receiver end. The generator voltage is stipulated in the given data, but the nature of the load is only partly defined by the given power factor and magnitude of the load. The total receiver load also includes the synchronous condenser or power-factor regulator, the function of which has been described above.

The next step in the computations is the determination of the required capacity of the synchronous condenser (power-factor regulator) necessary to maintain constant potential at the receiver end for a range in load from zero to maximum, and the determination of the best voltage relation between $E_G$ and $E_R$ which makes the required capacity of the synchronous condenser a minimum in comparison to all other possible relations of $E_G$ and $E_R$.

Reactive current required for constant voltage at both ends of the line.

$$i''_R = \frac{e_R(-a_1d_2 + a_2d_1)}{d_1^2 + d_2^2} + \left[\frac{e_G^2}{d_1^2 + d_2^2} - \left(\frac{e_R(a_1d_1 + a_2d_2)}{d_1^2 + d_2^2} + i'_R\right)^2\right]^{\frac{1}{2}} \text{ see equation (951)}$$

$$e_G = \frac{135{,}000}{\sqrt{3}} = 77{,}940$$

Substituting constants for $e_G = 77{,}940$ and

$e_R = 90$ per cent. of $e_G = 70{,}146$

$$i''_R = -709.6 + [674{,}150 - (99.18 + i'_R)^2]^{\frac{1}{2}}$$

$e_R = e_G = 77{,}940$

$$i''_R = -788.45 + [674{,}150 - (110.21 + i'_R)^2]^{\frac{1}{2}}$$

$e_R = 110$ per cent. of $e_G = 84{,}834$

$$i''_R = -858.20 + [674{,}150 - (119.95 + i'_R)^2]^{\frac{1}{2}}$$

$$e_G = \frac{160{,}000}{\sqrt{3}} = 92{,}380$$

Substituting constants for $e_G = 92{,}380$ and

$e_R = 90$ per cent. of $e_G = 83{,}142$

$$i''_R = -488.6 + [363{,}198 - (89.35 + i'_R)^2]^{\frac{1}{2}}$$

$e_R = e_G = 92{,}380$

$$i''_R = -542.90 + [363{,}198 - (99.27 + i'_R)^2]^{\frac{1}{2}}$$

$e_R = 110$ per cent. of $e_G = 101{,}618$

$$i''_R = -597.22 + [363{,}198 - (109.20 + i'_R)^2]^{\frac{1}{2}}$$

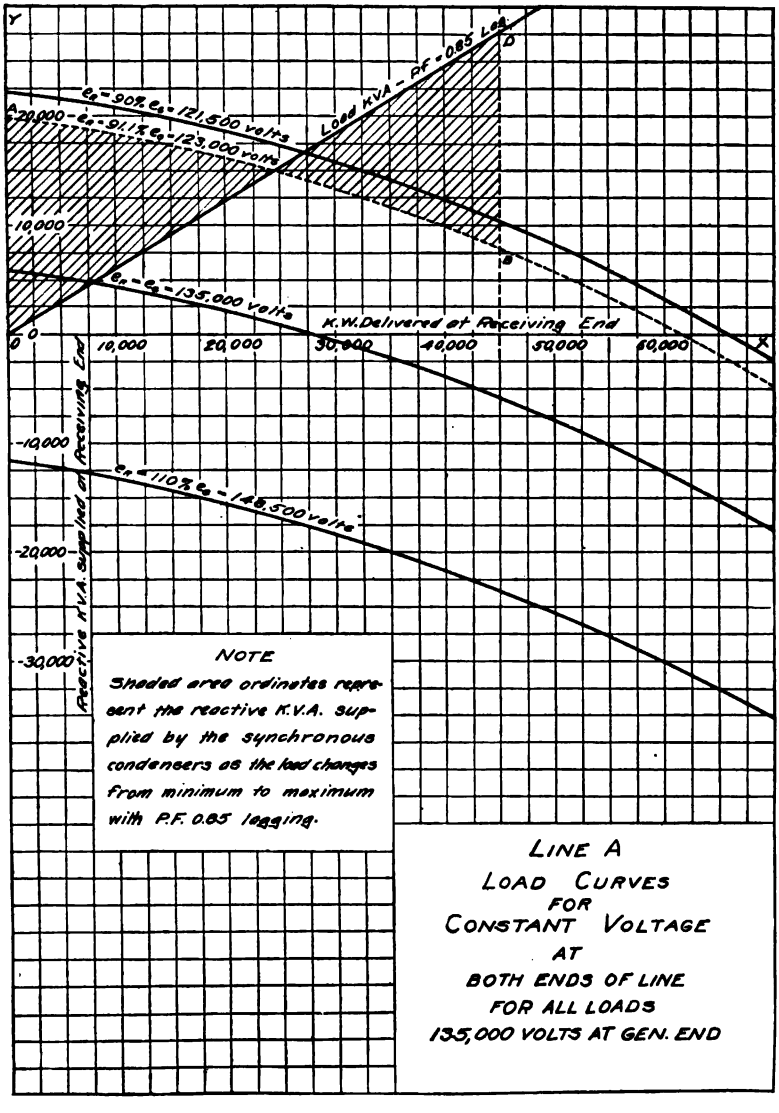

Y
A
B
D
X
Load K.V.A. - P.F. - 0.85 Lag
$e_s$ - 90% $e_r$ - 121,500 volts
$e_s$ - 91.1% $e_r$ - 123,000 volts
$e_s$ - $e_r$ - 135,000 volts
$e_s$ - 110% $e_r$ - 148,500 volts
K.W. Delivered at Receiving End
Reactive K.V.A. Supplied at Receiving End
20,000
10,000
0
-10,000
-20,000
-30,000
10,000
20,000
30,000
40,000
50,000
60,000
NOTE
Shaded area ordinates represent the reactive K.V.A. supplied by the synchronous condensers as the load changes from minimum to maximum with P.F. 0.85 lagging.
LINE A
LOAD CURVES
FOR
CONSTANT VOLTAGE
AT
BOTH ENDS OF LINE
FOR ALL LOADS
135,000 VOLTS AT GEN. END

FIG. 471.

—LINE *A*— —LINE *B*—

The values of $i_R''$ for a range of load from 0 to 70,000 kw. are tabulated below.

| $3\phi$ Load$_R$, Kw. | $e_R$ = 70,146 | | $e_R$ = 77,940 | | $e_R$ = 84,834 | |
|---|---|---|---|---|---|---|
| | $i'_R$ | $i''_R$ | $i'_R$ | $i''_R$ | $i'_R$ | $i''_R$ |
| 0 | 0 | 105.4 | 0 | 25.2 | 0 | −46.0 |
| 10,000 | 47.52 | 98.3 | 42.77 | 18.2 | 39.295 | −52.7 |
| 20,000 | 95.04 | 88.2 | 85.54 | 9.0 | 78.590 | −61.5 |
| 30,000 | 142.56 | 75.1 | 128.31 | −2.8 | 117.885 | −72.3 |
| 40,000 | 190.08 | 58.8 | 171.08 | −17.1 | 157.180 | −85.3 |
| 50,000 | 237.60 | 39.2 | 213.85 | −34.1 | 196.475 | −100.6 |
| 60,000 | 285.12 | 16.0 | 256.62 | −53.9 | 235.770 | −118.2 |
| 70,000 | 332.64 | −11.3 | 299.39 | −76.9 | 275.065 | −138.4 |

| $3\phi$ Load$_R$, Kw. | $e_R$ = 83,142 | | $e_R$ = 92,380 | | $e_R$ = 101,618 | |
|---|---|---|---|---|---|---|
| | $i'_R$ | $i''_R$ | $i'_R$ | $i''_R$ | $i'_R$ | $i''_R$ |
| 0 | 0 | 107.4 | 0 | 53.5 | 0 | −4.4 |
| 10,000 | 40.092 | 100.0 | 36.085 | 44.4 | 32.802 | −11.5 |
| 20,000 | 80.184 | 89.7 | 72.170 | 34.9 | 65.604 | −20.7 |
| 30,000 | 120.276 | 75.4 | 108.255 | 22.9 | 98.406 | −31.5 |
| 40,000 | 160.368 | 59.9 | 144.340 | 8.3 | 131.208 | −44.8 |
| 50,000 | 200.460 | 39.8 | 180.425 | −9.1 | 164.010 | −60.1 |
| 60,000 | 240.552 | 15.8 | 216.510 | −29.6 | 196.812 | −78.0 |
| 70,000 | 280.644 | −12.9 | 252.595 | −53.6 | 229.614 | −98.8 |

$$\text{Reactive kv.a.} = \text{kw.} \frac{i''_R}{i'_R}$$

The reactive kv.a. for the different values of $e_R$ are tabulated below.

| Kw. | $e_R$ = 70,146 | $e_R$ = 77,940 | $e_R$ = 84,834 |
|---|---|---|---|
| 0 | 22,180 | 5,892 | −11,683 |
| 10,000 | 20,686 | 4,255 | −13,411 |
| 20,000 | 18,560 | 2,107 | −15,650 |
| 30,000 | 15,802 | −655 | −18,397 |
| 40,000 | 12,373 | −3,998 | −21,706 |
| 50,000 | 8,249 | −7,973 | −25,600 |
| 60,000 | 3,367 | −12,602 | −30,080 |
| 70,000 | −2,378 | −17,980 | −35,220 |

| Kw. | $e_R$ = 83,142 | $e_R$ = 92,380 | $e_R$ = 101,618 |
|---|---|---|---|
| 0 | 26,790 | 14,826 | −1,491 |
| 10,000 | 24,943 | 12,305 | −3,506 |
| 20,000 | 22,371 | 9,672 | −6,310 |
| 30,000 | 18,805 | 6,345 | −9,603 |
| 40,000 | 14,940 | 2,300 | −13,656 |
| 50,000 | 9,927 | −2,522 | −18,320 |
| 60,000 | 3,941 | −8,202 | −23,777 |
| 70,000 | −3,218 | −14,853 | −30,120 |

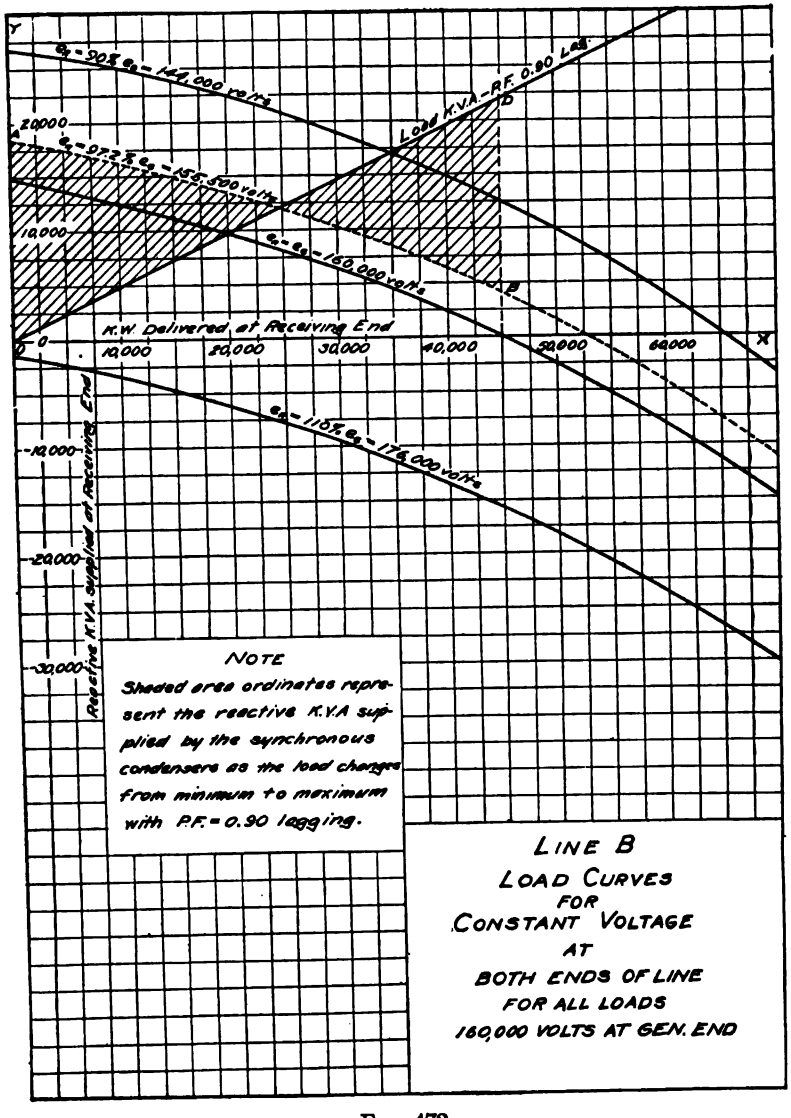

Y
$e_s$ = 90% $e_s$ = 142,000 volts
Load K.V.A. - P.F. 0.90 Lag.
D
20000
$e_s$ = 97.5% $e_s$ = 155,500 volts
A
10000
$e_s$ = $e_s$ = 160,000 volts
A.B
K.W Delivered at Receiving End
0
10,000
20,000
30,000
40,000
50,000
60000
X
$e_s$ = 115% $e_s$ = 174,000 volts
-10,000
-20,000
-30,000
Reactive K.V.A. supplied at Receiving End
NOTE
Shaded area ordinates represent the reactive K.V.A supplied by the synchronous condensers as the load changes from minimum to maximum with P.F. = 0.90 lagging.
LINE B
LOAD CURVES
FOR
CONSTANT VOLTAGE
AT
BOTH ENDS OF LINE
FOR ALL LOADS
160,000 VOLTS AT GEN. END

FIG. 472.

—LINE *A*—

The constant voltage curves drawn from the curves in Fig. 471 and the straight line giving the kv.a. of the load at 85 per cent. *P.F.* without the synchronous condenser, show that for equal voltages at both ends of the line, the reactive kv.a. must be 5,892 lagging at no load which must be all supplied by the synchronous condenser. At maximum load, the reactive kv.a. required are 6,000, current leading. The load supplies 27,500 kv.a., current lagging, therefore the load of the synchronous condenser would have to be 27,500 + 6,000 = 33,500 kv.a., current leading. This condition would be unsatisfactory. However, if the dotted curve, marked $e_R$ = 91.1 per cent. of $e_G$ (constructed by interpolation) represents the actual relationship between the generator and receiver voltage, synchronous condensers of an aggregate capacity of 20,000 kv.a. would suffice for the total load range from zero to maximum. The machines would be running under full-load lagging current when the power load is zero, and under full-load leading current when the power load is at the maximum. Therefore, for this particular line, with the generator voltage assumed as 135,000, the receiver voltage should be 123,000 for all loads. The voltage to neutral, $e_G$ = 77,940 volts; $e_R$ = 71,015 volts.

—LINE *B*—

The constant voltage curves drawn from the curves in Fig. 472 and the straight line giving the kv.a. of the load at 90 per cent. *P.F.* without the synchronous condenser, show that for equal voltages at both ends of the line, the reactive kv.a. must be 14,820 (current) lagging at no load, which must all be supplied by the synchronous condenser. At maximum load, the reactive kv.a. required are zero (*P.F.* = 1.00). The load supplies 22,000 kv.a., current lagging, therefore the load of the synchronous condenser would have to be 22,000 + 0 = 22,000 kv.a., current leading. This condition is not the most desirable. However, if the dotted curve marked $e_R$ = 97.2 per cent. of $e_G$ (constructed by interpolation) represents the actual relationship between the generator and receiver voltages synchronous condensers of an aggregate capacity of 18,500 kv.a. would suffice for the total load range from zero to maximum. The machines would be running under full-load lagging current when the power load is zero, and under full-load leading current when the power load is at the maximum. Therefore, for this particular line, with the generator voltage assumed as 160,000, the receiver voltage should be 155,500 for all loads. The voltage to neutral is $e_G$ = 92,380 volts; $e_R$ = 89,780 volts.

Using again formula (951) for this new voltage relation and solving for $i''_R$:

$i''_R = -718.3 + [674{,}150 - (100.42 + i'_R)^2]^{\frac{1}{2}}$ | $i''_R = -527.6 + [363{,}198 - (96.48 + i'_R)^2]^{\frac{1}{2}}$

—LINE *A*—

| $3\phi$ Load$_R$, kw. | $i'_R$ | $i''_R$ |
|---|---|---|
| 0 | 0 | 96.5 |
| 10,000 | 46.94 | 89.35 |
| 20,000 | 93.88 | 79.33 |
| 30,000 | 140.82 | 66.42 |
| 40,000 | 187.76 | 50.45 |
| 50,000 | 234.70 | 31.18 |
| 60,000 | 281.64 | 8.35 |
| 70,000 | 328.58 | −18.32 |

—LINE *B*—

| $3\phi$ Load$_R$, kw. | $i'_R$ | $i''_R$ |
|---|---|---|
| 0 | 0 | 67.3 |
| 10,000 | 37.13 | 60.1 |
| 20,000 | 74.26 | 50.4 |
| 30,000 | 111.39 | 38.1 |
| 40,000 | 148.52 | 23.0 |
| 50,000 | 185.65 | 4.9 |
| 60,000 | 222.78 | −16.5 |
| 70,000 | 259.91 | −41.6 |

With the knowledge of all load conditions at the receiver end, including the kv.a. of the synchronous condensers, the electrical conditions at the generator end may be computed. Of the conditions at the generator end, only the magnitude and frequency of the voltage are given. The computation carried out by the use of the voltage equation, therefore, is an effective check on the accuracy of the previous calculations.

Voltage at generator end with synchronous condensers operating at receiver end.

$$\dot{E}_G = e_R(a_1 - ja_2) + (i'_R + ji''_R)(d_1 - jd_2)$$

Zero load

$$\dot{E}_G = 68{,}850 - j294 + j96.5(12.74 - j94.07)$$
$$= 77{,}927 + j935$$

$$\dot{E}_G = 82{,}212 - j1{,}290 + j67.3(25.206 - j151.2)$$
$$= 92{,}380 + j406$$

10,000 kw.

$$\dot{E}_G = 68{,}850 - j294 + (46.94 + j89.35)(12.74 - j94.07)$$
$$= 77{,}853 - j3{,}571$$

$$\dot{E}_G = 82{,}212 - j1{,}290 + (37.13 + j60.1)(25.206 - j151.2)$$
$$= 92{,}235 - j5{,}389$$

| —LINE $A$— | —LINE $B$— |
|---|---|
| 20,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (93.88 + j79.33)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (74.26 + j50.4)(25.206 - j151.2)$ |
| $= 77{,}509 - j8{,}114$ | $= 91{,}705 - j11{,}246$ |
| 30,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (140.82 + j66.42)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (111.39 + j38.1)(25.206 - j151.2)$ |
| $= 76{,}893 = j12{,}696$ | $= 90{,}781 - j17{,}172$ |
| 40,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (187.76 + j50.45)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (148.52 + j23.0)(25.206 - j151.2)$ |
| $= 75{,}988 - j17{,}312$ | $= 89{,}434 - j23{,}168$ |
| 50,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (234.70 + j31.18)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (185.65 + j4.9)(25.206 - j151.2)$ |
| $= 74.773 - j21{,}975$ | $= 87{,}632 - j29{,}236$ |
| 60,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (281.64 + j8.35)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (222.78 - j16.5)(25.206 - j151.2)$ |
| $= 73{,}224 - j26{,}682$ | $= 85{,}332 - j35{,}389$ |
| 70,000 kw. | |
| $\dot{E}_G = 68{,}850 - j294 + (328.58 - j18.32)(12.74 - j94.07)$ | $\dot{E}_G = 82{,}212 - j1{,}290 + (259.91 - j41.6)(25.206 - j151.2)$ |
| $= 71{,}313 - j31{,}436$ | $= 82{,}473 - j41{,}639$ |

$$\dot{I}_G = e_R(c_1 - jc_2) + (i'_R + ji''_R)(a_1 - ja_2)$$

| —LINE $A$— | —LINE $B$— |
|---|---|
| Zero load | |
| $\dot{I}_G = 0.06 - j45.23 + j96.5(0.96959 - j0.0041387)$<br>$= 0.34 + j48.33$ | $\dot{I}_G = -0.48 - j96.54 + j67.3(0.91571 - j0.014365)$<br>$= 0.49 - j34.92$ |
| 10,000 kw. | |
| $\dot{I}_G = -0.06 - j45.23 + (46.94 + j89.35)(0.96959 - j0.0041387)$<br>$= 45.84 + j41.21$ | $\dot{I}_G = -0.48 - j96.54 + (37.13 + j60.1)(0.91571 - j0.014365)$<br>$= 34.38 - j42.04$ |
| 20,000 kw. | |
| $\dot{I}_G = -0.06 - j45.23 + (93.88 + j79.33)(0.96959 - j0.0041387)$<br>$= 91.29 + j31.30$ | $\dot{I}_G = -0.48 - j96.54 + (74.26 + j50.4)(0.91571 - j0.014365)$<br>$= 68.24 - j51.46$ |
| 30,000 kw. | |
| $\dot{I}_G = -0.06 - j45.23 + (140.82 + j66.42)(0.96959 - j0.0041387)$<br>$= 136.75 + j18.59$ | $\dot{I}_G = -0.48 - j96.54 + (111.39 + j38.1)(0.91571 - j0.014365)$<br>$= 102.07 - j63.25$ |
| 40,000 kw. | |
| $\dot{I}_G = -0.06 - j45.23 + (187.76 + j50.45)(0.96959 - j0.0041387)$<br>$= 182.18 + j2.91$ | $\dot{I}_G = -0.48 - j96.54 + (148.52 + j23.0)(0.91571 - j0.014365)$<br>$= 135.85 - j77.61$ |
| 50,000 kw. | |
| $\dot{I}_G = -0.06 - j45.23 + (234.7 + j31.18)(0.96959 - j0.0041387)$<br>$= 227.62 - j15.97$ | $\dot{I}_G = -0.48 - j96.54 + (185.65 + j4.9)(0.91571 - j0.014365)$<br>$= 169.59 - j94.72$ |

| —LINE $A$— | —LINE $B$— |
|---|---|
| | 60,000 kw. |
| $\dot{I}_G = -0.06 - j45.23 + (281.64 + j8.35)$ | $\dot{I}_G = -0.48 - j96.54 + (222.78 - j16.5)$ |
| $(0.96959 - j0.0041387)$ | $(0.91571 - j0.014365)$ |
| $= 273.05 - j38.30$ | $= 203.28 - j114.85$ |
| | 70,000 kw. |
| $\dot{I}_G = -0.06 - j45.23 + (328.58 - j18.32)$ | $\dot{I}_G = -0.48 - j96.54 + (259.91 - j41.6)$ |
| $(0.96959 - j0.0041387)$ | $(0.91571 - j0.014365)$ |
| $= 318.43 - j64.35$ | $= 236.92 - j138.36$ |

—LINE A—

Generator Voltage 135,000—Receiver Voltage 123,000 Load $P.F._R$ = 0.85 Lagging

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| KW. receiver 3$\phi$ load $P_R \times 3$ | 0 | 10,000 | 20,000 | 30,000 | 40,000 | 50,000 | 60,000 | 70,000 |
| $P_R = e_R i_R \times P.F._R$ | 0 | 3,333 | 6,667 | 10,000 | 13,333 | 16,667 | 20,000 | 23,333 |
| $i'_R$ | 0 | 46.94 | 93.88 | 140.82 | 187.76 | 234.70 | 281.64 | 328.58 |
| $i''_R$ | 96.50 | 89.35 | 79.33 | 66.42 | 50.45 | 31.18 | 8.35 | −18.32 |
| $i_R = [(i'_R)^2 + (i''_R)^2]^{1/2}$ | 96.50 | 100.93 | 122.90 | 155.70 | 194.41 | 236.74 | 281.78 | 329.09 |
| $i''_{load}$ | 0 | 29.11 | 58.22 | 87.33 | 116.44 | 145.55 | 174.67 | 203.77 |
| $i''_{s.c.} = i''_R - i''_{load}$ | 96.50 | 60.24 | 21.11 | −20.92 | −65.99 | −114.37 | −166.32 | −222.09 |
| $\dot{E}_G = e'_G - je''_G$ | 77,927 | 77,853 | 77,509 | 76,893 | 75,988 | 74,773 | 73,224 | 71,313 |
| | +$j$935 | −$j$3,571 | −$j$8,114 | −$j$12,696 | −$j$17,312 | −$j$21,975 | −$j$26,682 | −$j$31,436 |
| $e_G = [(e'_G)^2 + (e''_G)^2]^{1/2}$ | 77,940 | 77,940 | 77,940 | 77,940 | 77,940 | 77,940 | 77,940 | 77,940 |
| $E_G = \sqrt{3} e_G$ | 135,000 | 135,000 | 135,000 | 135,000 | 135,000 | 135,000 | 135,000 | 135,000 |
| $\dot{I}_G = i'_G - ji''_G$ | 0.34 | 45.84 | 91.29 | 136.75 | 182.18 | 227.62 | 273.05 | 318.43 |
| | +$j$48.33 | +$j$41.21 | +$j$31.30 | +$j$18.59 | +$j$2.91 | −$j$15.97 | −$j$38.30 | −$j$64.35 |
| $i_G = [(i'_G)^2 + (i''_G)^2]^{1/2}$ | 48.33 | 61.68 | 96.50 | 138.00 | 182.20 | 228.19 | 275.72 | 324.87 |
| $P_G = e'_G i'_G + e''_G i''_G$ | 72 | 3,421 | 6,822 | 10,278 | 13,783 | 17,371 | 21,014 | 24,731 |
| $P_G \times 3$ | 216 | 10,263 | 20,466 | 30,834 | 41,349 | 52,113 | 63,042 | 74,193 |
| $Q_G = e_G i_G$ | 3,767 | 4,807 | 7,521 | 10,755 | 14,200 | 17,783 | 21,490 | 25,320 |
| $Q_G \times 3$ | 11,301 | 14,421 | 22,563 | 32,265 | 42,600 | 53,349 | 64,470 | 75,960 |
| $P.F._G = P_G \div Q_G$ | −0.019 | −0.712 | −0.907 | −0.956 | −0.971 | −0.977 | −0.978 | +0.970 |
| Synchronous condenser load $e_R \times i''_{s.c.} \times 3$ kv.a | 20,560 | 12,833 | 4,497 | −4,457 | −14,058 | −24,368 | −35,435 | −47,310 |
| Resultant $P.F._R$ = cos $[\tan^{-1}(i''_R \div i'_R)]$ | 0.00 | 0.580 | 0.776 | 0.915 | 0.970 | 0.993 | 1.00 | −0.964 |
| Line loss, in per cent | 100.0 | 2.6 | 2.3 | 2.7 | 3.3 | 4.1 | 4.8 | 5.7 |

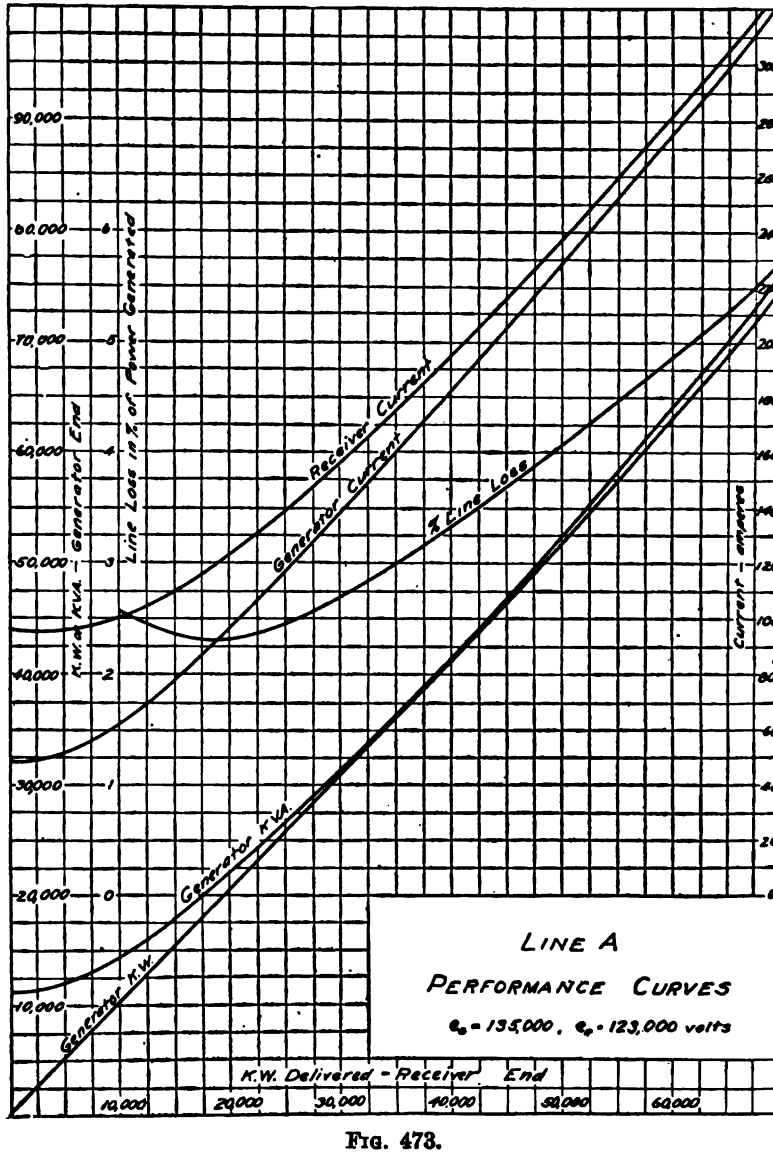

90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
K.W. & K.V.A. - Generator End
6
5
4
3
2
1
0
Line Loss in % of Power Generated
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
Current - amperes
Receiver Current
Generator Current
% Line Loss
Generator K.V.A.
Generator K.W.
LINE A
PERFORMANCE CURVES
e₀ = 135,000, e₂ = 123,000 volts
K.W. Delivered - Receiver End
10,000
20,000
30,000
40,000
50,000
60,000

FIG. 473.

—LINE *B*—

Generator Voltage 160,000—Receiver Voltage 155,500
Load $P.F._R$ = 0.90 Lagging

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| KW. receiver 3$\phi$ load $P_R \times 3$ | 0 | 10,000 | 20,000 | 30,000 | 40,000 | 50,000 | 60,000 | 70,000 |
| $P_R = e_R i_R \times P.F._R$ | 0 | 3,333 | 6,667 | 10,000 | 13,333 | 16,667 | 20,000 | 23,333 |
| $i'_R$ | 0 | 37.13 | 74.26 | 111.39 | 148.52 | 185.65 | 222.78 | 259.91 |
| $i''_R$ | +67.30 | +60.10 | +50.40 | +38.10 | +23.00 | +4.90 | −16.50 | −41.60 |
| $i_R = [(i'_R)^2 + (i''_R)^2]^{1/2}$ | 67.30 | 70.57 | 89.74 | 117.73 | 150.29 | 185.71 | 223.40 | 263.22 |
| $i''_{load}$ | 0 | +17.98 | +35.96 | 53.95 | 71.93 | 89.91 | 107.88 | 125.86 |
| $i''_{s.c.} = i''_R - i''_{load}$ | +67.30 | +42.12 | +14.44 | −15.85 | −48.93 | −85.01 | −124.38 | −167.46 |
| $E_G = e'_G - je''_G$ | 92,380 | 92,235 | 91,705 | 90,781 | 89,434 | 87,632 | 85,332 | 82,473 |
| | $+j406$ | $-j5,389$ | $-j11,246$ | $-j17,172$ | $-j23,168$ | $-j29,236$ | $-j35,389$ | $-j41,639$ |
| $e_G = [(e'_G)^2 + (e''_G)^2]^{1/2}$ | 92,380 | 92,380 | 92,380 | 92,380 | 92,380 | 92,380 | 92,380 | 92,380 |
| $E_G = e_G\sqrt{3}$ | 160,000 | 160,000 | 160,000 | 160,000 | 160,000 | 160,000 | 160,000 | 160,000 |
| $I_G = i'_G - ji''_G$ | 0.49 | 34.38 | 68.24 | 102.07 | 135.85 | 169.59 | 203.28 | 236.92 |
| | $-j34.92$ | $-j42.04$ | $-j51.46$ | $-j63.25$ | $-j77.61$ | $-j94.72$ | $-j114.85$ | $-j138.36$ |
| $i_G = [(i'_G)^2 + (i''_G)^2]^{1/2}$ | 34.93 | 54.31 | 85.47 | 120.08 | 156.45 | 194.25 | 233.48 | 274.36 |
| $P_G = e'_G i'_G + e''_G i''_G$ | 31 | 3,398 | 6,836 | 10,352 | 13,948 | 17,629 | 21,408 | 25,301 |
| $P_G \times 3$ | 93 | 10,194 | 20,508 | 31,056 | 41,844 | 52,887 | 64,224 | 75,903 |
| $Q_G = e_G i_G$ | 3,227 | 5,017 | 7,895 | 11,091 | 14,451 | 17,944 | 21,568 | 25,344 |
| $Q_G \times 3$ | 9,681 | 15,051 | 23,685 | 33,273 | 43,353 | 53,832 | 64,704 | 76,032 |
| $P.F._G = P_G \div Q_G$ | −0.0096 | −0.677 | −0.856 | −0.933 | −0.965 | −0.983 | −0.993 | −0.999 |
| Synchronous condenser load, $e_R \times i''_{s.c.} \times 3$ kv.a. | +18,125 | +11,343 | +3,889 | −4,269 | −13,178 | −22,895 | −33,500 | −45,100 |
| Resultant $P.F._R$ $= \cos[\tan^{-1}(i''_R \div i'_R)]$ | 0.00 | +0.525 | +0.830 | +0.946 | +0.987 | 1.00 | −0.997 | −0.986 |
| Line loss, per cent | 100.0 | 1.93 | 2.48 | 3.40 | 4.41 | 5.46 | 6.58 | 7.78 |

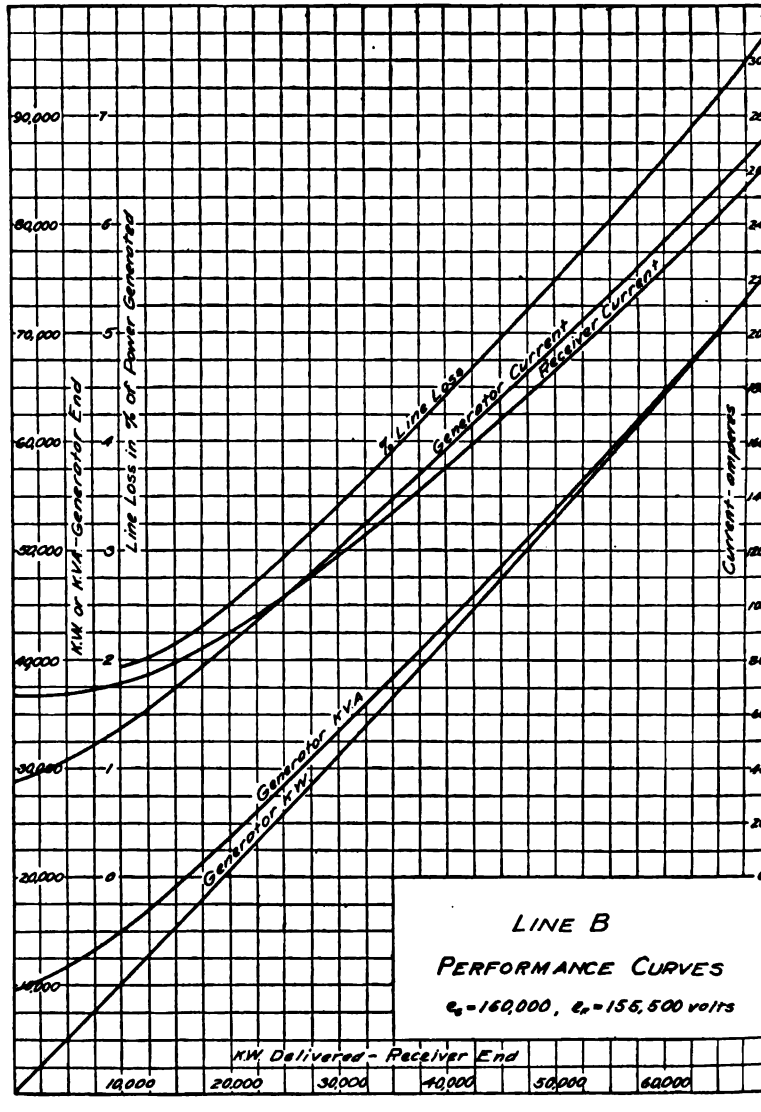

KW. or KVA-Generator End
Line Loss in % of Power Generated
% Line Loss
Generator Current
Receiver Current
Current-amperes
Generator KVA
Generator KW.
KW. Delivered - Receiver End
10,000
20,000
30,000
40,000
50,000
60,000
LINE B
PERFORMANCE CURVES
$e_g$ = 160,000, $e_r$ = 155,500 volts

FIG. 474.

(*g*) **Computation of Performance Curves for Line C.**—The specifications for Line C are the same as for Line B, except that it is twice as long. The computation illustrates the use of the polar notation in the solution of transmission-line problems.

For Line C:

$$r = 2 \times 26.74 = 53.5 \text{ ohms}$$
$$x = 2 \times 155.5 = 311.0 \text{ ohms}$$
$$z = 2 \times 157.78 = 315.6 \text{ ohms}$$
$$Z = 315.6 \angle 279.6°$$
$$b = 2 \times 1.1068 \times 10^{-3} = 2.214 \times 10^{-3} \text{ mho.}$$
$$Y = 2.214 \times 10^{-3} \angle 270.0°$$
$$\alpha = 2 \times 0.03552 = 0.07104 \text{ radian}$$
$$\beta = 2 \times 0.41638 = 0.8328 \text{ radian}$$
$$= 47.72°$$
$$\cosh \alpha = 1.0025$$
$$\sinh \alpha = 0.0711$$
$$\cos \beta = 0.6727$$
$$\sin \beta = 0.7398$$

The slide rule is considered as sufficiently accurate for the following computations:

$$\cosh (\alpha - j\beta) = \cosh \alpha \cos \beta - j \sinh \alpha \sin \beta$$
$$= 0.674 - j0.0526$$
$$= 0.676 \angle 355.5°$$
$$\sinh (\alpha - j\beta) = \sinh \alpha \cos \beta - j \cosh \alpha \sin \beta$$
$$= 0.0478 - j0.742$$
$$= 0.743 \angle 273.7°$$
$$\sqrt{Y/Z} = \sqrt{\frac{2.214 \times 10^{-3}}{315.6}} \angle \frac{270.0° - 279.6°}{2}$$
$$= 2.645 \times 10^{-3} \angle 355.2°$$
$$\sqrt{Z/Y} = 378 \angle 4.8°$$
$$a'_1 - ja'_2 = 0.676 \angle 355.5°$$
$$c'_1 - jc'_2 = (2.645 \times 10^{-3} \angle 355.2°)\ (0.743 \angle 273.7°)$$
$$= 1.967 \times 10^{-3} \angle 268.9°$$
$$d'_1 - jd'_2 = (378 \angle 4.8°)\ (0.743 \angle 273.7°)$$
$$= 281 \angle 278.5°$$

The complete equations for the line are therefore:

$$\dot{I}_G = (0.676 \angle 355.5°)\dot{I}_R + (1.967 \times 10^{-3} \angle 268.9°)\dot{E}_R$$
$$\dot{E}_G = (0.676 \angle 355.5°)\dot{E}_R + (281 \angle 278.5°)\dot{I}_R$$
$$\dot{I}_R = (0.676 \angle 355.5°)\dot{I}_G - (1.967 \times 10^{-3} \angle 268.9°)\dot{E}_G$$
$$\dot{E}_R = (0.676 \angle 355.5°)\dot{E}_G - 281 (\angle 278.5°)\dot{I}_G$$

In order to show the method of evaluating with numeric $E$ and $I$ consider the following problem:

What are the vector values of current and voltage at each end of the line when power factor control is used to maintain constant voltage (92380 to neutral) at both ends of the line? What are the power factors, line efficiency, etc.?

Select $\dot{E}_G = 92380$ as the reference vector.

Then $\dot{E}_R = 92380 \underline{/\psi}$ where $\psi$ is the angle of phase displacement with respect to the reference axis.

Substitute in the equations for $\dot{E}_G$ and $\dot{E}_R$ above and

$$92380 = (0.676 \underline{/355.5^\circ})(92380 \underline{/\psi}) + (281 \underline{/278.5^\circ})\dot{I}_R$$

and $$92380 \underline{/\psi} = (0.676 \underline{/355.5^\circ})(92380) - (281 \underline{/278.5^\circ})\dot{I}_G$$

From which $$\dot{I}_R = 329 \underline{/81.5^\circ} - 222 \underline{/77.0^\circ + \psi}$$

and $$\dot{I}_G = 222 \underline{/77.0^\circ} - 329 \underline{/81.5^\circ + \psi}$$

It will be noted that the loci of these currents are circles, that of $\dot{I}_R$ with its center at $329 \underline{/81.5^\circ}$ amperes, and that of $\dot{I}_G$ with its center at $222 \underline{/77.0^\circ}$ amperes from the origin.

An easy method of solution is to assume values for $\psi$ and solve for $\dot{I}_R$ and $\dot{I}_G$. When these are obtained, power, power factor, etc., may be readily calculated.

The figures for $\psi = 60.0^\circ$ follow:

$$\dot{I}_R = 329 \underline{/81.5^\circ} - 222 \underline{/77.0^\circ + 60.0^\circ}$$
$$= 48.6 + j325 + 162 - j151$$
$$= 210.6 + j174 = 274 \underline{/39.6^\circ}$$
$$\dot{E}_R = 92380 \underline{/60.0^\circ}$$

therefore $\theta_R = 20.4^\circ$ leading, cos $\theta_R = 0.937$ and sin $\theta_R = 0.349$

$$P_R = (92380 \times 274 \times 0.937)/1000$$
$$= 23700 \text{ kw. per phase}$$
$$\dot{I}_G = 222 \underline{/77.0^\circ} - 329 \underline{/81.5^\circ + 60.0^\circ}$$
$$= 50 + j216 + 258 - j205$$
$$= 308 + j9 = 308 \underline{/1.7^\circ} \text{ (lagging)}$$

$$\cos \theta_G = 0.9999, \sin \theta_G = 0.0297$$
$$P_G = (92380 \times 308 \times 0.9999)/1000$$
$$= 28450 \text{ kw. per phase}$$

Line efficiency $= 23700/28450 = 0.832 = 83.2$ per cent.

Solutions for other points may be obtained in the same way. If complete performance curves are desired it is convenient to plot curves of scalar values of current and their respective phase positions, and power as found above against $\psi$.

These are given in Fig. 475 for values of $\psi$ from 0° to 90° for Line C. Interpolated values from these curves may now be

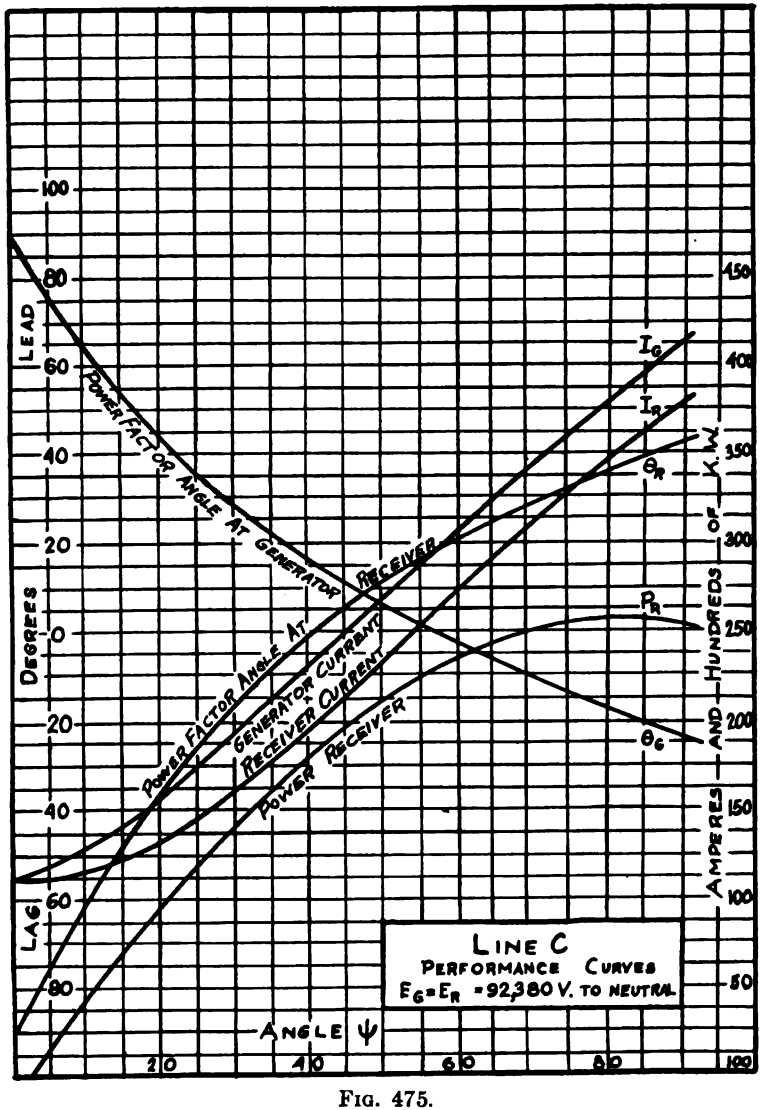

LEAD
DEGREES
LAG
100
80
60
40
20
0
20
40
60
80
POWER FACTOR ANGLE AT GENERATOR
POWER FACTOR ANGLE AT RECEIVER
GENERATOR CURRENT
RECEIVER CURRENT
POWER RECEIVER
$I_G$
$I_R$
$\theta_R$
$P_R$
$\theta_G$
ANGLE $\psi$
10
30
50
70
AMPERES — AND — HUNDREDS OF K.W.
150
200
250
300
350
400
100
150
50
LINE C
PERFORMANCE CURVES
$E_G = E_R$ = 92,380 V. TO NEUTRAL

FIG. 475.

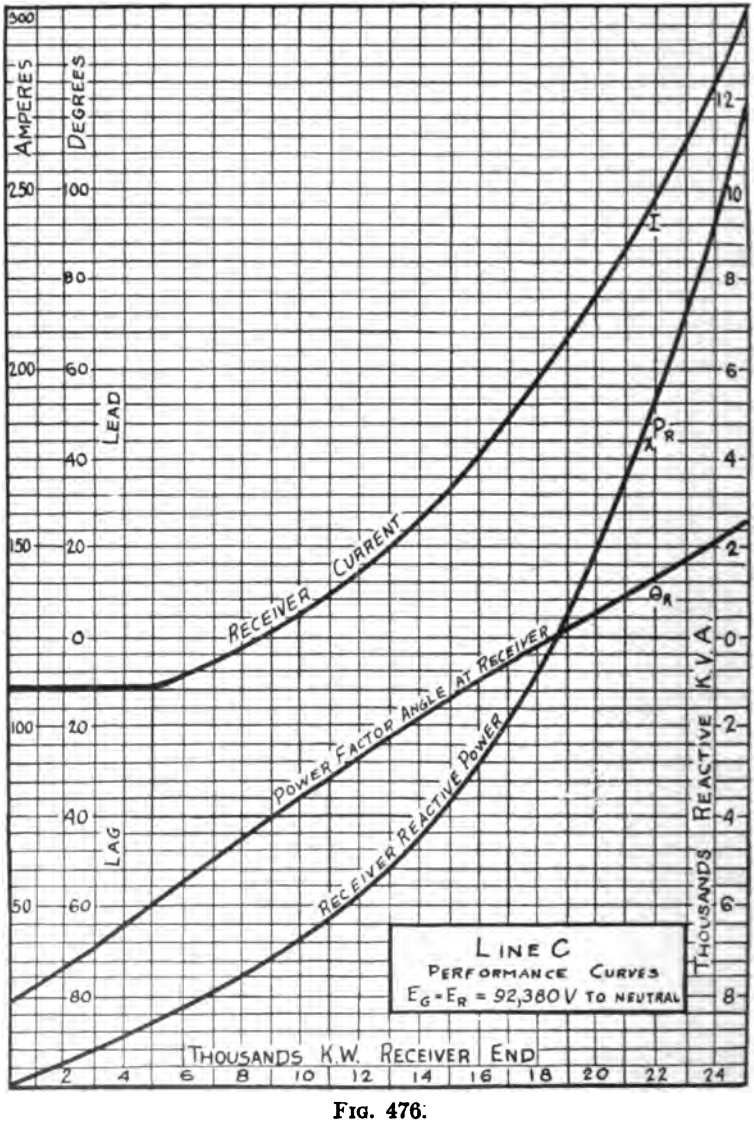

AMPERES
DEGREES
LEAD
LAG
RECEIVER CURRENT
POWER FACTOR ANGLE AT RECEIVER
RECEIVER REACTIVE POWER
THOUSANDS REACTIVE K.V.A.
THOUSANDS K.W. RECEIVER END
LINE C
PERFORMANCE CURVES
$E_G$-$E_R$ = 92,380 V TO NEUTRAL
I
$I_R$
$\theta_R$

FIG. 476.

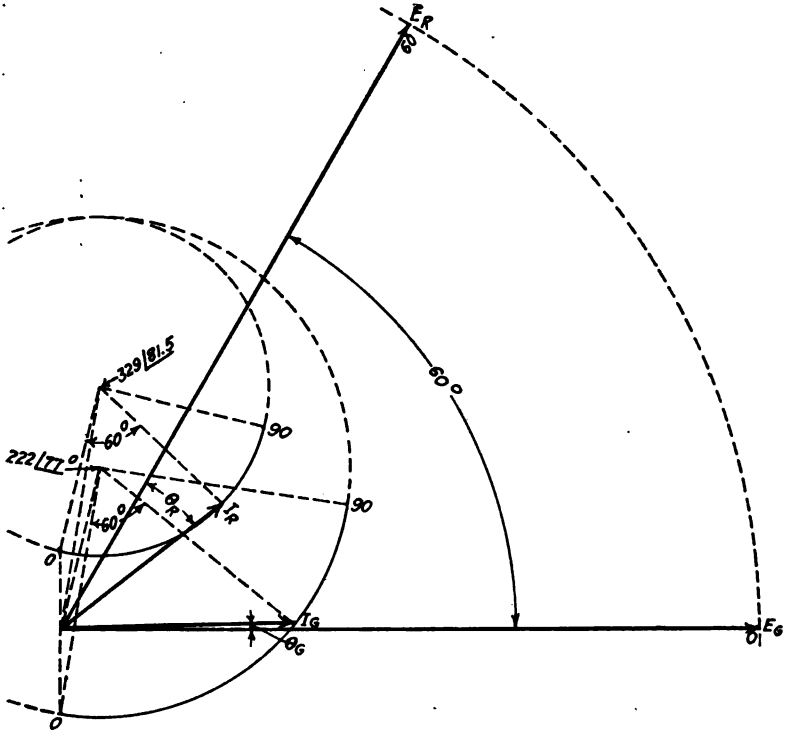



FIG. 477.—Vector diagram showing loci of currents and voltage for "Line C." Full lines for $\psi = 60°$.

used in plotting curves such as those in Fig. 476 against received power.

Vector diagrams showing the loci of $\dot{I}_R$, $\dot{E}_R$, and $\dot{I}_G$ are shown in Fig. 477. The full lines indicate the values for $\psi = 60.0°$.

(*h*) **Artificial Transmission Lines.**—It has been shown in Chaps. XXI and XXVII that the operating characteristics of transmission lines are determined by the electrical constants of the line; the resistance, inductance and condensance. The length of the line and the size and spacing of the conductors are merely factors in the equations giving the magnitude of the electrical constants of the line. The distance or space feature, therefore, is not an essential factor in the electrical phenomena. The operating characteristics can be accurately reproduced in laboratory structures occupying comparatively small space. The artificial transmission line at Union College,[1] housed in a small room, is electrically equivalent to 130 miles of an actual line of No. 1 A.W.G. copper with 72-in. spacing. The line has 400 units. Each unit consists of a glass cylinder 6¼ in. in diameter and 5 ft. long. The inside of the tube is lined with tin-foil, and on the outside is wound a helix of No. 8 A.W.G. copper wire. This arrangement gives uniformly distributed resistance, inductance and condensance. The total resistance for the 400 units is 93.6 ohms; the inductance, 0.394 henry; the condensance, 1.135 microfarads. Electrically the artificial line has the same constants as the actual 130-mile transmission line, and experiments have shown that the operating characteristics of the actual line can be accurately reproduced in the laboratory apparatus.

In the *lumpy* type, a less expensive and more compact design, the uniformly distributed inductance and condensance are represented by a number of small inductance coils and condensers, connected as indicated in Fig. 468.

Some of Dr. A. E. Kennelly's researches on long transmission lines were made on a lumpy, artificial line[2] equivalent to 1,500 miles of No. 3/0 A.W.G. aluminum cable, spaced 90.5 in. The line had 30 units, each consisting of four small air-core solenoids and a condenser. Although each unit was equivalent to 50 miles of line, it occupied less than 1 cu. ft. of space.

[1] *Trans.* A.I.E.E., XXX, 245; XXXI, 887.

[2] *Trans.* A.I.E.E., XXXI, 1131; *Elec. World*, Aug. 10, 1912; June 14, 1914; Aug. 8, 1914.

As the lumpy type only approximates a uniform distribution of the line resistance, inductance and condensance, the size of each unit must be small in comparison to the total length of the line.

In Fig. 478 is shown a 10-mile unit of the lumpy type having adjustable line constants. The artificial transmission line at the University of Washington[1] has 20 units similar to Fig. 478. The electrical constants of each unit are adjustable within the following limits:

Resistance, minimum value = 2.59 ohms.

Inductance, maximum value = 0.021 henry.

Condensance, from 0.1 to 1.0 microfarad.

The resistance may be increased to any desired amount by moving the clamp on the resistance loop or by inserting resistance



FIG. 478.—One unit of 200-mile Artificial transmission line. (*University of Washington.*)

elements between the units; the inductance may be decreased by turning the right-hand coil and by taps in the lower coil; and the condensance may be varied in steps by using ten or a less number of condensers in series. Adjustments can be made so as to give to the artificial line the electrical constants equivalent to an actual transmission line of any size of wire up to No. 4/0 A.W.G. hard-drawn copper and for any spacing up to 120 in.

For Experiments see page 553.

[1] *Trans.* A.I.E.E., XXXV, p. 1137.

# EXPERIMENTS

Experiments were prepared and arranged by my Colleague, Assistant Professor Leslie F. Curtis.

## Experiments to Chapter II

The oscillograph is the most satisfactory instrument for the study of wave forms. (See Chap. VIII.) The student should become thoroughly familiar with its principles and the interpretation of oscillograms. In the following experiment the work is intended to give a conception of the successive instantaneous values of alternating current and voltage, and the angular phase displacement between waves when these instantaneous values are considered to be recurrent harmonic functions. For these experiments it is desirable to use power from an approximately sine-wave voltage supply.

Note that the oscillograph vibrators are essentially sensitive D'Arsonval galvanometers and are used with multipliers to indicate voltage and with shunts to indicate current. The direction of the deflection on the ground glass or film indicates the sign of the instantaneous value of the wave being measured. The vibrators should be calibrated with the multiplier or shunts as used in volts or amperes per millimeter deflection.

**1. Alternating Waves with the Oscillograph.**—Obtain tracings of simultaneous values of alternating current and voltage when the circuit measured contains:

(*a*) resistance
(*b*) inductance
(*c*) condensance
(*d*) resistance and inductance
(*e*) resistance and condensance
(*f*) inductance and condensance (protected by resistance on the supply side of the meters)

Letter or number the points between the units and record voltages as $E_{ab}$ or $E_{12}$, etc., indicating that the direction from *a* to *b* or from 1 to 2 is an assumed positive direction around the circuit. When more than one device is connected obtain voltage waves across each as well as of the total. Show that the algebraic sum of the instantaneous values of the component voltages is the same as the total voltage.

Calculate the maximum and effective values of each wave from the tracings and check the latter with meter readings.

Interpret the spacing between waves in terms of time and angular phase displacement.

## Experiments to Chapter IV

The object of the following experiments is to show the relations between the effective values of current and voltage in simple series circuits of constant impedance and to give methods of calculating the constants of these circuits.

The supply voltage should be as nearly a sine wave as possible and of constant frequency. The resistances used should have negligible temperature coefficients within the range of current used and the inductances should have air cores. Fairly large currents (5 amperes or more) are desirable so that voltmeter currents will be negligible.

**1. Resistance and Inductance.**—Connect a non-inductive resistance in series with a coil having a reactance of not less than half or more than twice the value of the resistance in ohms. Letter the terminals and the junction between the units to correspond to a recorded diagram. Place meters for determining the component and total voltages, the current and frequency.

Tabulate several sets of readings for different values of current. Record voltages $E_{ab}$, $E_{bc}$, etc., indicating the points between which the readings were taken and the assumed positive directions around the circuit by the subscripts. In laboratory work it is desirable to use the notation with distinctive subscripts, as, for example, $E_{ab}$, $Z_{ab}$, etc., corresponding to the circuit diagram until the final interpretation of results.

Calculate the component and total impedances from the voltmeter and ammeter readings, find the mean values and plot the impedance triangle for the circuit. Letter the vertices of the triangle to correspond to the letters on the circuit diagram.

The value of the non-inductive resistance is the scaled value between the letters assigned to the terminals of that unit as plotted. The resistance of the coil may be found from the component of the impedance of the coil parallel to the non-inductive resistance. Check the scaled values of these resistances on the impedance diagram with the resistances between the same points when measured by direct current.

The reactance of the coil is the component of the coil impedance at right angles to the resistance line. From the reactance and frequency calculate the inductance of the coil and check the resultant figure with that calculated from the dimensions of the coil.

Express the impedance of the circuit and its component parts in the form of $r \pm jx$.

Draw a current and voltage vector diagram for the circuit for a current of 10 amperes.

**2. Resistance and Condensance.**—As far as is practicable follow the procedure of Experiment 1, using a condensance in place of the inductance.

## Experiments to Chapter V

The object of the following experiments is to show the relations between the effective values of current and voltage in parallel circuits of constant admittance. The same precautions regarding wave form should be observed as in the experiments on series circuits.

**1. Resistance and Inductance.**—Connect a non-inductive resistance in parallel with a coil having a reactance of not less than half or more than twice the value of the resistance in ohms. Place meters to determine the component and total currents, the voltage and frequency.

Tabulate several sets of readings in a manner similar to that used for series circuits using distinctive subscripts for the current in each case.

Calculate the component and total admittances from the meter readings and plot the admittance triangle. From the diagram scale the component and total conductances and the susceptance. Check the component values with those calculated from the resistance and reactance data.

Express the component conductances and susceptances and total admittances in the form of $g \mp jb$.

Draw a current and voltage vector diagram for the circuit at 100 volts.

**2. Resistance and Condensance.**—As far as is practicable follow the procedure of Experiment 1, using a condensance in place of an inductance.

## Experiments to Chapter VI

While the experiments outlined below are particularly adapted to the study of series-parallel circuits, the methods employed are suitable for routine commercial tests. The student should master the checks afforded by wattmeter measurement upon the phase angles between each of the component currents and the voltages.

**1. Series-parallel Circuits.**—Take any complex circuit such as that in Fig. 72. Letter the branch points and if desired number the units as shown. Measure the currents in all of the branches. Measure the voltages between all of the lettered points.

With a wattmeter having its current coil in the supply lead and with one side of its potential coil connected to the same side of the line take readings when the "high" side of the potential coil is tapped to all of the branch points. Change the current coil to one of the branch circuits and repeat.

Plot a topographic voltage diagram, the distances between points on the plot being proportional to the voltages between the points in the circuit. Locate the phase angles of the current vectors by means of the wattmeter, voltmeter and ammeter readings where $W = EI \cos \theta$. (See Chap. VII.) (Note that $\theta$ is not necessarily a power factor angle as $E$ may be only a portion of the total voltage.) Complete the current diagram by plotting all of the ammeter readings, observing Kirchoff's law for currents. If desired the vectors indicated in the topographic plot may be transferred to a vector diagram with a common origin for the coordinates. The phase angles and lengths will be the same in either plot.

**2. Circle Diagram with Resistance Variable.**—Arrange meters for determining the vector relations and the constants of a series circuit containing resistance and reactance. Keep the supply voltage, frequency and reactance fixed and vary the resistance through as wide a range as possible.

Show the loci of the component voltages and current by vector diagram using the total voltage as the reference vector.

Tabulate and plot to rectangular coordinates all of the readings and calculated constants of the circuit with the resistances as abscissæ.

**3. Circle Diagram with Reactance Variable.**—Repeat Experiment 2 with reactance variable and the resistance constant.

If possible keep the resistance of the reactance coil constant by keeping the total amount of wire in series the same for all readings.

**4. Voltage Resonance with Variable Frequency.**—Voltage resonance is obtained in a series circuit when the inductive and condensive reactances neutralize, thus making the total voltage equal to the resistance drop. A protective resistance in series with the circuit may be necessary to prevent excessive voltage across the condensance at resonance.

Adjust the inductance and condensance so that resonance may be obtained within the available range of frequency. Hold the voltage across the two units in series at a constant value (as high as is safe for the condensance at resonance) and take readings of component voltages, current and their phase relations for a wide range of frequency.

Calculate the resistance, reactance, impedance and power factor for the component and total circuits.

Show the loci of the voltages and current by vector diagram using the total voltage as the reference vector.

Plot curves in rectangular coordinates showing the variation of current, voltages, reactance, impedance and power factor with frequencies as abscissæ.

**5. Current Resonance with Variable Frequency.**—Current resonance is obtained in a parallel circuit when the inductive and condensive susceptances neutralize to give a minimum line current.

Adjust an inductance in parallel with a condensance so that resonance may be obtained within the available range of frequency. Hold the voltage across the two units at a constant value and take readings to obtain component and total currents, voltage and their phase relations for a wide range of frequency.

Calculate the conductance, susceptance and admittance for the component and complete circuits. Plot in rectangular coordinates the currents and the constants of the circuits with frequencies as abscissæ.

Show the loci of the currents by vector diagram using the voltage as the reference vector.

## Experiments to Chapter VII

**1. Wattmeter in Single-phase Measurements.**—(*a*) Measure the current and power input with an ammeter and wattmeter

in several of the circuits used in previous experiments. Check the resistance as calculated by other methods with $P/I^2$. Calculate the power factors and the reactive volt-amperes.

(*b*) Show how to correct for errors introduced by losses in the current and potential coils of the wattmeter.

(*c*) Show how to correctly connect a wattmeter provided with a compensating coil.

(*d*) Connect an ammeter, wattmeter and voltmeter to a load and determine its power factor. Leaving the voltmeter connected across the potential coil of the wattmeter, connect a 2-microfarad telephone condenser between these two potential circuits in parallel and the "high" side of the load. Take the new readings of the meters.

Show that the wattmeter reads $EI \cos \theta$ where $E$ is the voltage across the potential coil, $I$ is the current through the series coil, and $\theta$ is the phase displacement of these vectors. In making the calculations, assume that the potential circuits of the voltmeter and wattmeter are non-inductive. Show how this method[1] may be used to determine whether a current lags or leads.

## Experiment to Chapter VIII

**1. Indicating Instruments.**—The object of the following experiment is to show how to correct for the errors introduced by the instruments.

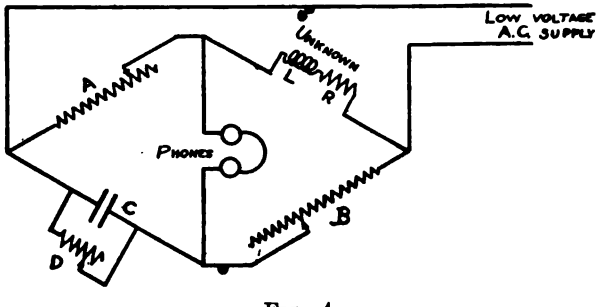



FIG. *A*.

(*a*) Determine for several types of alternating current instruments the effect of stray fields, the errors when used on direct current circuits and the methods of eliminating the above.

[1] *Jour. of Elec.*, June 1, 1919, p. 530.

(*b*) Determine the resistance and inductance of a voltmeter, ammeter and wattmeter suitable for measuring small amounts of power at low power factors. An impedance bridge is convenient for this measurement. A convenient diagram for this is shown in Figure *A*.

When no sound is heard in the telephone receiver,

$$R = \frac{AB}{D} \text{ and } L = ABC$$

(*c*) Show by measurement and calculation how readings of the meters measured above should be corrected. Simultaneous readings should be taken of all the instruments, the coils of which are to be treated like fundamental circuits with known constants. A careful diagram of connections must be made to show whether the instruments are in series or parallel with each other and the load. Vector diagrams should be drawn.

## Experiments to Chapter IX

The phase relations between voltages in polyphase circuits, especially if from meter readings, are best indicated by a topographic diagram. The current relations are best indicated by a current triangle for each branch point, the phase angles being checked against the voltage diagram.

An important consideration of these vector diagrams is the phase sequence or the order in which the individual vectors reach a maximum. It is possible to make two plots from voltmeter readings, one the reverse of the other, only one of which is correct. It is usually necessary to add to the system a vector of known phase position and include it in the diagram before the correct plot can be selected.

The following experiments are typical of a large number which may be arranged to show the interrelations of polyphase voltage and current vectors.

**1. Delta-, and Star-connected Loads.**—(*a*) Make measurements of line currents, line and phase voltages, and phase rotation, with star-connected non-inductive load resistances for a general case (unequal adjustments of load resistances). Make a topographic plot of the voltage vectors, being careful to show the correct phase rotation. On the same sheet make a current triangle from the ammeter readings assuming that the phase currents and voltage are in time-phase.

Predict what will happen to the shape of the diagram and check the result by actual measurement; when one leg of the star is short circuited, when one leg is open circuited, and when all legs are of equal resistance.

(*b*) Make measurements of line voltages and line and phase currents with a delta-connected set of unequal load resistances and make a complete voltage and current diagram, as in (*a*).

Predict what will happen to the shape of the plot and check by meter readings; when one leg is open circuited, when two legs are open circuited, and when all legs are of equal resistance.

(*c*) Show that for balanced resistances and line voltages, the voltage to neutral is $1/\sqrt{3}$ times the line voltage for a star connection and the phase current is $1/\sqrt{3}$ times the line current for a delta connection. Show that for the same line voltages and currents the vector diagram for these vectors is independent of the type of connection.

**2. Polyphase Windings.**—It is desirable to have for this experiment an alternator with six or more separate circuits each giving the same effective voltage but differing in phase position. A six pole alternator having, for example, seventy-two slots with a distributed winding may be easily modified to provide twelve separate circuits of the same number of turns but differing in phase position by one slot pitch. A smaller number may be used but it will be noted that to produce three-phase connections a multiple of three coils must be used and that for two-phase connections a multiple of two.

Number or letter all of the terminals and determine the effective value of the voltage generated by each of the separate coils with definite field current and speed. Inter-connect the terminals from one end of each coil, making a careful record of the numbers so connected. Measure the voltages between all of the other terminals. It should be noted that, if twelve separate coils are used, a total of seventy-eight separate readings may be obtained.

Connect between two of the terminals which are at a decided difference of potential a phase splitting device such as that shown in Fig. 117 and take a sufficient number of readings to determine the phase sequence of the original vectors.

Plot a topographic diagram of the voltages as read, numbering the points to correspond to the interconnection used. Include the readings taken with the phase splitting device.

From this diagram show what connections may be made to produce symmetrical two-phase three-wire, three-phase delta, three-phase star, and single-phase windings, using all of the coils in such a manner that the maximum voltage combinations are obtained. Check these calculated results by meter readings at no-load.

Assume current ratings for the coils and calculate the relative volt-ampere ratings of the complete machine for each of the types of connection used.

## Experiments to Chapter X

It is desirable to study the performance of wattmeters in polyphase work on unbalanced circuits before proceeding to the balanced circuit which is more usually met in practice.

**1. Three-phase Vectors and Power.**—Arrange an unbalanced, star-connected lamp bank load from a supply of unbalanced three-phase voltage. Place ammeters and the current coils of wattmeters in each line wire. Connect the "low" side of each wattmeter potential coil to the adjacent line and the "high" side to a flexible lead common to all the wattmeters. Use a voltmeter with flexible leads, which may be tapped in at any point.

With a fixed load read all of the meters with the flexible leads tapped to all of the branch points. These branch points should be lettered or numbered and the readings marked accordingly. There should be six voltmeter readings and four readings (one of which should be zero) for each wattmeter.

Plot a topographical vector diagram using all of the possible checks in determining the positions of the vectors. It should be noted that each wattmeter reading may be used in the calculation of the cosine of the included angle between the voltage and current applied to its coils. This may be used to check the angles plotted from the voltmeter readings alone.

Assume a "neutral" point at any point whatsoever on the same sheet with the topographical plot. Draw lines from this point representing the voltage from this point to each of the line wires. Calculate the total power as being equal to the summation of the products of these voltages to "neutral," the line currents and the cosines of the respective included angles. Take care to observe the proper algebraic signs. Check this calcu-

lated value with the sum of the wattmeter readings when the flexible lead is tapped to any one branch point in the system.

For the same assumed "neutral" point as above calculate the total reactive power as being the summation of the products of the voltages to "neutral," the line currents and the sines of the respective included angles. This should be zero for the lamp bank load.

Show that two wattmeters may be used to measure the total power in any polyphase three-wire system, balanced or unbalanced.

**2. Two-wattmeter Method on Balanced Circuits.**—(*a*) Arrange two wattmeters to measure the total power in a balanced three-phase system, the power factor of which may be varied over a wide range.

Calculate the power factor as the total power divided by the voltamperes and by the "tangent method." Show why the latter method is the more accurate and preferable for most work.

Show why both methods fail in the calculation of the power factor of unbalanced circuits.

(*b*) Develop a method whereby the ratio of two wattmeter readings in a balanced two-phase three-wire system may be used to calculate the power factor. Check by actual measurement.

## Experiments to Chapter XI

**1. Ironclad Circuits.**—With an iron-cored inductance show the variation of impressed voltage, coil impedance, inductance and effective resistance with exciting current at constant frequency. Use a separately excited generator with no resistance in the leads for the supply in order to prevent a distortion of the voltage wave form. Plot curves with exciting current as a base and be sure to obtain the high point on the inductance curve.

Show the effect of iron loss on the shape of the impedance triangle in one representative case. Compare the wattmeter readings with the copper loss and compute the iron loss.

**2. Mutual Inductance.**—Place two air-cored coils in such a position that their mutual inductance is as great as possible. One coil will be used as the primary and the other as the secondary of an air-cored transformer. Number the terminals, paying particular attention to the beginnings of the windings when considered as helices along the same axis.

Place voltmeters, ammeters and wattmeters for finding the simultaneous phase positions and values for the vectors of both coils. An ammeter and the current coil of a wattmeter should be placed in series with each coil. One terminal of the first coil should be connected to one terminal of the second outside of the meters. Voltmeter readings may be taken across each coil and across the combination of two coils. Checks upon the angles indicated by the above may be had by calculating their cosines from wattmeter readings taken when the potential coil of one wattmeter is tapped successively to two of the voltages in question.

Determine the complete vector diagram for no load upon the coil selected as the secondary.

Keeping the supply voltage constant, repeat for several values of non-inductive load on the secondary up to short circuit.

Determine the loci of the vectors in a topographical plot for the various loads.

Calculate the resistances and self-inductances of the coils and their mutual inductance. Calculate the equivalent resistance and self-inductance when the whole circuit, including the load, is considered as a unit for one adjustment.

**3. Steinmetz's Coefficient and Exponent.**—Iron loss may be expressed by the equation

$$W_i = V(afB^x + bB^2f^2)$$

where $V$ is the volume of iron in cubic centimeters, $B$ is the maximum flux density in lines per square centimeter, $f$ is the frequency, and $a$, $b$, and $x$ are empirical figures.

For the purpose of this experiment, the eddy-current loss, $P_e = bVB^2f^2$, will be separated from the hysteresis loss, $_hP = aVB^xf$, and the constants $a$ and $x$ of the latter will be determined.

A suitable apparatus for this experiment is a ring made up of annular punchings of large diameter from the sample to be tested and uniformly wound with two exactly similar distributed coils. One of the coils is to be used as an exciting winding and should be connected in series with an ammeter and the current coil of a wattmeter. The other coil is to indicate the voltage induced by the flux and is connected to a voltmeter. The wattmeter potential coil is connected across the primary.

The induced voltage in either coil is

$$E = 4.44NABf10^{-8}$$

Thus

$$B = KE/f$$

Thus by keeping $E/f$ constant, $B$ may be kept constant.

Take readings of voltage, current, frequency and watts over as wide a range of frequency and voltage as possible. Several runs should be made either with constant flux density or with constant frequency. It is usually easier with the apparatus available to make several runs (6 or 8) at different constant frequencies.

Determine the copper loss in the primary and subtract it from the wattmeter readings and obtain the iron loss.

A table made up with the values in the order given will be useful: $f$, $E$, $I$, $W_0$, $B$, $RI^2$, $W_i$, and $W_i/f$.

Plot curves with $W_i/f$ as ordinates and $B$ as abscissæ for each value of $f$. Interpolate in the above and find the values of $W_i/f$ for six or eight values of $B$ and plot against $f$. These curves should be straight lines and will give a means of separating the eddy current and hysteresis losses.

Since at any given flux density (as for each one of the curves above),

$$W_i/f = aVB^x + bVB^2f$$

in which $W_i$ and $f$ are the only variables, $aVB^x$ is the intercept of these curves with the $Y$ axis. Values of $aVB^x$ might now be plotted against $B$, but the curves would give no means of determining $a$ and $x$. It is best to plot $log\ (aVB^x)$ against $log\ B$. This curve should be a straight line of slope $x$ and intercept $log\ (aV)$ on the $Y$ axis. These values give the desired solution.

Similar methods may be used for finding the constants in the expression for eddy currents.

## Experiments to Chapter XII

**1. Preliminary Study.**—(*a*) *Polarity.*—Check the marking of leads of a standard single-phase distributing transformer according to the Standardization Rules of the A.I.E.E. (600 to 607). The transformer should be excited by applying low voltage to one of the high-tension coils.

(*b*) *Ratio.*—Determine the ratio of the transformer, and, if it is provided with more than one primary or secondary winding, all of the possible ratios using standard connections. The ratio of the transformer may be taken as the voltage ratio at rated voltage and no load. (Standardization Rules 200 to 214.)

**2. Losses and Efficiency.**—(*a*) *Core Loss.*—Excite the low-tension winding of the transformer from a separately excited generator without resistance in the load circuit at normal frequency. Keep the secondary circuit open and take readings of meters for a wide range of voltage up to 110 per cent. of normal.

Plot volts and watts core loss against amperes as abscissæ. Determine if any correction should be made for the primary copper loss.

(*b*) *Copper Loss.*—Short circuit the low-tension winding through an ammeter and supply low voltage at normal frequency to the high-tension winding. If more than one high or low-tension coil is provided, connect them in series. Take readings for a range of current up to 200 per cent. of normal rated load current.

Calculate the equivalent resistance and reactance (referred to the high-tension winding) for each of the readings. Check this resistance with the value calculated from the ohmic resistances of the windings and the ratio of transformation.

Plot the total copper loss and the equivalent resistance and reactance drop in volts against secondary load current.

(*c*) *Efficiency.*—Calculate and plot the efficiency for all loads up to 150 per cent. of rated load at unity power factor with normal secondary voltage.

Calculate and plot the efficiency for a series of power factors (current leading and lagging) at full kv.-a. load and with normal secondary voltage.

**3. Regulation.**—(*a*) Using the calculations for the equivalent resistance and reactance drop in volts from the previous experiment, calculate and plot the primary voltage necessary to give normal secondary voltage for all loads up to 150 per cent. of rated load at unity power factor. Neglect the increase of the primary current due to the exciting current.

Repeat for rated load at various power factors, current lagging and leading. Calculate and plot the regulation of the transformer for all power factors. (Standardization Rule 565.)

(*b*) Check the regulation and efficiency as previously calculated by input-output readings at approximately full load and unity power factor. Note that a slight error in the meter readings may produce a big discrepancy in the results.

**4. Transformer Connections.**—(*a*) Connect the high- and low-tension windings of a standard distributing transformer in series.

Determine what ratios may be obtained when using it as an auto-transformer. Measure the line, coil and load currents as well as the voltages. Make out a table giving these ratios and the corresponding kv.-a. ratings of the apparatus when used at a voltage and current not exceeding normal, for any winding.

(*b*) Show how to connect the low-tension coils of two standard distributing transformers as a V-connected auto-transformer for reducing a three-phase supply to one-half voltage. Give circuit and vector diagrams.

(*c*) Connect three transformers with their high-tension windings in star and low-tension in delta, and vice versa. Give vector diagrams and check the expected voltage ratios by actual readings. Show what happens if one phase of the primary or secondary of the delta-star connected transformers is reversed.

Connect the transformers in delta-delta. How may the connections be checked in a delta connected secondary before the third winding is connected?

**5. Instrument Transformers.**—(*a*) Connect the primaries of two current transformers in two leads of a three-phase load circuit. Place an ammeter in series with each secondary terminal. Connect a third ammeter outside of the other two meters and in parallel with the two secondary circuits. Show that the meters give readings proportional to the current in the three mains irrespective of conditions of balance.

(*b*) Connect two potential transformers in "V" on an unbalanced three-phase supply. Show that three voltmeters may be used on the secondaries to give readings proportional to the primary line voltages.

(*c*) Connect the current coil of a wattmeter in one lead of a balanced three-phase load. Connect two potential transformers between this lead and the other two mains. Show how to connect the potential coil of the wattmeter in series with the secondaries of both potential transformers so that the wattmeter will read either proportional to the total power or to the reactive volt-amperes.

## Experiments to Chapter XIII

**1. Operation by Loading.**—Load an induction motor (preferably a squirrel cage machine) by means of a prony brake when supplied from a constant-frequency and constant-voltage source. Take a series of readings of voltage, current, watts,

frequency, speed, scale reading and slip up to 150 per cent. of rated load. Correct the scale reading for the dead weight of the brake arm which is to be taken as the mean of readings with positive and negative friction at rest. Calculate torque, watts output, power factor and efficiency. Plot performance curves against watts output as abscissæ.

When the motor is near synchronism the slip may be very satisfactorily measured by means of a disc attached to the shaft and painted with alternate black and white sectors, as many of one color as there are poles in the machine. The apparent rotation of the disc when viewed by the light of an arc lamp supplied from the same source as the motor is the slip in revolutions per minute.

If the speed is far below synchronism the slip may be most readily calculated from the actual speed of the machine as read with a tachometer.

**2. Method by Losses.**—Any induction machine may be used for the following experiment. Two standard methods for the calculation of the performance curves of induction motors require the measurement of the friction and core losses and the equivalent resistance and reactance of the stator and rotor. The experiment outlined below gives the determination of these quantities; the calculation of the characteristics is left for Experiments 3 and 4.

(*a*) At rated frequency take no load (running light) readings for as wide a range of voltage as possible. If the machine is a single-phase motor, read volts, amperes and watts, but if a polyphase motor, volts and watts (two meters). Line ammeter readings should be taken with three-phase machines merely to determine if the load is balanced and as a rough check on the calculated current. Calculate the power factor from the ratio of the wattmeter readings, and the equivalent single-phase current from the total watts, power factor and line voltage. Measure the equivalent single-phase resistance of the stator. Then treat as a single-phase machine.

Deduct the stator copper loss from the input giving the iron and friction losses. Plot equivalent single-phase exciting current, power factor, and core loss against volts. Interpolate on the curve for values at normal voltage.

(*b*) With blocked rotor, take readings at rated frequency with low voltage on the stator. This is readily obtained from a

separately driven under-excited alternator. Reduce readings to equivalent single-phase quantities as in (*a*).

Calculate the equivalent single-phase resistance of the whole machine from the total power and the equivalent single-phase current. Subtract the equivalent single-phase stator resistance from this figure to give the rotor resistance. Calculate the power factor and equivalent rotor reactance.

(*c*) With the machine loaded take voltmeter and wattmeter readings at normal frequency and voltage up to 150 per cent. of rated load. If a single-phase motor is used ammeter readings must also be taken.

**3. Circle Diagram.**—Using the data from Experiment 2, calculate and plot the performance curves of the machine up to the pull-out point by the circle diagram method. Check with values obtained as in Experiment 1.

**4. McAllister's Method.**—Using necessary data from Experiment 2, calculate and plot the performance curves of the motor by the method outlined on page 200. Check by other methods.

**5. Starting Torque of Wound-Rotor Motor.**—(*a*) Find the primary current and power factor, the torque and torque efficiency at standstill for different balanced star-connected rotor resistances and plot against equivalent single-phase rotor circuit resistance.

Use as large currents as are permissible. If it is not possible to use full rated voltage on account of excessively large line current, use lower voltage and reduce readings to equivalent values at full voltage before plotting. Be sure that the rotor is in exactly the same position for each reading. Take all precautions to reduce friction to a minimum and to correct for it in the brake readings.

(*b*) Find the variation in starting torque for a given voltage and rotor resistance with the position of the rotor. Take about ten or twelve readings equally spaced over one pole pitch.

**6. Split-phase Starting.**—(*a*) Connect one phase of a three-phase induction motor in parallel with a resistance and reactance in series, and supplied from a readily controlled supply of voltage at about one-half the rated voltage of the machine. Connect the third lead to the point between the resistance and reactance through a switch. Arrange a sufficient number of meters to obtain a complete diagram of all the currents and voltages.

Determine the best adjustment of resistance and reactance for starting. There are two conditions to be satisfied; an adjustment to give rotation with minimum line current and one to give shortest time to full speed.

It is suggested that with some definite value of resistance and reactance, the voltage be brought up from a low value until the rotor begins to turn. Record the critical line current. Make other trials with new values of resistance until rotation is obtained with the smallest line current. Leaving the resistance in this position, adjust the reactance for a still lower minimum.

Since the reactance of the rotor varies with the speed of rotation, it may be that this adjustment will not bring the machine up to full speed in a reasonably short time. If not, adjust the units for minimum time to full speed.

(*b*) With the most satisfactory adjustment from (*a*) take readings for vector diagrams with constant supply voltage (one-half to three-quarters normal):

1. With the motor at standstill and with the resistance and reactance across the line but not interconnected with the motor.
2. With the motor blocked and the third lead connected.
3. With the motor at full speed and with the third lead still connected.
4. With the motor alone at full speed.

Note that there are six possible current, and six possible voltage vectors for each connection, some of which may be zero, or duplicates of other vectors. Plot all four vector diagrams and interpret.

## Experiments to Chapter XIV

**1. Saturation and Short Circuit Curves.**—(*a*) Determine the no-load saturation curve for a three-phase machine with ascending and descending values of field current up to 150 per cent. of normal rated voltage. Take readings of all three line voltages and plot the mean values against field current. Keep the speed constant at its rated value.

(*b*) Short circuit the terminals of the machine through star-connected ammeters and take the short-circuit curve up to 200 per cent. of normal rated armature current. Reduce the mean readings to equivalent single-phase values and plot against field current on the same sheet with (*a*).

(*c*) Connect the generator to an unloaded synchronous motor.

Vary the field currents of both machines to hold normal full load armature current, reading terminal voltage and field current. This gives the zero-power-factor saturation curve (see A.I.E.E. Standardization Rule 585). Plot on the same sheet with (*a*).

(*d*) Determine the equivalent single-phase resistance of the armature. From (*a*) and (*b*) estimate the equivalent single-phase impedance drop and calculate the equivalent single-phase reactance drop. Plot *RI* and *XI* against equivalent single-phase load current.

(*e*) Estimate the zero-power-factor saturation curve by the method in A.I.E.E. Rule 586. Compare with that taken in (*c*).

**2. Regulation.**—(*a*) Using the data from Experiment 1, give a diagram for finding the no-load voltage for any load current at any power factor at rated terminal voltage.

(*b*) Plot the no-load voltage against power factor for rated terminal voltage at rated current.

(*c*) Plot the no-load voltage against load current for rated terminal voltage at unity, 0.5 lagging and 0.5 leading power factors.

(*d*) Check the unity power-factor curve in (*c*) by readings for the highest load possible.

(*e*) Give the regulation curve of the machine for all power factors.

**3. Efficiency.**—(*a*) Determine the losses of a synchronous generator as enumerated in A.I.E.E. Standardization Rule 441 and explained in Rules 420 to 458. Plot curves of core loss against internal generated voltage. Plot load and copper losses against equivalent single-phase load current. Give values for windage and friction losses.

On account of the small value of the load loss, the tests should be made only after the copper has reached a uniform temperature. Friction and windage should be measured both before and after the run. The resistances of the generator and driving motor should be measured hot immediately after the run. For the simplest calculation of losses in the driving motor, hold constant speed by adjusting the impressed voltage, keeping the motor field current and therefore the motor core loss constant. The same precautions must be observed in taking the generator core loss.

(*b*) From the experiment on regulation calculate the field current necessary to produce rated terminal voltage for various

loads at specified power factor. Plot field current, field losses and field rheostat losses against load current.

(*c*) Calculate the efficiency of the machine as a generator at specified power factor from 0 to 150 per cent. of rated load.

(*d*) Check by loading the machine (input-output method) to as high a load as possible.

**4. Heat Runs** (see A.I.E.E. Standardization Rules 300 to 392).—Before starting measure the cold resistances and the temperature of the windings. Thermometers may be held in place by small lumps of oil putty over the stem and bulb. They should be arranged to read the temperatures at various places on the following parts: armature coils, laminations, frame, ventilating ducts, field coils, collector rings, commutator, leading and trailing pole tips, bearings and room. Thermometers on accessible stationary parts are read every half-hour during the run, others after the run is completed.

Load the machine as in Rules 322 and 325. Keep a "log" of the test, recording all available readings every half-hour. Calculate the temperature of field coils by the resistance method (Rule 348) for each reading.

If the temperatures do not become constant during the run, estimate the final temperature rise by exterpolating on the logarithmic curve of temperature rise plotted against time.

At the end of the run immediately make measurements to calculate the temperature rise of all windings by the resistance method. Record final temperatures by thermometer, allowing sufficient time for each thermometer to reach its maximum reading after the machine is stopped. It is convenient to record the reading of each thermometer in chalk every few seconds until successive readings show a fall of temperature. This may require several minutes after shutdown owing to the stopping of ventilation.

Apply the proper "Hottest Spot" correction for each part, and determine whether or not the machine is properly rated.

**5. Synchronizing.**—(*a*) Synchronize one phase of a three-phase generator with a single-phase supply. It is convenient to use a direct current motor for driving. Synchronize with lamps arranged for dark and for bright indication, with and without potential transformers.

(*b*) Check the sequence of phases of the three-phase machine and supply, and synchronize as a polyphase generator with the

lamps arranged to indicate synchronism when dark. Repeat when the lamps are arranged to show the rotating effect.

(*c*) Synchronize using a synchroscope with and without potential transformers.

**6. Parallel Operation.**—Wire a three-phase generator and a balanced star-connected lamp bank load in parallel with the laboratory supply or with another three-phase machine. Use a sufficient number of meters to obtain the vector diagram of all the currents and voltages.

(*a*) Carry the entire load from the laboratory supply and synchronize the alternator. Draw vector diagram.

(*b*) Change the alternator field excitation and note the meter indications. Show the change by vector diagram.

(*c*) Regulate the driving torque of the generator until it carries its share of the load at unity power factor and note the meter indications. Note that unity power factor is most readily indicated when both wattmeters read alike. Plot the vectors.

(*d*) Note the shift in the vectors when the field is changed while the machine is loaded.

## Experiments to Chapter XV

**1. Characteristics and Circle Diagram.**—Determine and plot three "V" curves for as widely different constant loads as possible, plotting amperes armature and power factor as ordinates against field amperes as abscissæ.

If the supply fluctuates either in voltage or frequency, reliable readings, especially at no load, cannot be obtained. A separately excited laboratory generator connected only to the motor is the best source of power.

Figure the power factor by the tangent method as differences in the generated wave forms of motor and generator will give rise to wattless circulating currents of a higher frequency than the fundamental. These affect the r.m.s. value of the current but not the power.

Plot the loci of the current readings in a vector diagram. These should be portions of circle diagrams.

## Experiments to Chapter XVI

**1. Preliminary.**—The measurements in the first experiment on the synchronous converter need not be made with great

accuracy. The test is to make the student familiar with the features of converters.

(*a*) Start the converter as a direct current shunt motor in the usual way. Measure the direct and alternating terminal voltages and determine the voltage ratio. Does a change in the strength of the field current alter this?

(*b*) Apply a balanced load to the alternating current side keeping the direct voltage and field current constant. Determine the ratio of the alternating current to the increase in direct current from no load.

(*c*) Synchronize the machine with a supply of the rated voltage and frequency. Find the effect of a change in the field current when the machine is loaded on the direct current side.

(*d*) Start as an induction (hysteresis) motor at part voltage with the field switch open. Note the line current and the commutator voltage before and after connecting the field across the commutator brushes. Note the operation of the machine when this connection is reversed. It is convenient to use a direct-current voltmeter with the zero in the center of the scale across the commutator brushes.

Determine the effect of changing the resistance in the field circuit for both positions of the field switch. Note that with proper manipulation of the switch and rheostat, the direct voltage may be made to reverse polarity, or to build up as in a shunt generator.

**2. Characteristics.**—Use a separately controlled alternating supply for driving a converter at rated voltage and frequency. Determine the characteristics of the converter with the field rheostat adjusted so as to give unity power factor at full load. The field circuit should be supplied from the direct-current side of the machine. Plot supply amperes, power factor, efficiency and direct voltage against load amperes.

**3. Compounding.**—Run the converter from an alternating supply of rated voltage and frequency. Adjust the field rheostat so as to give unity power factor at the converter terminals at one-half load for the following cases:

(*a*) When using only the shunt field and with no reactance in the supply leads.

(*b*) With the series field helping the shunt.

(*c*) With both fields and with reactance in each supply lead to give from 10 per cent. to 15 per cent. reactance drop in the coils at one-half load.

Take readings and plot slip ring voltage, commutator voltage, supply current, converter and supply power factors and overall efficiency against load current.

**4. Three-wire Generator.**—(*a*) Arrange to run the converter from a normal alternating supply. If it is a single-phase machine connect two low-tension coils of a transformer in series across the supply leads. Bring out the transformer neutral as the neutral of a direct-current three-wire system. If it is a three-phase machine connect the low-tension coils of three single-phase transformers in distributed or interconnected star to the supply and bring out the neutral.

Connect meters in the supply and load circuits in the usual way and in one compensator lead place a direct-current ammeter, a universal alternating and direct-current ammeter and a current transformer and secondary ammeter. The field should be excited from the direct-current brushes.

Load one side of the direct-current three-wire system and determine the direct-current line voltages and voltages to neutral, alternating, direct and total compensator currents, supply current, power, power factor, etc. Plot curves against load current.

(*b*) Show how to use the machine as a "two to one" or a "one to two" direct-current "transformer." Take a few typical readings in each case.

## Experiments to Chapter XVII

A compensated series motor of the same general type as used for railway work is most desirable for these experiments. If special types are available the experiments should be modified to suit the apparatus.

**1. Preliminary.**—(*a*) Remove the power brushes and place a special pair of narrow brushes in two holders and connect them to a voltmeter. These may be metal strips but must be narrow enough to prevent the short circuiting of adjacent commutator segments.

Disconnect the compensating field and excite the main field from a constant low-voltage alternating supply.

Block the rotor and determine the value and phase position of the brush voltages with reference to the exciting field voltage and current for all possible brush holder positions. The phase

relations may be determined in a similar way to those in the experiment on Mutual Inductance (Chap. XI, Exp. 2).

Plot results against angular brush shift. Give a complete vector diagram for one of the above positions.

(*b*) Drive the armature by a belt at a suitable speed with the same excitation as above and repeat (*a*). Compare the space-phase and time-phase relations of the voltages induced by the speed and transformer fields.

**2. Series Motor Characteristics.**—Carefully set the power brushes in the running position. This may be done by exciting the main field at standstill as in Experiment 1 and finding the position for zero voltage between brushes. Connect the fields in series with the armature. The proper polarity of the compensating field is obtained when the drop across the armature and compensating field is a minimum when carrying alternating current.

(*a*) Compare the starting torque for alternating and direct currents and plot against amperes.

(*b*) Take the characteristics of the motor when loaded on a supply of normal voltage and frequency. Plot speed, current, torque and power factor against watts output.

(*c*) Determine the resistances of the fields and armature and take a sufficient number of voltage readings across the fields and armature to plot a complete vector diagram at full load.

**3. Other Single-phase Motors.**—Connect the machine used in the previous experiments as an inductively-compensated series-excited motor or as a repulsion motor. Place a sufficient number of meters to obtain the values and phase positions of all the currents and voltages.

Plot typical vector diagrams. Plot load characteristics.

## Experiments to Chapter XVIII

**1. Induction Generator.**—Wire a synchronous motor, a balanced lamp bank load and an induction machine in parallel. To start the set first drive the synchronous machine as a generator. Drive the induction generator and connect it in parallel with the synchronous machine, being careful to obtain the proper direction of rotation and speed. Throw the belt off of the pulley of the synchronous machine. Adjust the field of the synchronous motor to obtain the desired line voltage.

The induction machine will supply the losses in the synchronous machine which will in turn supply the magnetizing component of the exciting current for the induction generator. In ordinary operation, induction generators are connected in parallel with large synchronous generators. For this test the unloaded synchronous motor is selected so that the current in it will be as small as possible, making the readings much easier to interpret.

The meters should be so placed that the phase relations of all the currents and voltages may be obtained when the set is loaded.

(*a*) Load the set with the field of the synchronous machine and the speed (of the induction machine) constant and adjusted to give normal voltage and frequency at no load.

(*b*) Repeat, keeping the frequency and field excitation constant.

(*c*) Repeat, keeping the frequency and terminal voltage constant.

With watts useful output as abscissæ, plot frequency, speed, voltage, field current, induction generator current and power factor, and synchronous motor current and power factor in each case.

**2. Phase Converter.**—Start a three-phase induction motor on a polyphase supply and then disconnect one line lead. Connect balanced load resistances to the motor terminals.

Take readings for the magnitude and phase relations of all the currents and voltages. It is important to take readings on all three phases as the values will not be the same.

Plot curves of line current and power factor, load currents and voltages, and motor currents against total watts output.

Show a complete vector diagram for the above at full load.

## Experiments to Chapter XIX

A small paper condenser is a convenient test sample for the investigation of dielectrics. One made of tinfoil and a few square centimeters of 0.002-in. varnished paper giving a condensance of from 0.1 to 1.0 m.f. will serve the purpose. It should be tightly compressed.

**1. Leakage Resistance.**—(*a*) Use a high-resistance direct current voltmeter in series with the condenser on a direct current

supply. Vary the impressed voltage across the voltmeter and condenser in series from a low value to 500 volts, reading the voltmeter and supply voltage. Measure the resistance of the voltmeter. Calculate and plot ohms insulation resistance against voltage across the dielectric.

(*b*) Arrange the condenser with a voltmeter in series on a constant supply of about 150 volts. Place the condenser in a small oven or thermostat. Slowly raise the temperature to 150°C., taking frequent measurements of insulation resistance. Allow the condenser and oven to slowly cool and repeat the measurements. Plot curves of leakage resistance against temperature.

**2. Effective Capacitance and Dielectric Losses.**—The impedance bridge is the most satisfactory apparatus for the investigation of dielectric losses due to alternating currents. A vacuum tube may be used to supply sufficient energy for the operation of the bridge at any desired frequency and a telephone receiver as detector. The apparatus should be grounded and the bridge as a whole shielded electrostatically to prevent unbalance by the capacitance of the body. A large air condenser may be used as a standard if carefully constructed to eliminate leakage.

(*a*) Investigate the effective resistance and condensance (considered as a series circuit) of a small mica condenser. Plot curves against frequency up to the limit of audibility.

(*b*) Repeat with a paper condenser having rather low insulation resistance or with an artificial leak in parallel. Show that at high frequencies the relative effect of the leakage is reduced and that the principal losses are due to dielectric hysteresis. Plot effective resistance and condensance against frequency.

## Experiment to Chapter XX

A 100-kv. transformer is suitable for the following experiment. It should be provided with ample protection to prevent contact with live terminals. The high-tension winding should be protected against high-frequency spark discharges by water tube resistances in series with the test circuit. It is convenient to provide a tertiary winding for a voltmeter which will give a reading proportional to the no-load high-tension voltage. This may be calibrated by comparison with a standard sphere gap.

**1. Corona and Spark Over.**—(*a*) Determine the critical visual corona and spark over voltages for parallel wires in air. Plot kilovolts against spacing in centimeters.

Read the room temperature and barometric pressure. Calculate the critical voltages from the empirical equations and compare with the measured values.

The critical corona voltage may be detected by careful visual observation in a dark room or by the sound emitted.

(*b*) Use a stroboscope driven by a small synchronous motor on the same supply as the transformer to observe the difference in corona during different parts of the cycle. This may be conveniently driven in front of the lens of a surveyor's telescope for observation from a distance.

## Experiments to Chapter XXII

**1. Phase Control.**—Stimulate the effect of a short single-phase transmission line by lumped resistance and reactance. Drive an idle synchronous motor in parallel with a lamp bank load at the end of this line. Control the field of the motor so that:

(*a*) $E_g = 0.9E_r$

(*b*) $E_g = E_r$

(*c*) $E_g = 1.1E_r$

Hold $E_g$ constant.

Plot generator current and power factor, synchronous motor current and overall efficiency against load current or watts output in each case.

## Experiments to Chapter XXIII

**1. Oscillograms.**—Take several oscillograms of current in circuits containing inductance and condensance adjusted for the resonance of one of the higher harmonics of a commercial voltage wave.

A variometer of a maximum inductance of 200 m.h. in series with an adjustable condenser of from 1 to 4 m.f. is convenient. Approximate resonance of the harmonics may be determined by the sound in a telephone receiver in series with a 0.001 m.f. condenser placed in parallel with the adjustable condenser.[1]

[1] Method outlined in *Bull.* No. 8, University of Washington, Engineering Experiment Station.

Take one oscillogram of open circuit voltage. When in resonance read the effective value of the current and take a film. Add a small measured non-inductive resistance in series (2 or 3 ohms) and repeat. Measure the condensance used to produce resonance.

Calibrate the vibrator of the oscillograph in amperes per millimeter deflection. Note that for frequencies above a few hundred cycles errors are introduced by mechanical resonance of the vibrator strip. Determine whether such correction should be applied to the oscillograms taken.

**2. Wave Analysis.**—Analyze the films taken in Experiment **1.** Calculate the maximum and effective values of each of the harmonic components of the complex wave. Check the effective value of this wave as determined by the ammeter readings with the square root of the sum of the squares of the components.

From the frequency of the harmonic in resonance and the condensance, calculate the inductance of the complete circuit. From the reduction in the harmonic current upon the addition of the known resistance, calculate the effective resistance of the circuit to the harmonic.

Calculate the impedances of the circuit to the fundamental and to the harmonic in resonance. Assume that the effect of resistance on the impedance to the fundamental is negligible. Calculate the generated voltages corresponding to the measured component currents and calculated impedances. Check these values with those scaled from the film of open-circuit voltage.

## Experiments to Chapter XXVII

**1. Line Constants.**—Measure the resistance, inductance and condensance of each unit of an artificial transmission line and adjust to correspond to the constants of some actual three-phase line (one wire to neutral).

Measure the currents and voltages and their phase relations at both ends of the line at open circuit and short circuit. Be careful to obtain the relations of the vectors at one end with respect to those at the other.

Calculate the constants of the line for use in Equations 940 and 941 from these readings. Note that on open circuit $\dot{I}_r$ is zero and on short circuit $\dot{E}_r$ is zero. Therefore from the open-circuit readings

$$a'_1 - ja'_2 = \frac{\dot{E}_g}{\dot{E}_r}$$

and (1)

$$c'_1 - jc'_2 = \frac{\dot{I}_g}{\dot{E}_r}$$

and from the short-circuit readings

$$a'_1 - ja'_2 = \frac{\dot{I}_g}{\dot{I}_r}$$

and (2)

$$d'_1 - jd'_2 = \frac{\dot{E}_g}{\dot{I}_r}$$

Compare the measured values of these constants with those calculated from the constants of the individual units.

**2. Voltage Distribution and Charging Current.**—Impress a sine wave of voltage of constant amplitude and frequency upon the open line used in the previous experiment. Place meters at the generator end of the line and arrange to determine the values and phase relations of the current and voltage at several equidistant points along the line. Check the phase relations with the generator or voltage current as well as between the vectors at the different points.

Plot charging current and voltage against distance from the receiving end of the line in per cent. of the total length of the line. Plot a vector diagram showing the loci of the current and voltage.

**3. Loading and Power Factor.**—Place meters at each end of the line.

(*a*) Load the line at unity power factor with constant generator voltage and frequency. Plot generator current and power factor and load voltage against load current.

(*b*) Load the line, adjusting the load power factor (lagging or leading) for each load so that the receiver voltage is the same as the generator voltage. A small synchronous motor in parallel with a lamp-bank makes a convenient load.

Plot generator current and power factor and load current, load power factor and wattless current or reactive power against watts load. Plot representative vector diagrams.

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